depend parametrically on the intermolecular potential. The first term, not containing $\hbar$, is the well-known classical expression for each of the transport properties. Obviously, this series converges only in the temperature region where the deviations from classical theory are relatively small.

If the intermolecular potential function may be written as $\varphi(r)=\epsilon f(\sigma/r)$, in which ϵ and σ are units of energy and length, respectively, then the quantities σ and ϵ characterizing the potential field can be used to define "molecular units" for each of the transport quantities. In classical theory[6] the thus "reduced" transport coefficients are universal functions of the reduced temperature, $T^*=\kappa T/\epsilon$. When quantum corrections must be taken into account, one may, in general, expect deviations from the classical result depending on the quantum mechanical parameter, $\Lambda^*=[h/\sigma(m\epsilon)^{\frac{1}{2}}]$, characteristic for each substance. The results of this paper are in agreement with this "quantum mechanical principle of corresponding states":[7] the series development for each of the transport properties is a power series in Λ^{*2}/T^*, the coefficients being functions of T^* only.

A detailed account of this work, accompanied by numerical results, will be published elsewhere. One of the authors (R.B.B.) wishes to acknowledge the financial assistance provided him by the Fulbright exchange program.

[1] S. Chapman and T. G. Cowling, *The Mathematical Theory of Non-Uniform Gases* (Cambridge University Press, London, 1939).
[2] J. de Boer, Physica **10**, 348 (1943).
[3] J. de Boer and E. G. D. Cohen, Physica (to be published).
[4] R. Miyako, Proc. Phys. Math. Soc. Japan **24**, 852 (1942).
[5] B. Kahn, dissertation, Utrecht (1938).
[6] J. de Boer and J. van Kranendonk, Physica **14**, 442 (1948); Hirschfelder, Bird, and Spotz, J. Chem. Phys. **16**, 968 (1948).
[7] J. de Boer, Physica **14**, 139 (1948).

Limits on the Energy of the Antiferromagnetic Ground State

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received July 16, 1951)

THE energy of the ground state of a simple antiferromagnetic lattice remains one of the unsolved problems of quantum mechanics, except in the simplest case of atoms of angular momentum quantum number $S=\frac{1}{2}$ on a linear chain.[1] For such a lattice, the hamiltonian is effectively

$$H=J\Sigma_i\Sigma_j\mathbf{S}_i\cdot\mathbf{S}_j. \tag{1}$$

The double sum runs over all pairs of neighbors i and j. Hereafter, it will be understood that atoms labeled i are on sublattice (1), while those labeled j are on sublattice (2). We shall assume the lattice is sufficiently simple, as, for example, is the simple or body-centered cubic, that it can be divided into two sublattices, the nearest neighbors of atoms on one sublattice laying all on the other and *vice versa*. The naive expectation is then that the spins on sublattice (1) lie all in one direction, while the spins on (2) all lie antiparallel to those on (1). The average energy for such a situation, represented by the wave function

$$\Psi=\Pi_i\psi_i(+S)\Pi_j\psi_j(-S) \tag{2}$$

(where here again i and j are atoms on sublattices (1) and (2), respectively), is easily obtained, and is

$$(\Psi, H\Psi)=-\tfrac{1}{2}NZJS^2, \tag{3}$$

where N is the number of atoms and Z the nearest neighbor number. Unfortunately, the state (2) is not an eigenstate, and so all that can be said is that, by the variational principle, the energy E_g of the ground state lies lower than Eq. (3):

$$E_g<-\tfrac{1}{2}NZJS^2. \tag{4}$$

A second, lower bound may be placed on the ground state energy in the following way: For the two-sublattice situation, it is clear that the hamiltonian (1) may be decomposed into a sum of terms:

$$H=\Sigma_i H_i, \tag{5}$$
$$H_i=J\Sigma_j\mathbf{S}_i\cdot\mathbf{S}_j. \tag{6}$$

The sum in Eq. (6) runs over all atoms j of sublattice (2) which are neighbors to the atom i of sublattice (1). Now the diagonal energies of the H_i are well known.[2] The lowest energy is

$$E_i=-JS(ZS+1), \tag{7}$$

which is obtained when the cluster of neighbor spins S_j is given its maximum angular momentum $\Sigma_j S_j=ZS$, and S_i is then set as nearly antiparallel to the surrounding cluster as possible. Unfortunately, there is no way to make up a full eigenfunction for the lattice out of such least-energy wave functions—for instance, we find that $(S_{jz})_{Av}\neq-(S_{iz})_{Av}$, which shows that such a wave function would not be self-consistent. On the other hand, one can easily see that the least eigenvalue of the total hamiltonian must be greater than the sum of the least eigenvalues of its parts:

$$E_g>\Sigma_i\{-JS(ZS+1)\},$$
$$E_g>-\tfrac{1}{2}NZJS^2(1+1/ZS). \tag{8}$$

This statement follows from the fact that the ground-state energy, or least eigenvalue E_g, must be the sum of diagonal elements of H_i by Eq. (5), but the variational theorem says that all possible diagonal elements of any matrix are greater than or equal to the least element. Physically, this means that the interaction with the rest of the lattice necessarily "spoils" the wave functions for the clusters which lead to the energy (7), and increases the energies of the clusters.

Thus, the bounds on the energy of the lowest state are

$$-\tfrac{1}{2}NJZS^2>E_g>-\tfrac{1}{2}NJZS^2(1+1/ZS). \tag{9}$$

In most practical cases of interest, these are very strong bounds: MnF_2, for instance, has $Z=8$, $S=5/2$, so that the possible fractional variation for E_g is 1/20. This in turn is a good indication that the wave function of the ground state is usually not greatly different from the "naive" expectation (2).

In Hulthén's case,[1] $1+1/ZS$ is 2. He found the corresponding factor for his rigorous ground state to be 1.773:

$$E_g(S=\tfrac{1}{2}, Z=2)=-1.773(\tfrac{1}{2}NZ)JS^2, \tag{10}$$

while a somewhat better upper bound than Eq. (4), 1.5, may be obtained with a simple "bond eigenfunction." We see that the limits are correct, if somewhat wide, in this, the worst possible case.

[1] L. Hulthén, Arkiv. Mat. Astron. Fysik **26A**, No. 11 (1938).
[2] See, for instance, P. R. Weiss, Phys. Rev. **74**, 1493 (1948).

The New State in Li^7 at Low Bombarding Energy*

R. W. GELINAS, C. M. CLASS, AND S. S. HANNA
Department of Physics, Johns Hopkins University, Baltimore, Maryland
(Received July 20, 1951)

GOVE and Harvey[1] have revealed the existence of a state in Li^7 at 4.77 ± 0.10 Mev from observations on the $Be^9(d, \alpha')Li^{7*}$ reaction and on the inelastic scattering of protons, deuterons, and alphas by Li^7, at bombarding energies of 7.9, 14, and 31 Mev. In a survey of the $Be^9(d, \alpha)Li^7$ reaction,[2] at bombarding energies of 0.68 and 1.03 Mev, we examined the alpha-spectrum at 90° using magnetic analysis.[3] Above $E_{ex}=2.5$ Mev in Li^7 a continuum of alpha-particles is encountered, and in the neighborhood of $E_{ex}=4.7$ Mev Li^{+++} recoils from the doublet ground-state transition interfered with our observations. With photographic detection, discrimination between the lithium particles and alphas is difficult because they have practically the same range in emulsion. The two lithium groups are separated by only 135

kev on the alpha-energy scale. We have now reexamined the region around $E_{ex}=4.7$ Mev, using thinner targets and more favorable bombarding energies and target orientations than in the survey work.

With magnetic analysis, alpha-particles corresponding to a 4.77-Mev state will coincide with the ground state. Li^{+++} recoils at a bombarding energy of 1.64 Mev and with the first excited state recoils at 1.20 Mev. Interference from deuterons scattered elastically from Be, C, and O will be experienced at bombarding energies of 1.84, 1.55, and 1.37 Mev, respectively. We have selected a bombarding energy near 0.5 Mev, which produces a separation of about 200 kev between the predicted alpha-position and the closest lithium recoils, making possible a search on either side of the expected location.

The beryllium target (on solid backing) was set at an angle of 22° to the deuteron beam. This orientation produces a relatively large spread in energy in the impinging deuterons; but the lithium recoils receive only 2/11 of the bombarding energy, and for them the effective thickness is only 8 percent above that for normal emergence. The oblique setting also has the advantage of increasing the yield from a given thin target. In the first run, the results of which are shown in the lower part of Fig. 1, a fairly thick target of approximately 3000A was used. The spectrum was calibrated with the proton peak from $Be^9(d, p')Be^{10*}$. The breadth of this peak is due in large measure to the target orientation which is not favorable for the proton reaction. Despite the poor resolution a group of particles can be seen partially resolved on the low energy side of the lithium recoils.

The second run, shown in the upper part of Fig. 1, was made with a considerably thinner target (conveniently prepared from the stock of evaporated targets on hand by dissolving away a portion of the beryllium in very dilute sulfuric acid). With the increased sharpness of the lithium peaks it was possible to use them to calibrate the plate. The energy scale obtained agreed satisfactorily with that from the first run when compared on the basis of the magnetic field settings. The group of particles on the low energy side of the recoils is well resolved and falls in the right region to be identified with the alpha-group observed by Gove and Harvey[1] at a bombarding energy of 14 Mev. Using the Q values of $Be^9(d, p')Be^{10*}$ and $Be^9(d; Li, Li^*)He^4$ given in the literature,[4] we obtain a Q of 2.53 ± 0.02 Mev and an excitation energy in Li^7 of 4.62 ± 0.02 Mev. The intensity of the transition is estimated to be about 25 percent that of the ground state alpha-transition at this bombarding energy. In comparing the two spectra in Fig. 1, one observes the shift in peak maxima arising from target thickness. The rising edge on the high energy side of the ground state lithium group, however, agrees satisfactorily between the two runs. In computing energy values we have attempted to utilize the high energy edge of the peaks in a consistent fashion.

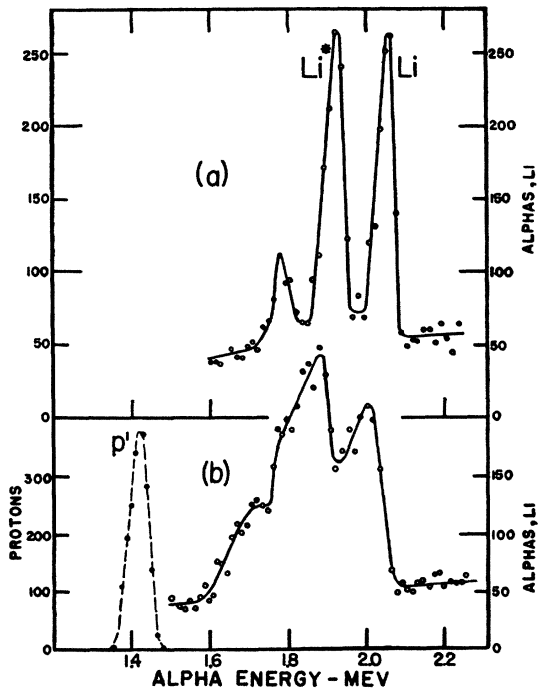

FIG. 1. Magnetic spectra of Be^9+d, in the neighborhood of $E_{ex}=4.7$ Mev in Li^7. The background arises from $Be^9(d, \alpha)He^4+H^2$, the proton peak from $Be^9(d, p')Be^{10*}$, and the lithium peaks from $Be^9(d; Li, Li^*)He^4$. The lithium particle energy is 9/7 the alpha-energy. (a) $E_d=0.47$ Mev. The target, approximately 1000A, is set at 22° to the deuteron beam. Magnetic field $=12.3$ kilogauss. (b) $E_d=0.48$ Mev. The target, approximately 3000A, is set at 25° to the deuteron beam. Magnetic field $=11.6$ kilogauss.

* Assisted by a contract with the AEC.

[1] H. E. Gove and J. A. Harvey, Phys. Rev. **82**, 299, 658 (1951).
[2] R. W. Gelinas and S. S. Hanna, Phys. Rev. **82**, 298 (1951).
[3] For a description of the instrument see D. R. Inglis, Phys. Rev. **78**, 104 (1950).
[4] W. W. Buechner and E. N. Strait, Phys. Rev. **76**, 1547 (1949); W. Whaling and C. W. Li, Phys. Rev. **81**, 150 (1951); Strait, Van Patter, Buechner, and Sperduto, Phys. Rev. **81**, 747 (1951).

Stars Produced in Nuclear Emulsions by Carbon Ions from the Cyclotron

JAMES F. MILLER
Crocker Laboratory, University of California, Berkeley, California
(Received July 20, 1951)

NUCLEAR emulsions have been exposed to carbon ions accelerated in the 60-inch cyclotron at Crocker Laboratory,[1–5] and a study of salient features is being made. This note reports early results on the observation of inelastic nuclear events with the atoms of the emulsion. Ilford E-1 and D-1 nuclear emulsions were used. In the E-1 emulsion, the track of a carbon ion has a solid core, covered with electron spurs less than 2 microns long in the early part of the track and none in the later part. The track tapers appreciably and may have gaps near the end. In the D-1 emulsion the track is less dense and the tapering less noticeable.

The plates are exposed in the cyclotron vacuum to the external beam, with nothing interposed between them and the beam. A set of three slits over a length of 26 inches serves to exclude all but a few extraneous charged particles. The plates are inclined so that the tracks dig into the emulsion at an angle of five degrees. Both C^{12} and C^{13} ions, completely stripped, have been used. C^{12} ions of 120-Mev energy have a range in the emulsion of 175 microns; C^{13} ions of 130-Mev energy have a range of 190 microns. Carbon ions ionized twice, rather than six times, can be accelerated in the cyclotron, using the cyclotron frequency as the third harmonic of their orbital frequency. These C^{2+} appear in great quantity in the internal cyclotron beam, and of these a small fraction is often found in the external beam. Their energy is one-ninth that of the C^{6+}, and their range in the emulsion is about 2.5 microns.

Inelastic events in which charged particles are emitted are the only kind that can be definitely identified, and only these are included here. From these events can be calculated a composite cross section for all the elements of the emulsion. In Ilford emulsions, the stated composition gives, out of 100 atoms, approximately 13 atoms each of silver, bromine, and oxygen, 17 of carbon, 41 of hydrogen, and 3 of minor constituents.

In calculating a cross section, the hydrogen atoms have been included here as capable of producing a star in contrast to the usual procedure when bombardment is with protons or neutrons. The full length of the carbon ion track has been used in the calculation. With C^{12} ions, 102 events have been found out of 181,000 tracks; with C^{13}, 108 events out of 188,000. These correspond to