

equilibrium orientations in its cubic cage of bromide ions. In the simplified model used for calculation, each deuterium was treated as a spherical cloud whose center was distributed with uniform probability around a circle centered on the cube body diagonal, the root-mean-square radius of the cloud and the half-angle subtended by the circle at the nitrogen atom being treated as parameters whose final values were 0.21A and  $10^\circ$ , respectively. Models involving ordered structures of symmetry  $T_d^1$  with or without anisotropic temperature motion and models with freely rotating ammonium ions failed to give agreement with the data. Also eliminated were models in which the ammonium ions rotate about their twofold axes, space group  $O_h^1$ , and about their threefold axes with and without randomness of orientation, space group  $T_d^1$  and  $O_h^1$  (simulated), respectively.

Thus, in room-temperature  $\text{ND}_4\text{Br}$  there exists orientational disorder of the ammonium ions, and the crystal belongs to symmetry class  $O_h$ , simulating the space-group symmetry  $O_h^1$ . The reported chloride structure, in contrast, is ordered, with symmetry  $T_d^1$ , although involving temperature motion similar to that found in  $\text{ND}_4\text{Br}$ . This difference between the room temperature phases of  $\text{ND}_4\text{Br}$  and  $\text{ND}_4\text{Cl}$  is surprising and calls for further study to confirm its reality. A disagreement exists on the value of the coherent scattering cross section of deuterium (5.2 barns<sup>6</sup> in this work versus 5.8 used in reference 4). We find that use of the latter value does not change our conclusions with regard to the nature of the structure; however definitely better agreement is achieved with the value 5.2.

Our study yields the values  $0.99 \pm 0.02\text{A}$  for the length of the N-D link, equal within experimental uncertainty in both phases. Further work on the atomic arrangements in the two remaining phases is planned. A complete description will be published elsewhere.

\* This work was performed for the Atomic Energy Commission.

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### The Spins and Parities of the 3.7-3.9-Mev Doublet in $\text{C}^{13}$

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(Received August 1, 1951)

THE energy of the nuclear level of  $\text{C}^{13}$ , following after the 3.11-Mev state, has been variously given in the literature as 3.7 and 3.9 Mev. Thus Heydenburg *et al.*<sup>1</sup> in a study of the protons from  $\text{C}^{12}(d, p)\text{C}^{13}$  give the energy value of the state as 3.91 Mev, while recently Malm and Buechner,<sup>2</sup> in a careful measurement of the alpha-particles from  $\text{N}^{15}(d, \alpha)\text{C}^{13}$ , found a value of 3.68 Mev. Actually both of these values are nearly correct. In a detailed study of the protons from the  $\text{C}^{12}(d, p)\text{C}^{13}$  reaction, using an 8-Mev deuteron beam and employing the photographic emulsion technique for recording the protons (the experimental arrangement is described by Rotblat *et al.*)<sup>3</sup> two groups of protons, of slightly different ranges, were found to be present in this energy region, indicating the existence of two closely lying levels in  $\text{C}^{13}$ . The protons of longer range have a much lower intensity than those of the shorter range; at some angles the intensity of the former is only 2 percent of that of the latter. For this reason the longer range group tends to be lost in the "tail" of the more intense group. Nevertheless, after measuring a large number of tracks at 20 angles of emission and analyzing the histograms of the proton groups, the existence of both levels has been established beyond doubt. The  $Q$ -values for the two states were found to be

-0.967 and -1.168 Mev; assuming a  $Q$ -value for the ground state of 2.716 Mev<sup>4</sup> we obtain for the energy values of the two states 3.683 and 3.884 Mev. It is possible that these correspond to the states which were observed by Creagan<sup>5</sup> from the  $\text{B}^{10}(\alpha, p)\text{C}^{13}$  reaction and for which he gave values 3.76 and 4.00 Mev. It is also interesting to note that, unlike the  $\text{C}^{12}(d, p)$  process in which the 3.7-Mev level is formed relatively rarely, in the  $\text{N}^{15}(d, \alpha)$  reaction this level appears to be formed predominantly.

Apart from establishing the existence of the doublet, the angular distributions of the two groups of protons have also been investigated. According to Butler's<sup>6</sup> stripping process the spins and parities of the corresponding nuclear states can be determined

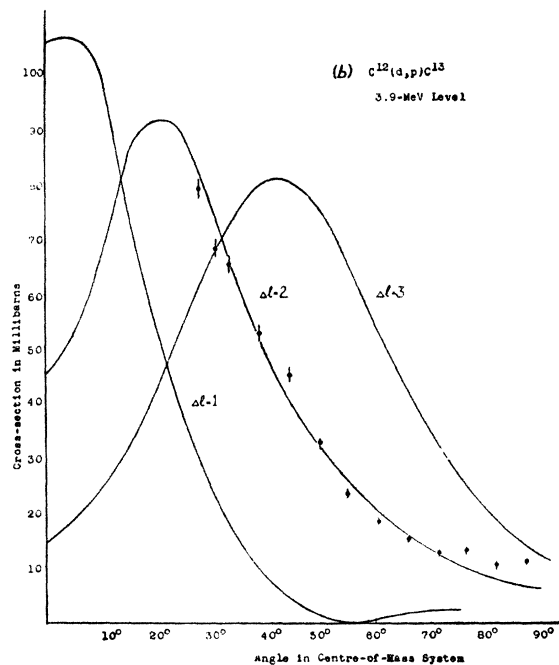
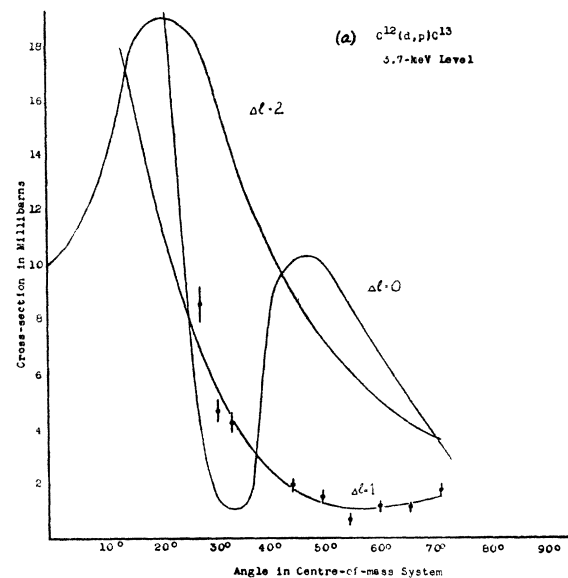


FIG. 1. Angular distributions of protons from  $\text{C}^{12}(d, p)\text{C}^{13}$  in the center-of-mass system. The full curves give the theoretical distributions for various values of the angular momentum transfer. The experimental points are given with their probable errors. Figure 1(a) is for the formation of  $\text{C}^{13}$  in the 3.7-Mev state and Fig. 1(b) for the 3.9-Mev state.

from the angular distributions. The analysis of the histograms into the two components gives the number of protons in each group, and from this the differential cross sections can be calculated.<sup>7</sup> Unfortunately the distribution could not be studied in the most interesting region, at small angles, owing to the overlap of a group of protons from the elastic scattering of deuterons on hydrogen (the target gas was acetylene), but the accuracy with which the cross sections were determined at the larger angles was sufficient to establish the spins and parities of the two states. The observed angular distributions up to angles of about  $90^\circ$  are indicated in Figs. 1(a) and (b) for the 3.7- and 3.9-Mev levels, respectively. The full curves are the distributions calculated on the basis of Butler's theory for various values of the angular momentum transfer. The experimental points are shown together with the statistical errors; these do not include possible errors due to the separation of the groups into two components. It is seen that very good agreement is obtained with  $\Delta l=1$  for the 3.7-Mev state and  $\Delta l=2$  for the 3.9-Mev state. Since  $C^{12}$  has zero spin and even parity in the ground state, it follows that the 3.7-Mev state of  $C^{13}$  has a spin of  $1/2$  or  $3/2$  and odd parity, while the 3.9-Mev state has spin  $3/2$  or  $5/2$  and even parity.

Goldhaber and Williamson<sup>8</sup> have recently investigated the  $C^{12}(p, p)$  reaction and found an indication of resonances at 1.68 and 1.73 Mev, pointing to the existence of two levels of  $N^{13}$  at an energy around 3.5 Mev; their analysis shows these to be  $P_{3/2}$  and  $D_{5/2}$  states. This fits in very well with our values found for the 3.7- and 3.9-Mev states of  $C^{13}$ . The agreement of the spins and parities of the ground and first excited states of  $C^{13}$  and  $N^{13}$  has already been established.<sup>9</sup> The present findings of doublets in  $C^{13}$  and  $N^{13}$ , together with the agreement of their spins and parities, is an excellent confirmation of the correspondence of energy states in mirror nuclei.

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### Isotope Shift in the Ce II Spectrum and the Magic Number 82\*

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(Received August 1, 1951)

THE isotope shift in the spectrum of Ce II was studied,<sup>1</sup> using an enriched sample of cerium ( $Ce^{138}$  4.4 percent,  $Ce^{140}$  92.0 percent,  $Ce^{142}$  3.5 percent), a hollow-cathode discharge tube,<sup>2</sup> and a Fabry-Perot etalon.

$\lambda 4628$  ( $4f^2 6s^2 H_{9/2} - 4f^2 6p^2 I_{9/2}^o$ ) was studied first, since it is not disturbed by other cerium lines. The  $Ce^{142}$ -component was well resolved from the strong  $Ce^{140}$ -component. However, the  $Ce^{138}$ -

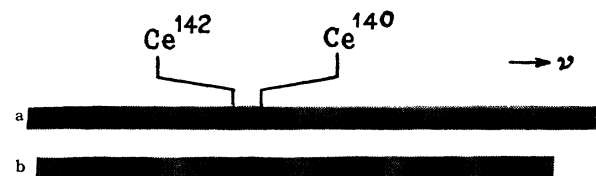


FIG. 1. Fabry-Perot patterns of Ce II  $\lambda 4628$ . (a) Natural cerium ( $Ce^{140}$  92.0 percent,  $Ce^{142}$  3.5 percent); 20-mm etalon. (b) Enriched sample ( $Ce^{138}$  4.4 percent,  $Ce^{140}$  92.0 percent,  $Ce^{142}$  3.5 percent); 30-mm etalon.

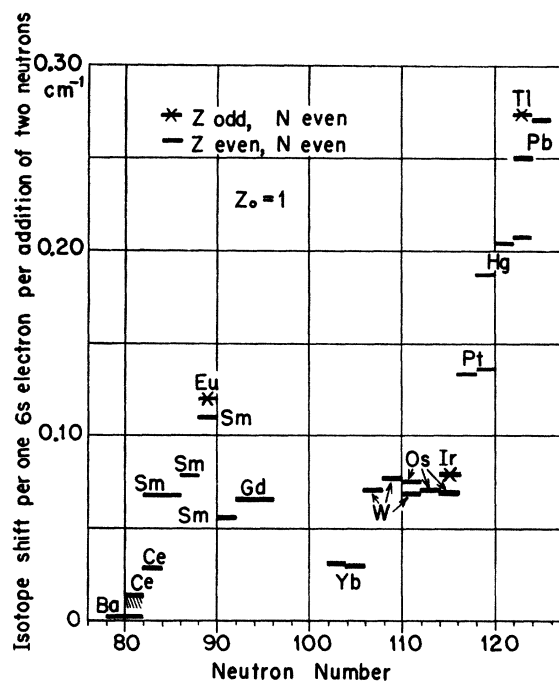


FIG. 2. Plot of the isotope shift per one 6s electron in the neutral atom per addition of two neutrons as a function of the neutron number.

component was not resolved, although  $Ce^{138}$  is somewhat more abundant than  $Ce^{142}$ . The distance 140-142 was found to be  $0.055 \text{ cm}^{-1}$ , which agrees with the measurement of Brix and Frank<sup>4</sup> within experimental error. The upper limit of the distance 138-140 was found to be about  $0.029 \text{ cm}^{-1}$ . Figure 1 illustrates some of the spectrograms of  $\lambda 4628$ . Here  $\lambda 4528$  ( $4f^2 6s^2 H_{13/2} - 4f^2 6p^2 I_{13/2}^o$ ) was also found to have the same structure as  $\lambda 4628$ .

This result shows that the upper limit of the distance 138-140 is about half of the distance 140-142. Since the neutron number ( $N$ ) of the isotope 140 is 82, the above-mentioned anomaly can be very probably ascribed to the stable nuclear structure connected with the magic number 82.<sup>5</sup>

The isotope shift in the spectra of heavy elements was treated theoretically by Rosenthal and Breit<sup>6</sup> and by Crawford and Shawlow.<sup>7</sup> It is characteristic of similar theories that the isotope shift is proportional to  $\psi^2(0)$ , the square of the nonrelativistic wave function at the position of the nucleus. It is expressed by

$$\psi^2(0) = Z_i Z_0^2 (dn^*/dn) / [\pi a_H^3 n^{*3}],$$

where the symbols have their usual meaning.

Using the same considerations as those of Crawford and Shawlow<sup>7</sup> and the above-mentioned formula, we can calculate from experimental data, for most heavy elements, the isotope shift per one 6s electron in the neutral atom ( $Z_0=1$ ), per addition of two neutrons, the accuracy being 10-15 percent. Figure 2 represents the above-mentioned quantity<sup>8</sup> as a function of the neutron number. The isotope  $Sm^{146}$  does not exist, so the value for the isotope  $Sm^{144} - Sm^{148}$  is divided by 2 and plotted. Since no gross multiplet analysis of Nd I is available, no value for the Nd isotopes is plotted. However, a rough estimate, obtained by using the data of Klinkenberg,<sup>9</sup> shows that the plots for Nd fall fairly close to those of Sm, and the irregularity for the pair N88-90 is especially conspicuous for Nd also.

Figure 2 shows that there is a distinct rise in the diagram when we pass from  $N=80$  to  $N=82$ , and the above-mentioned anomaly in the isotope shift for  $N=88-90$  is especially large, an idea which was expressed by Klinkenberg. Investigation of the physical meaning of the neutron number 88 or 90 will be important.