Electron-Gamma-Angular Correlation Measurements in Radioactive Decay*

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(Received June 7, 1951)

The beta-gamma angular correlations in the decay of a number of nuclides have been studied as a function of the beta-ray energy. Isotropic correlations were found in the decay of Co^{55}, Co^{56}, Co^{60}, Na^{22}, Na^{24}, Cs^{134}, and I^{124}. Non-isotropic correlations have been measured in the decay of Rb^{86}, Sb^{124}, and I^{126}. The interpretation of these results in terms of the properties of the emitted radiations and the nuclear energy states is discussed. A non-isotropic conversion electron-gamma-angular correlation was found in the decay of Mo^{93}.

1. INTRODUCTION

IN recent years angular correlation measurements have taken their place with the measurement of internal conversion coefficients, lifetimes, and spectral shapes in the determination of the angular momentum and parity of nuclear levels.[1] In this paper we shall describe measurements of beta-gamma and conversion electron-gamma correlations.

The theory of beta-gamma angular correlation has been dealt with most extensively by Falkoff and Uhlenbeck[2] who have calculated explicit beta-gamma correlation functions for all possible cases involving an angular momentum change of one or two for the beta-ray and the gamma-ray, assuming a zero nuclear charge and assuming a single nuclear matrix element is responsible for the beta-transition. Lennox and co-workers[3] have calculated explicit correlation functions for some special cases assuming a finite nuclear charge and mixtures of matrix elements. The theory of conversion electron-gamma angular correlation has been studied by Ling and Uhlenbeck.[4] They have computed these correlations for K shell conversion using various approximations for the atomic wave functions.

In all of these theoretical studies it is assumed that the lifetime of the intermediate state of the nucleus is so short that no appreciable reorientation takes place before the emission of the second radiation. It is also assumed that the efficiency of the radiation detectors are polarization and spin insensitive. Goertzel[5] has made a theoretical study of the effect of reorientation of the nucleus by the atomic magnetic field and by externally applied magnetic fields.

Some general conclusions of the theoretical studies which apply to all types of angular correlations discussed in this paper are stated here:

1. The correlation function will always be of the form:

$$W(\theta)=1+a_1\cos^2\theta+a_2\cos^4\theta+\cdots.$$

Only even powers of $\cos\theta$ appear.

2. The coefficients a_1, a_2, etc., depend on the angular momenta of the three nuclear states involved and on the nature of the transitions. For a gamma-transition the coefficients depend on the multipole order of the transition but not on parity change. For a beta-transition the coefficients depend on the particular matrix element responsible for the transition. The conversion electron-gamma-correlation coefficients depend on the atomic shell in which the conversion takes place and on the multipole order and parity change of the transition.

3. No higher power of $\cos\theta$ can appear than $2L$, where L is the smaller of the angular momenta carried away by the emitted particles.

4. No higher power of $\cos\theta$ can appear than $2J$, where J is the angular momentum of the intermediate state. For example, if $J=0$ or $J=\frac{1}{2}$ there will be no correlation.

Some further theoretical conclusions applicable only in the beta-gamma case should also be given here:

1. If the beta-transition is allowed there will be no correlation.

2. If the beta-transition is forbidden but the shape of the spectrum is strictly identical with the allowed shape there will be no correlation.

3. For the axial vector and tensor interactions the highest power of $\cos\theta$ that can appear in $2F$ where F is the degree of forbiddenness. For example, for first-forbidden axial vector or tensor interactions $\cos^4\theta$ terms do not appear even though these transitions may involve an angular momentum change of two.

4. The beta-gamma angular correlation depends on the maximum energy of the beta-spectrum involved. If beta-rays of all energies are detected the so-called integral correlation will be observed. If beta-rays of a particular energy only are detected the so-called differential correlation will be observed. The differential correlation depends on the energy of the beta-rays being observed and vanishes as the energy of the beta-rays being observed tends to zero.

5. The correlation computed assuming the actual value of Z for cases where matrix elements like $\int \sigma \times \mathbf{r}$ are involved is substantially lower in absolute value

* This work has been supported in part by the joint program of the ONR and AEC.

[1] E. L. Brady and M. Deutsch, Phys. Rev. **78**, 558 (1950).

[2] D. L. Falkoff and G. E. Uhlenbeck, Phys. Rev. **79**, 334 (1950).

[3] M. Fuchs and E. S. Lennox, Phys. Rev. **79**, 221 (1950); E. S. Lennox, private communication.

[4] D. S. Ling, thesis, University of Michigan (1948), unpublished.

[5] G. Goertzel, Phys. Rev. **70**, 897 (1946).

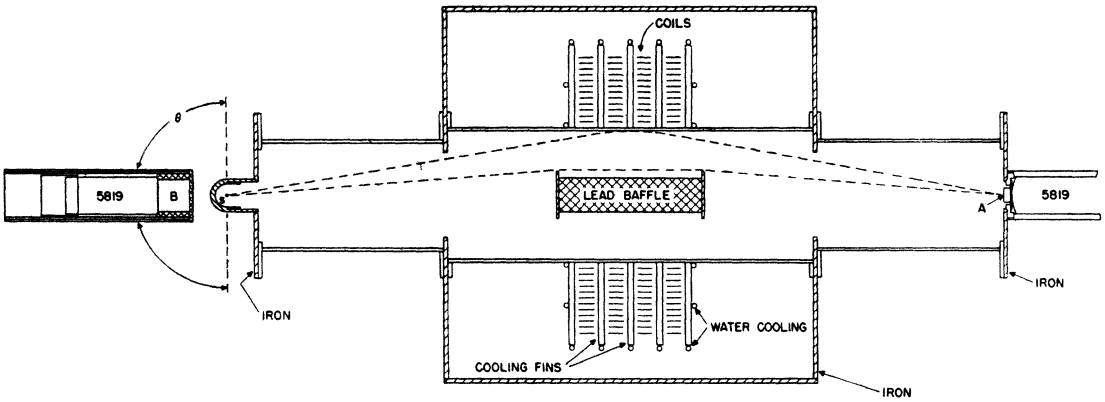

FIG. 1. Apparatus for observing beta-gamma angular correlations. S is the source, A is the thin anthracene crystal of the beta-counter, and B is the NaI crystal of the gamma-counter.

than that computed from the $Z=0$ approximation. When matrix elements like B_{ij} are involved the correlation is nearly independent of Z.

2. EXPERIMENTAL PROCEDURE

The simplest experimental arrangement would consist of a source set up between a beta-counter and a gamma-counter, one of which could be rotated around the source. Appropriate electronic equipment to operate the counters and record the coincidences would also be needed. Several such experimental arrangements have been described recently.[6] In these arrangements the energy of the beta-particles being observed is fixed by the amount of absorber in front of the counter and if a scintillation counter is used by the height of pulse accepted by the counting equipment. For this research this simple arrangement was modified by inserting a thin-lens beta-ray spectrometer between the source and the beta-counter to select the energy of the beta-rays being observed. By selecting beta-particles in a narrow range of energy we are able to measure the differential correlation. A sketch showing the position of the source, counters, and the spectrometer is given in Fig. 1.

At energies near the maximum beta-energy the differential correlation is expected to be larger than the integral correlation and might be easier to find. Furthermore, if a correlation is found we are able to measure the differential correlation as a function of energy. This provides additional information when comparing the experiments with the theory.

The use of a lens spectrometer has certain advantages from an experimental point of view. Its use allows the beta-counter to be placed at some distance from the gamma-counter. Since the electrons follow curved paths in the spectrometer, the direct line of travel from the source to the beta-counter can be filled with sufficient lead-absorber to prevent any gamma-rays from being counted in the beta-counter. In the decay of a nucleus such as Co^{60} where a gamma-gamma correlation exists, a spurious beta-gamma correlation might be observed if the gamma-ray efficiency of the beta-counter were not very small. If the nucleus being studied is a positron emitter the possibility of observing a spurious beta-gamma correlation is much increased since the two gamma-rays emitted when the positron is annihilated always come off with an angle of 180 degrees between them. In any case eliminating gamma-counts in the beta-counter reduces the chance coincidence rate. Locating the beta-counter some distance away from the gamma-counter also eliminates the possibility of registering a coincidence when a gamma-ray is Compton scattered from one counter to the other.

The spectrometer used in this experiment was of the conventional thin lens type.[7] This type of spectrometer was chosen because of its simplicity and because it can be adjusted so that the angular spread of electrons transmitted and the momentum resolution are satisfactory for angular correlation experiments. Furthermore, its use allows both the source and the detectors to be placed outside the magnetic field. The coil, wound around the center of the spectrometer chamber, is surrounded by $\frac{3}{16}$-inch steel plate and the spectrometer endplates are made of steel so as to reduce the magnetic field at the positions of the counters. This is necessary since the amplification of the photomultipliers used in the scintillation counters is very sensitive to changes in the magnetic field. The spectrometer baffles were set so that the momentum resolution $\Delta p/p$ was about 0.15. With this baffle setting the transmission of the spectrometer for electrons of a given energy was about $\frac{3}{4}$ percent.

The beta-counter consists of a small crystal of anthracene about $\frac{3}{4}$-inch square and $\frac{1}{8}$-inch thick and an RCA 5819 photomultiplier. The crystal is mounted on the vacuum side of a glass plate which seals a hole in

[6] R. L. Garwin, Phys. Rev. **76**, 1876 (1949); Grace, Allen, and Halban, Nature **164**, 538 (1949); T. B. Novey, Phys. Rev. **78**, 66 (1950); R. Stump and S. Frankel, Phys. Rev. **79**, 243 (1950); S. L. Ridgeway, Phys. Rev. **79**, 243 (1950); **78**, 821 (1950); J. R. Beyster and M. L. Wiedenbeck, Phys. Rev. **79**, 728 (1950); **79**, 169 (1950); I. Shaknov, Phys. Rev. **82**, 333 (1951).

[7] Deutsch, Elliott, and Evans, Rev. Sci. Instr. **15**, 178 (1944).

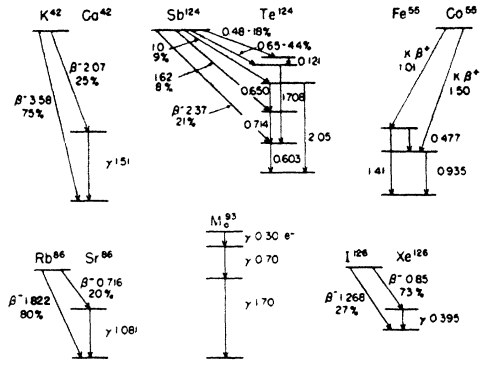

FIG. 2. Decay schemes. References to original work on these may may be found in reference 12.

the end plate. The photomultiplier is mounted outside the vacuum (see Fig. 1). The gamma-counter in some of the earlier experiments consisted of a cylinder of anthracene 1.8 inches in diameter and 1.7 inches high and a 5819. Later the efficiency of the counter was increased by replacing the anthracene by a cylinder of thallium activated sodium iodide. The sides of the crystal are protected from stray and scattered radiation by $\frac{1}{4}$-inch of lead. Sufficient lead is placed between the source and the gamma-counter to prevent the detection of gamma-rays scattered through large angles near the source. Both photomultipliers are surrounded by Permalloy cylinders to reduce the effect of magnetic fields. The tubes with their Permalloy shields are placed in grounded brass cylinders which serve to eliminate electrical pick-up and stray light.

Pulses from the multipliers are amplified by a pair of Model 501 amplifiers.[8] The coincidence circuit consists of a pair of blocking oscillators giving pulses about 0.1 μsec long. The blocking oscillator pulses are fed to separate grids of 6AS6 coincidence tube. Coincidences and the single counts in each channel are recorded simultaneously.

It is evident that to observe an angular correlation we must observe the direction of the particles just as they come from the nucleus. Any scattering between source and detector will reduce the observed correlation. In any angular correlation experiment where gamma-rays are involved care must be taken to avoid counting Compton scattered radiation since the energy and intensity of the scattered radiation is strongly dependent on angle. As noted previously we have placed sufficient lead in front of the gamma-counter to keep Compton scattered quanta out of it.

The scattering of the beta-particles in the source and the source backing is an even more serious problem. We have used the thinnest possible sources consistent with a reasonable counting rate. Most of the sources were vacuum evaporated onto a very thin backing, assuring uniform source thickness. A few were made by precipitating the source material from a drop of liquid. The sources are mounted inside an aluminum hemisphere at one end of the spectrometer so that the gamma-rays pass through the same thickness of aluminum regardless of their angle of emergence (see Fig. 1). Except for the source backing there is no scattering material within a half-inch of the source. The sources were from $\frac{1}{8}$ to $\frac{3}{8}$ inch in diameter on a backing of $\frac{1}{4}$ mg/cm^2 aluminum foil or on thinner collodion films.

The linearity of momentum of the focused electrons with current was proved by measuring several conversion and photoelectron lines of known energy. Some departure from linearity might be expected because of the steel used for magnetic shielding. Apparently the steel is far enough away from the coil to have little effect on the field inside the spectrometer tank.

With an external source of gamma-radiation the counting rates in both counters were substantially independent of a spectrometer current showing that sufficient magnetic shielding was present. Variations of about 2 percent in the gamma-counting rate as a function of angle were noted. This is attributed to a slight misalignment of the source and the axis of rotation of the counter and also to scattering by the spectrometer end plates. The effect of this variation on the measured correlation was corrected for in first order by dividing all coincidence rates by the single counting rate in the gamma-counter. The angle of the gamma-counter was changed frequently during a run to minimize errors arising from changing efficiencies, spectrometer current, resolving time, etc. The principal proof of the symmetry of the apparatus was provided by the numerous symmetrical (or zero) angular correlations observed in the decay of various nuclides.

The angular correlation was corrected for finite angular resolution assuming a theoretical correlation of the form $1+a\cos^2\theta$. For our arrangement all measured values of the angular correlation must be raised by 11 percent in absolute value. The energy dependence of the correlation was corrected for the finite momentum resolution of the spectrometer by using the observed dependence as a first approximation.

3. BETA-GAMMA ANGULAR CORRELATION RESULTS

We subtract the chance rate from the gross coincidence rate at a given angle and divide the result by the gamma single rate at that angle to obtain a number $N(\theta)$. We define $\epsilon_\theta=[N(\theta)-N(90°)]/N(90°)$. Then $E(\theta)=A\epsilon_\theta+R$, where A is the angular resolution correction factor which for our arrangement was 1.11, and R is a small correction for the momentum resolution of the spectrometer. $E(\theta)$ is to be compared with $[W(\theta)-1]$ as computed theoretically. Most of our data was taken at 180° and 90° and hence we obtain $E(180°)$ which is to be compared with the theoretical value, $(a_1+a_2+\cdots)$. In most cases enough data was

[8] W. C. Elmore and M. Sands, *Electronics; Experimental Techniques*, National Nuclear Energy Series, V-1 (McGraw-Hill Book Company, Inc., New York, 1949).

taken at 135° to show that no large $\cos^4\theta$ terms were present. The errors given in this paper are the standard deviation based only on the number of counts.

A. Rb^{86}

The decay scheme is shown in Fig. 2. Our results for E_θ plotted *vs* the energy of the focused electrons are given in Fig. 3. The solid points were obtained using sources of RbCl 0.5 mg/cm² thick and 1.1 cm in diameter vacuum evaporated onto ¼ mg/cm² aluminum foil. The source material was made available to us through the kindness of Dr. C. S. Wu, who had obtained it from Chalk River. Our earlier results obtained from sources about 6 mg/cm² thick made from lower specific activity RbCl from Oak Ridge are plotted in Fig. 3 as open circles. The standard deviations of these points are approximately the same as for the solid circles at corresponding energies. It can be seen that at 525 kev both sources give the same result while at lower energies the correlation measured with the thicker source is smaller than that with the thinner sources. We take this to mean that at 525 kev and probably also at 330 kev our thin sources were thin enough to allow us to measure the true correlation. Below 300 kev our values are probably lower than the true value.

In all cases the large amount of Cs^{134} present in the source material was removed by repeated cesium silicotungstate precipitation. The absence of Cs^{134} in the purified Rb^{86} was proved by noting the absence of gamma-gamma-coincidences.

The integral correlation in Rb^{86} of +0.13 was calculated assuming an allowed spectrum shape[9] and a differential correlation passing through our observed points with the thinner sources.

The high energy beta-spectrum has a shape corresponding to the B_{ij} matrix element.[9,10] Since Sr^{86} is an even-even nucleus its ground state is expected to have spin zero and hence the ground state of Rb^{86} probably has spin two. The low energy spectrum has a shape

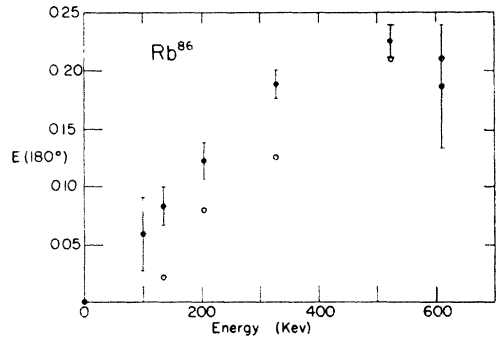

FIG. 3. Beta-gamma angular correlation of Rb^{86}. Solid circles are data obtained with the thin sources. Open circles are data obtained with thick sources. Standard deviations in the thick source data are approximately the same as for the solid circles at corresponding energies.

[9] H. R. Muether and S. L. Ridgeway, Phys. Rev. **80**, 750 (1950).
[10] Zaffarano, Kern, and Mitchell, Phys. Rev. **74**, 682 (1948).

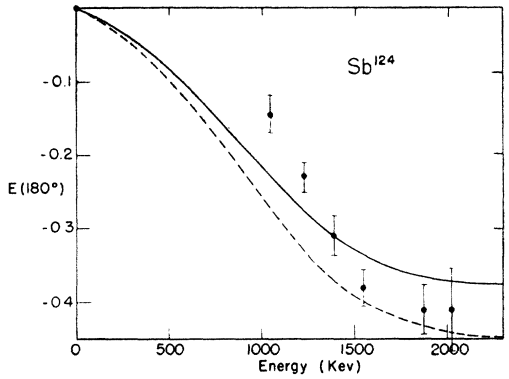

FIG. 4. Beta-gamma angular correlation of Sb^{124}. The solid line is the theoretical correlation for spins 3–2–0 and the dotted line the theoretical correlation for spins 1–1–0 with the B_{ij} matrix element responsible for the beta-transition in both cases.

consistent with an allowed transition or with some types of first-forbidden transitions.[9] An allowed transition is ruled out by the existence of a beta-gamma angular correlation. The excited level in Sr^{86} is probably one or two.

The experimental correlation is not consistent with any theoretical correlation from reference 2. Lennox and co-workers[11] have been unable to fit the data with any correlation calculated assuming a single matrix element operative but taking the nuclear charge into account. They are currently trying to fit the data with mixtures of matrix elements.†

B. Sb^{124}

Ridgeway,[6] and Beyster and Wiedenbeck[6] have reported an integral beta-gamma correlation in the decay of Sb^{124}. Our results for Sb^{124} as a function of energy are given in Fig. 4. The source material was obtained from Oak Ridge. Three different sources were used. Each consisted of metallic antimony about 1.5 mg/cm² vacuum evaporated onto a 3 mg/cm² mica backing. Although no thick sources were made for comparison it is felt that the values given in Fig. 4 are nearly the true correlation since the high electron energy reduces the scattering. Assuming the scheme given by Kern *et al.*,[12] shown in Fig. 2, the correlation above 1620-kev beta-energy involves only the 2.29-Mev negatron spectrum and the 602-kev gamma-ray. Below 1620 kev a second spectrum begins to contribute beta-gamma-coincidences to the measured rate. At about this energy the correlation is seen to drop rapidly in absolute value presumably because these additional coincidences are isotropic or perhaps even have a

[11] E. S. Lennox (private communication).
† *Note added in proof:* M. Fuchs [thesis, University of Michigan (1951), unpublished] has fitted our experimental value of the correlation in Rb^{86} by assigning an angular momentum of 2 to the excited state in Sr^{86} and using a mixture of the B_{ij}^{β} and $\int\beta\alpha$ matrix elements.
[12] K. Way *et al.*, *Nuclear Data*, National Bureau of Standards Circular 499 (1950).

TABLE I. Isotropic correlations.

Decaying nuclide	Energy (kev)	$E_\theta(180°)$
Co^{60}	111	-0.028 ± 0.02
	252	$+0.005\pm0.017$
	277	-0.023 ± 0.025
Cs^{134}	467	$+0.017\pm0.03$
Na^{22}	277	-0.021 ± 0.027
	436	-0.005 ± 0.041
Na^{24}	1045	$+0.02\ \pm0.019$
$Co^{55}-Co^{56}$ (source A)	609	$+0.026\pm0.024$
	1074	$+0.028\pm0.024$
Co^{56} (source B)	1074	$+0.013\pm0.02$

positive correlation. Langer *et al.*[13] were unable to determine whether or not the 1.69-Mev spectrum has a forbidden shape. They found the shape of the 2.29-Mev negatron spectrum to correspond to the first-forbidden matrix element B_{ij} ($\Delta J=0, \pm1, \pm2$ and parity change). Assuming all beta-transitions eventually lead to the 602-kev level they find the 602-kev gamma-ray to be electric dipole. In Fig. 4 the solid line is the theoretical correlation for angular momenta 3–2–0 and the dotted line that for 1–1–0 with the beta-transition due entirely to the B_{ij} matrix element and zero nuclear charge in both cases. As we have said the correlations involving only B_{ij} type transitions are nearly insensitive to Z. The scheme 3–1–0 with B_{ij} matrix element proposed by Langer *et al.*[13] leads to a theoretical value of $E(180°) = -0.14$ at maximum beta-energy and seems definitely ruled out by our results. The 1–1–0 scheme would seem to fit our data best since we might expect that any scattering effects would reduce the observed correlation. However, in this case the transition direct to the ground state of Te^{124} would be allowed or first forbidden only. The scarcity of these direct transitions could be accounted for by assuming that the gamma-transition is magnetic dipole and that the $L=1$ first-forbidden matrix elements $\int \beta\sigma\times\mathbf{r}$ and $\int \alpha$ are small compared with the B_{ij} matrix element. The correlation resulting from the 3–2–0 scheme nearly fits the observed points and this scheme would explain satisfactorily the absence of direct transitions to the ground state. Perhaps a mixture of matrix elements with only a small amount of an $L=1$ matrix element with the B_{ij} matrix element would raise the correlation slightly without changing the characteristic B_{ij} shape noticeably. The 3–2–0 scheme would obviously require the gamma-ray to be quadrupole. No other plausible spin assignments with unmixed matrix elements and $L=1$ or 2 yield acceptable correlation functions for Sb^{124}. No gamma-gamma angular correlation has been found in the decay of Sb^{124} by Beyster and Wiedenbeck[14] or by the present authors. A further study is necessary before anything definite can be said about the spins and parities of the levels involved in the decay of Sb^{124}.

[13] Langer, Moffat, and Price, Phys. Rev. **79**, 808 (1950).
[14] J. R. Beyster and M. L. Wiedenbeck, Phys. Rev. **79**, 169 (1950).

C. I^{126}

The decay scheme given in Fig. 2 is due to Mitchell *et al.*[15] We have found a beta-gamma angular correlation of about 0.13 for 460-kev beta-rays in the decay of I^{126}. The source was made by evaporating a drop of water solution of NaI to dryness on a very thin collodion film. The average thickness of the source was about 2 mg/cm^2. The source material was obtained by an (α,n) reaction on antimony in the MIT cyclotron. The decay scheme is similar to that of Rb^{86} and from our preliminary data the correlations of the two nuclides appear to be similar. Additional measurements of the I^{126} correlations are being made in this laboratory.

D. Co^{55}, Co^{56}, Co^{60}, Na^{22}, Na^{24}, Cs^{134}, I^{124}

No beta-gamma angular correlation was found in the decay of any of the above nuclides. The results are given in Table I.

The Co^{60} source was vacuum evaporated onto a collodion backing 30 μg/cm^2 thick. The source material was $CoCl_2$ of specific activity 0.77 mc/mg obtained from Oak Ridge. The source strength was about 15 μc and the area $\frac{1}{3}$ cm^2 making the thickness about 60 μg/cm^2. The beta-spectrum has an allowed shape.[16]

The Cs^{134} source was vacuum evaporated onto a backing of $\frac{1}{4}$ mg/cm^2 aluminum foil and was about 2 mg/cm^2 thick. Waggoner *et al.*[16] have found some evidence that the spectrum deviates slightly from the allowed shape.

The Na^{22} source was vacuum evaporated onto a thin collodion film and was less than 100 μg/cm^2 thick. The spectrum has an allowed shape.[12]

The Na^{24} source was vacuum evaporated onto $\frac{1}{4}$ mg/cm^2 aluminum foil and was about 20 mg/cm^2 thick. The spectrum has an allowed shape.[17]

Approximate decay schemes of Co^{55} and Co^{56} are shown in Fig. 2. As far as is known the shape of the spectrum of Co^{56} is allowed.[18] However, Deutsch and Hedgran[19] have obtained data which indicates that the K-capture-positron ratio in Co^{55} is not that expected for an allowed transition. Co^{55} and Co^{56} were made simultaneously by bombarding iron with deuterons in the MIT cyclotron. The purified Co activity was vacuum evaporated onto a thin film of collodion. From the half-life curve it was estimated that source A contained about equal activity of Co^{55} and Co^{56}. A month later when all the Co^{55} had decayed source B was made to measure Co^{56} alone. No correlation was found in either case. Only a fairly large correlation due to Co^{55} would have been found in source A because of the presence of the Co^{56}. Both sources were less than 10 mg/cm^2.

[15] Mitchell, Mei, Maienschen, and Peacock, Phys. Rev. **76**, 1450 (1950).
[16] Waggoner, Moon, and Roberts, Phys. Rev. **80**, 420 (1950).
[17] K. Siegbahn, Phys. Rev. **70**, 127 (1946).
[18] L. G. Elliott and M. Deutsch, Phys. Rev. **64**, 321 (1943).
[19] M. Deutsch and A. Hedgran, Phys. Rev. **75**, 1443 (1949).

The source of I^{126} (see C) when first prepared contained a large amount of I^{124}. We found no gamma-rays in coincidence with the 2.20-Mev positron spectrum of I^{124} in contradiction with the decay scheme of Mitchell *et al.*[12] Preliminary results indicate that coincidences involving the 1.50-Mev positrons are isotropic.

4. CONVERSION ELECTRON-GAMMA ANGULAR CORRELATION

A conversion electron-gamma angular correlation was looked for in the decay of Hf^{181} and Mo^{93}. With L and M conversion electrons from the 130- and 134-kev gamma-transitions of Hf^{181} focused in the spectrometer we found the electron-gamma coincidence rate to be isotropic. The source used was very thick and it is probable that any correlation present was obscured by the scattering.

The decay scheme of Mo^{93} given by Kundu *et al.*[20] is shown in Fig. 2. With a strong electron line at 240 kev focused in the spectrometer we found a correlation of $E(180°)=+0.33$. Data at angles between 180° and 90° indicate that terms higher than $\cos^2\theta$ may be present. Interpretation of these results is not possible at this time because both K and L conversion electrons were counted and because two gamma-rays follow the conversion electron. We have measured a gamma-gamma angular correlation of about $E(180°)=+0.20$ in the decay of this nuclide.

It is a pleasure to acknowledge the advice of Professor J. W. Irvine on the chemical separations and the collaboration of Dr. J. W. Shearer in the design of the spectrometer.

[20] Kundu, Hult, and Pool, Phys. Rev. **77**, 71 (1950).

PHYSICAL REVIEW VOLUME 83, NUMBER 6 SEPTEMBER 15, 1951

The Schrödinger Equation in Quantum Electrodynamics

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(Received April 27, 1951)

Concluding a program begun in earlier papers, this paper is occupied with the analysis of the hamiltonian occurring in the Schrödinger equation of quantum electrodynamics in a new representation called the intermediate representation. It is proved that the hamiltonian is free of divergences, either explicit or implicit. The intermediate representation therefore provides a complete divergence-free formulation of quantum electrodynamics, applicable to all physical situations. The divergences which are eliminated are the well-known ultraviolet divergences. The hamiltonian is still expressed as a power series in the fine-structure constant; the convergence of the series is not proved.

I. INTRODUCTION

THIS paper is one of a series[1] dealing with the applications of the idea of renormalization in quantum electrodynamics. It is a direct continuation of (D), and the notations of (D) will generally be used without explanation. The purpose of the paper is to prove that, after the state-vector is transformed by the unitary operator $S(t)$ defined by Eq. (D35), the Schrödinger equation of quantum electrodynamics is free of divergences.

The Schrödinger equation after transforming with $S(t)$ is

$$i\hbar(\partial\Phi/\partial t)=H'(t)\Phi, \tag{1}$$

$$H'(t)=S^{-1}(t)[H_1(t)-i\hbar(d/dt)]S(t). \tag{2}$$

Here $H_1(t)$ is given by Eqs. (D10)–(D14). The physical meaning of $H'(t)$ is discussed in (E). In this paper Secs. II–VI will be occupied with the proof that $H'(t)$ is divergence-free. In Sec. VIII the form of $H'(t)$ will be described, and the problems connected with the practical use of the Schrödinger equation (1) will be briefly considered.

[1] F. J. Dyson, Phys. Rev. **75**, 486 and 1736 (1949); Phys. Rev. **82**, 428 (1951); Phys. Rev. **83**, 608 (1951); Proc. Roy. Soc. **A207**, 395 (1951). These five papers will be referred to hereafter as (A), (B), (C), (D), (E), respectively. References to equations and sections in them will be made as follows: Eq. (D35) means Eq. (35) in (D); Sec. CII means Section II in (C).

II. THE STRUCTURE OF THE TRANSFORMED HAMILTONIAN

The series expansion of $S(t)$ given by Eq. (D35) depends on t in two ways. First, t appears as the upper limit of the multiple integrals; second, t appears in the functions $g(t-t_j)$, contained in $H_g(x, x_j)$ according to Eqs. (D21)–(D30). The total differentiation with respect to t in Eq. (2) can thus be separated into two partial differentiations,

$$(d/dt)=(\partial/\partial t)+(\Delta/\Delta t), \tag{3}$$

where $(\Delta/\Delta t)$ operates only on the g-functions. The operation of $(\partial/\partial t)$ on $S(t)$ gives

$$i\hbar(\partial S/\partial t)=\left[\int H_g(x, x')d_3x'\right]S, \tag{4}$$

where the integration with respect to x' extends over