overestimate at small scattering angles; more accurately, the inelastic scattering, instead of remaining nearly isotropic, approaches 0 at small angles. To obtain the above numbers, the zero range deuteron wave function was employed; when the more refined Hulthén wave function is used, the elastic scattering is reduced and the inelastic scattering is increased.

Details will be given in a future paper with B. Segall.

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* Supported in part by the joint program of the ONR and AEC.

[1] Ph.D. thesis of author, University of Illinois, 1951.

[2] B. Ferretti and S. Gallone, Phys. Rev. **77**, 153 (1950). Also see the recent work of Fernbach, Green, and Watson, Phys. Rev. **82**, 980 (1951).

Charge Exchange Scattering of π-Mesons in Deuterium

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SINCE ordinary scattering[1] and absorption[2] of fast π-mesons of spin zero in deuterium have already been studied in the weak-coupling approximation, the calculation of the exchange scattering cross section in deuterium,

$$\pi^{+}+d\rightarrow p+p,$$

completes the list of important meson reactions in the two-body system. After carrying out this calculation, we find the deuteron cross section to be directly expressible in terms of the corresponding single-nucleon cross section. An outline of the argument follows.

It was first found that the result obtained from a second-order perturbation calculation, with the most general type of coupling, is consistent with the result obtained by the impulse approximation method,[3] in which one assumes that the scattering amplitude of the deuteron as a whole can be taken to be a linear superposition of the single-nucleon amplitudes. The essential point here is simply that the lifetime of the intermediate state is very short compared with the period of the deuteron. Thus, it is plausible to assume the validity of the impulse approximation even if the meson-nucleon coupling is not weak.

The results of applying the impulse approximation are particularly simple if the deuteron crosss section is expressed as a function of the nuclear recoil, $\mathbf{K}_f=\mathbf{q}_0-\mathbf{q}_f$, where $\mathbf{q}_0$ and $\mathbf{q}_f$ are initial and final meson momenta. The probability for scattering per unit interval of K_f^2 is given by

$$d\sigma/d\mathrm{K}_f^2=\{\tfrac{1}{3}\int d\mathbf{k}_f|I_s|^2+\tfrac{2}{3}\int d\mathbf{k}_f|I_T|^2\}d\sigma_A/d\mathrm{K}_f^2+\{\int d\mathbf{k}_f|I_T|^2\}d\sigma_B/d\mathrm{K}_f^2, \quad (1)$$

where the integration variable $\mathbf{k}_f$ is the final relative momentum of the nucleon system and the integration is to be carried out only over those values of $\mathbf{k}_f$ compatible with the conservation of energy. In Eq. (1), I_S and I_T are the singlet and triplet overlap integrals defined by

$$I_S=\int\psi_f^{e*}\exp(\tfrac{1}{2}i\mathbf{K}_f\cdot\mathbf{r})\psi_i d\mathbf{r},\qquad I_T=\int\psi_f^{o*}\exp(\tfrac{1}{2}i\mathbf{K}_f\cdot\mathbf{r})\psi_i d\mathbf{r},$$

where ψ_i, ψ_f^e, and ψ_f^o are the nuclear wave functions for the initial state (deuteron), final state (di-proton) with even parity, and final state with odd parity, respectively. The quantities $d\sigma_A/d\mathrm{K}_f^2$ and $d\sigma_B/d\mathrm{K}_f^2$ are the single-nucleon exchange cross sections corresponding to spin flip and no spin flip, respectively.

From the above expression, one can easily deduce the important result that the deuteron cross section cannot be larger than the neutron cross section. Evaluation of the integrals in Eq. (1) has been performed in both the plane wave and closure approximations;[4] however, for the sake of brevity, only the closure result, which is found to be a reasonable approximation, will be presented here. The formula for the cross section is

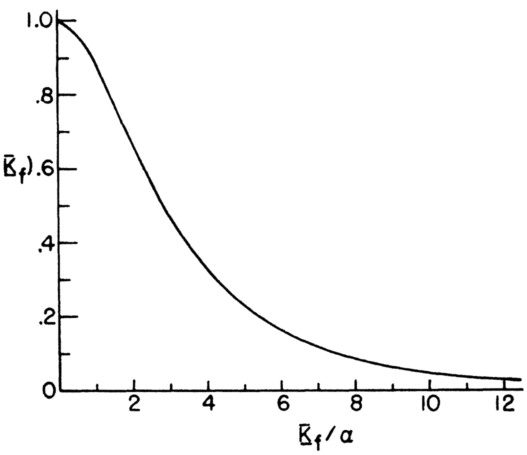

FIG. 1. $F(\mathrm{K}_f)$ *versus* K_f.

$$d\sigma/d\mathrm{K}_f^2=[1-\tfrac{1}{3}F(\mathrm{K}_f)](d\sigma_A/d\mathrm{K}_f^2)+[1-F(\mathrm{K}_f)]d\sigma_B/d\mathrm{K}_f^2. \quad (2)$$

The function $F(K_f)$, which is defined by

$$F(\mathrm{K}_f)=\int\exp(i\mathbf{K}_f\cdot\mathbf{r})\psi_i^2(\mathbf{r})d\mathbf{r},$$

is plotted against K_f in Fig. 1. Since $F(\mathrm{K}_f)$ is very nearly equal to unity for values of K_f in the neighborhood of zero, only the spin flip terms will contribute appreciably for small nuclear recoils.

TABLE I. Weak-coupling single-nucleon cross sections. In this table g is the charged meson coupling parameter; g_N and g_P are coupling parameters of the neutral meson to the neutron and proton, respectively; and $(\mu)^{-1}$ is the meson Compton wavelength.

Charged $\pi\downarrow$ \ Neutral $\pi\rightarrow$	Scalar coupling	Spin coupling
Scalar coupling	$d\sigma_A=0$	$d\sigma_A=\frac{g^2}{16\pi}\frac{q_f^2}{\mu^2}\frac{(g_P-g_N)^2}{q_0^2\hbar^2\omega_0^2}d\mathrm{K}_f^2$
	$d\sigma_B=\frac{g^2}{16\pi}\frac{(g_P-g_N)^2}{q_0^2\hbar^2\omega_0^2}d\mathrm{K}_f^2$	$d\sigma_B=0$
Spin coupling	$d\sigma_A=\frac{g^2}{16\pi}\frac{(g_P-g_N)^2}{\mu^2\hbar^2\omega_0^2}d\mathrm{K}_f^2$	$d\sigma_A=\frac{g^2}{16\pi}\frac{(g_P+g_N)^2}{\mu^4 q_0^2\hbar^2\omega_0^2}(\mathbf{q}_0\times\mathbf{q}_f)^2d\mathrm{K}_f^2$
	$d\sigma_B=0$	$d\sigma_B=\frac{g^2}{16\pi}\frac{(g_P-g_N)^2}{\mu^4 q_0^2}\frac{(\mathbf{q}_0\cdot\mathbf{q}_f)^2}{\hbar^2\omega_0^2}d\mathrm{K}_f^2$

This can easily be understood in terms of the Pauli principle. For, in the case of no change in spin, the final state consists of two identical nucleons with their spins aligned. This is forbidden unless the nuclear system is kicked into a state of odd orbital angular momentum.

As an example, the relevant weak-coupling single-nucleon cross sections for the various possible combinations of parities of spin zero neutral and charged mesons are listed in Table I. In Table II

TABLE II. Total cross sections.

Meson energy (Mev)	Deuteron $\sigma_A\times10^{27}$ cm^{-2} $\left[\frac{g^2(g_P+g_N)^2}{(4\pi\hbar c)^2}\right]$	Deuteron $\sigma_B\times10^{27}$ cm^{-2} $\left[\frac{g^2(g_P-g_N)^2}{(4\pi\hbar c)^2}\right]$	Single nucleon $\sigma_A\times10^{27}$ cm^{-2} $\left[\frac{g^2(g_P+g_N)^2}{(4\pi\hbar c)^2}\right]$	Single nucleon $\sigma_B\times10^{27}$ cm^{-2} $\left[\frac{g^2(g_P-g_N)^2}{(4\pi\hbar c)^2}\right]$
28	19.6	4.06	24.5	14.6
56	59.4	16.0	70.1	34.7
90	120	34.0	133	68.2

are tabulated the total cross sections at several energies for the case of spin coupling for the π^+ and π^0 mesons for both the deuteron and single-nucleon reactions. Elaboration and proof of these results will be given in a future paper by J. S. Blair and the author.

The author gladly expresses his indebtedness to G. F. Chew for his suggestion of the problem and his constant help and encouragement. He also wishes to thank J. S. Blair for his helpful suggestions.

[1] J. S. Blair, Phys. Rev. **83**, 1246 (1951).
[2] W. Cheston, Phys. Rev. **82**, 335 (1951).
[3] The application of the impulse approximation is similar to the treatment of photomeson production in deuterium by Chew and Lewis, and by Feshbach and Lax (to be published).
[4] Fernbach, Green, and Watson, Phys. Rev. **82**, 980 (1951), have evaluated the impulse approximation in a similar way for the case of ordinary scattering with no spin flip.

On the Magnetic Moments of Odd-Odd Nuclei

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IT was pointed out by Feenberg[1] and others that the magnetic moments of odd-odd nuclei are very close to the values calculated with the assumption that the two odd nucleons move in a central field and their angular momenta combine in the j-j coupling scheme. This excellent agreement is rather astonishing in view of the large deviations of the magnetic moments of odd-even nuclei from the Schmidt values (also in the case of light nuclei).

The aim of this note is to point out that the apparent contradiction between these two facts disappears upon consideration of the mutual compensation of the changes in the magnetic moment due to the odd proton and odd neutron. This compensation can occur in many reasonable theories which explain the deviations from the Schmidt values. It represents itself in a simple way if that deviation can be expressed as a change of the g factor which belongs to the j of the odd nucleon. This is the case with the theory which assumes that the anomalous magnetic moment of the proton and neutron is quenched when they are bound in the nucleus.[2,3]

If the two odd nucleons have the same l and j (according to the shell model), their angular momenta will have equal projections on their resultant. We may assume that the changes in the individual g factors are then roughly equal in magnitude (they have opposite signs) if there are as many protons as neutrons. As a result, the changes would cancel and the magnetic moment would have the calculated value. In case the odd proton and odd neutron do not have the same l and j, the situation will not be the same; but it is still possible to estimate the sign and order of magnitude of the deviation from the calculated value.

Turning to the experimental material (Table I), we find that in all the cases where the proton and neutron configurations are the same, the calculated and experimental values agree within a few percent. The only case where the proton and neutron are in different orbits is K^{40}, and only in this case is the deviation not small. Elementary calculation shows that the deviation in this case should be

$$\Delta\mu = (8/5)[(4/7)\Delta\mu_N + (1/3)\Delta\mu_P].$$

The deviation found could have well been caused by changes in the individual magnetic moments $\Delta\mu_N$ and $\Delta\mu_P$ of the sign and order of magnitude found for odd $f_{7/2}$ neutron and $d_{3/2}$ proton-nuclei. The magnetic moment of Lu^{176} can also be explained on these lines, but the lack of exact knowledge of the spin and the ambiguity of the orbit assignment in this region do not allow a clear discussion. The magnetic moment of Li^6 agrees better with the value calculated in LS-coupling; this result is not unexpected for this light nucleus. (If LS-coupling is assumed, the simple cancellation described above would take place if the deviations from the Schmidt values can be expressed as a change in the g_s or g_l of the odd nucleon.) It may be noted that the assignment $d_{3/2}$ in the case of Na^{22} will not give a good agreement.

TABLE I. Calculated and observed magnetic moments.

Nucleus	I obs	Neutron state	Proton state	Coupling scheme	Ground state	Magnetic moment Calculated	Magnetic moment Observed[a]
Li^6	1	$p_{3/2}$	$p_{3/2}$	jj	$J=1$	0.63	0.8221
		p	p	LS	${}^3S_1$	0.88	
B^{10}	3	$p_{3/2}$	$p_{3/2}$	jj	$J=3$	1.88	1.801
		p	p	LS	${}^3D_3$	1.88	
N^{14}	1	$p_{1/2}$	$p_{1/2}$	jj	$J=1$	0.37	0.403
Na^{22}	3	$d_{5/2}$	$d_{5/2}$	jj	$J=3$	1.73	1.7454
		$(d_{5/2})^3$, $J_N=\frac{3}{2}$	$(d_{5/2})^3$, $J_P=\frac{3}{2}$	jj	$J=3$	1.73	
K^{40}	4	$f_{7/2}$	$d_{3/2}$	jj	$J=4$	−1.68	−1.29

[a] *Nuclear data*, Natl. Bur. Standards (U. S.) Circ. 499 (1950).

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[1] E. Feenberg, Phys. Rev. **76**, 1275 (1949).
[2] A. de Shalit, Helv. Phys. Acta, to be published.
[3] F. Bloch, Phys. Rev. **83**, 839 (1951).

The Systematics of Nuclear Energies

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THE Weizsäcker semi-empirical formula for nuclear energies may, by straightforward analysis, be placed in the form,

$$E(A, D) = A\epsilon_v(A) + J(A)[D - D_v(A)]^2 + H(A)\delta, \qquad (1)$$

where $D = N - Z$ and δ is ± 1 or 0 according to the nuclear type. On the basis of this expression the Q value of a nuclear reaction,

$$Q = E(A, D) + e(a, d) - E(A', D') - e(a', d'), \qquad (2)$$

may be written in the convenient form,

$$Q = Q_1 + Q_2 + Q_3, \qquad (3)$$

where

$$Q_1 \approx e - e' - (a - a')\bar{S}_v, \qquad (4)$$
$$Q_2 = \delta H - \delta' H' \approx (\delta - \delta')\bar{H}, \qquad (5)$$
$$Q_3 = \theta^2 J - \theta'^2 J' \approx (\theta^2 - \theta'^2)\bar{J}, \qquad (6)$$
$$S_v = d(A\epsilon_v)/dA, \qquad (7)$$

and

$$\theta = D - D_v(A). \qquad (8)$$

The bar signifies that the functions are to be evaluated at

$$\bar{A} = \tfrac{1}{2}(A + A') = A + \tfrac{1}{2}(a - a'), \qquad (9)$$

and $\approx$ signifies "very nearly equal to."

The key functions ϵ_v, D_v, J, H, and S_v, although quite complicated from the computational standpoint, have relatively small variations. Thus graphical representations of these functions, once prepared, are adequate for most purposes.

Alternatively, one may replace the complicated key functions derived from the Weizsäcker surface by the simple analytical functions:

$$E_v(A) = A\epsilon_v(A) = c_1 - c_2 A + c_3 A^2, \qquad (10)$$
$$J(A) = c_4/A, \qquad (11)$$
$$H(A) = c_5/A^{\frac{3}{4}}, \qquad (12)$$
$$D_v(A) = \gamma A^2/(A + B), \qquad (13)$$

and

$$S_v(A) = -c_2 + 2c_3 A. \qquad (14)$$

A graphical analysis of Rosenfeld's[1] table of nuclear energies, the known pairing correction function, and other empirical con-