depend parametrically on the intermolecular potential. The first term, not containing $\hbar$, is the well-known classical expression for each of the transport properties. Obviously, this series converges only in the temperature region where the deviations from classical theory are relatively small.

If the intermolecular potential function may be written as $\varphi(r)=\epsilon f(\sigma/r)$, in which ϵ and σ are units of energy and length, respectively, then the quantities σ and ϵ characterizing the potential field can be used to define "molecular units" for each of the transport quantities. In classical theory[6] the thus "reduced" transport coefficients are universal functions of the reduced temperature, $T^*=\kappa T/\epsilon$. When quantum corrections must be taken into account, one may, in general, expect deviations from the classical result depending on the quantum mechanical parameter, $\Lambda^*=[h/\sigma(m\epsilon)^{\frac{1}{2}}]$, characteristic for each substance. The results of this paper are in agreement with this "quantum mechanical principle of corresponding states":[7] the series development for each of the transport properties is a power series in Λ^{*2}/T^*, the coefficients being functions of T^* only.

A detailed account of this work, accompanied by numerical results, will be published elsewhere. One of the authors (R.B.B.) wishes to acknowledge the financial assistance provided him by the Fulbright exchange program.

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Limits on the Energy of the Antiferromagnetic Ground State

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(Received July 16, 1951)

THE energy of the ground state of a simple antiferromagnetic lattice remains one of the unsolved problems of quantum mechanics, except in the simplest case of atoms of angular momentum quantum number $S=\frac{1}{2}$ on a linear chain.[1] For such a lattice, the hamiltonian is effectively

$$H=J\Sigma_i\Sigma_j\mathbf{S}_i\cdot\mathbf{S}_j. \tag{1}$$

The double sum runs over all pairs of neighbors i and j. Hereafter, it will be understood that atoms labeled i are on sublattice (1), while those labeled j are on sublattice (2). We shall assume the lattice is sufficiently simple, as, for example, is the simple or body-centered cubic, that it can be divided into two sublattices, the nearest neighbors of atoms on one sublattice laying all on the other and *vice versa*. The naive expectation is then that the spins on sublattice (1) lie all in one direction, while the spins on (2) all lie antiparallel to those on (1). The average energy for such a situation, represented by the wave function

$$\Psi=\Pi_i\psi_i(+S)\Pi_j\psi_j(-S) \tag{2}$$

(where here again i and j are atoms on sublattices (1) and (2), respectively), is easily obtained, and is

$$(\Psi, H\Psi)=-\tfrac{1}{2}NZJS^2, \tag{3}$$

where N is the number of atoms and Z the nearest neighbor number. Unfortunately, the state (2) is not an eigenstate, and so all that can be said is that, by the variational principle, the energy E_g of the ground state lies lower than Eq. (3):

$$E_g<-\tfrac{1}{2}NZJS^2. \tag{4}$$

A second, lower bound may be placed on the ground state energy in the following way: For the two-sublattice situation, it is clear that the hamiltonian (1) may be decomposed into a sum of terms:

$$H=\Sigma_i H_i, \tag{5}$$
$$H_i=J\Sigma_j\mathbf{S}_i\cdot\mathbf{S}_j. \tag{6}$$

The sum in Eq. (6) runs over all atoms j of sublattice (2) which are neighbors to the atom i of sublattice (1). Now the diagonal energies of the H_i are well known.[2] The lowest energy is

$$E_i=-JS(ZS+1), \tag{7}$$

which is obtained when the cluster of neighbor spins S_j is given its maximum angular momentum $\Sigma_j S_j=ZS$, and S_i is then set as nearly antiparallel to the surrounding cluster as possible. Unfortunately, there is no way to make up a full eigenfunction for the lattice out of such least-energy wave functions—for instance, we find that $(S_{jz})_{Av}\neq-(S_{iz})_{Av}$, which shows that such a wave function would not be self-consistent. On the other hand, one can easily see that the least eigenvalue of the total hamiltonian must be greater than the sum of the least eigenvalues of its parts:

$$E_g>\Sigma_i\{-JS(ZS+1)\},$$
$$E_g>-\tfrac{1}{2}NZJS^2(1+1/ZS). \tag{8}$$

This statement follows from the fact that the ground-state energy, or least eigenvalue E_g, must be the sum of diagonal elements of H_i by Eq. (5), but the variational theorem says that all possible diagonal elements of any matrix are greater than or equal to the least element. Physically, this means that the interaction with the rest of the lattice necessarily "spoils" the wave functions for the clusters which lead to the energy (7), and increases the energies of the clusters.

Thus, the bounds on the energy of the lowest state are

$$-\tfrac{1}{2}NJZS^2>E_g>-\tfrac{1}{2}NJZS^2(1+1/ZS). \tag{9}$$

In most practical cases of interest, these are very strong bounds: MnF_2, for instance, has $Z=8$, $S=5/2$, so that the possible fractional variation for E_g is 1/20. This in turn is a good indication that the wave function of the ground state is usually not greatly different from the "naive" expectation (2).

In Hulthén's case,[1] $1+1/ZS$ is 2. He found the corresponding factor for his rigorous ground state to be 1.773:

$$E_g(S=\tfrac{1}{2}, Z=2)=-1.773(\tfrac{1}{2}NZ)JS^2, \tag{10}$$

while a somewhat better upper bound than Eq. (4), 1.5, may be obtained with a simple "bond eigenfunction." We see that the limits are correct, if somewhat wide, in this, the worst possible case.

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The New State in Li^7 at Low Bombarding Energy*

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(Received July 20, 1951)

GOVE and Harvey[1] have revealed the existence of a state in Li^7 at 4.77 ± 0.10 Mev from observations on the $Be^9(d, \alpha')Li^{7*}$ reaction and on the inelastic scattering of protons, deuterons, and alphas by Li^7, at bombarding energies of 7.9, 14, and 31 Mev. In a survey of the $Be^9(d, \alpha)Li^7$ reaction,[2] at bombarding energies of 0.68 and 1.03 Mev, we examined the alpha-spectrum at 90° using magnetic analysis.[3] Above $E_{ex}=2.5$ Mev in Li^7 a continuum of alpha-particles is encountered, and in the neighborhood of $E_{ex}=4.7$ Mev Li^{+++} recoils from the doublet ground-state transition interfered with our observations. With photographic detection, discrimination between the lithium particles and alphas is difficult because they have practically the same range in emulsion. The two lithium groups are separated by only 135