Neutral Photomeson Production and Nucleon Isobars

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The magnitude of the neutral photomeson cross section, which is comparable with the cross section for charged mesons, is not given correctly by weak coupling perturbation theory. In fact, the phenomenon appears to depend on a nonlinear behavior of the meson fields corresponding to intermediate or strong coupling. Accordingly, the effect of the existence of nucleon isobars, which is one of the most characteristic features of strong coupling theory, has been investigated through the use of a classical model and is found to give the general features and magnitude of the neutral cross section correctly. The isobars are predicted to be unstable with an excitation energy of three or four hundred Mev.

I. INTRODUCTION

THE experimental measurements on the production of mesons by photons[1] show (1) that the total cross sections for charged and neutral meson production are about the same and are approximately equal to 10^{-28} cm^2 for a bremsstrahlung spectrum with maximum energy of 320 Mev; (2) that the neutral cross section rises much more steeply with energy than does the charged; and (3) that the angular distribution for charged mesons is roughly isotropic for angles between 45 and 135 degrees in the laboratory system. The angular distribution for the neutral mesons is only very approximately known but appears to be somewhat peaked forward in the laboratory system for mesons of 60 to 70 Mev.

The production of charged mesons might be reasonably expected to depend only on the extent of the charge cloud about the nucleon, the probability of a meson being present, and the strength of the interaction of the photon with the meson charge, i.e.,

$$\sigma(\text{charged})\simeq\pi(\hbar/\mu c)^2(g^2/\hbar c)(e^2/\hbar c)\sim 2(g^2/\hbar c)10^{-28}\text{ cm}^2. \quad (1)$$

For $g^2/\hbar c=\frac{1}{2}$, this is indeed the approximate magnitude of the cross section observed. One can similarly argue that if neutral mesons are only weakly affected by the interaction of the photon with the charged mesons, the neutral meson can be produced only through the interaction of the photon with the nucleon magnetic moment. Since this is characterized by a length $\hbar/Mc$, where M is the nucleon mass, the cross section would be

$$\sigma(\text{neutral})\simeq\pi(\hbar/Mc)^2(g^2/\hbar c)(e^2/\hbar c)\sim(\mu/M)^2\sigma(\text{charged}). \quad (2)$$

Since $\mu/M\simeq 1/7$, one would expect on the basis of this elementary argument to find σ(neutral) one or two orders of magnitude smaller than σ(charged), in contradiction with the observed approximate equality.

A detailed calculation of this effect has been made by Kaplon[2] using weak coupling theory and including the effects of the nucleon anomalous moments by the introduction of a Pauli term[3] of the form,

$$\sigma_{\mu\nu}F_{\mu\nu}=\boldsymbol{\sigma}\cdot\mathbf{H}-i\boldsymbol{\alpha}\cdot\mathbf{E} \quad (3)$$

in the equation of motion of the nucleon field. He finds for symmetrical pseudoscalar theory that the ratio σ(neutral)/σ(charged) is 0.138 for production from protons and 0.062 for production from neutrons at 250 Mev, where the experimental value is approximately 0.6. The theoretical values of Kaplon are therefore considerably lower than those observed, particularly for neutrons. It is possible to obtain agreement with experiment by increasing the neutral coupling constant; we shall, however, restrict ourselves to the symmetrical form of the theory.

The anomalous magnetic moments which make the values computed by Kaplon appreciably larger than the estimate in Eq. (2) are of course themselves thought to be due to charged meson currents; and thus Kaplon's calculations represent one mechanism by which the electromagnetic effects of charged mesons can serve to increase the photoeffect of neutral mesons. We have thought it of interest to investigate other such effects, which will not appear when the photoeffect is treated by perturbation theory in its lowest order, because they correspond to essentially nonlinear terms in the equation. Indeed it appears that the neutral meson distribution can be strongly excited by the interaction of the photon with the charged mesons and with the nucleon magnetic moment. A possible source of such nonlinear behavior of the charged and neutral meson fields might be the existence of unstable nucleon isobars which could be easily excited by a resonant interaction at the frequencies of the electromagnetic radiation. The excited states of the nucleon would then decay with

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[1] J. Steinberger and A. S. Bishop, Phys. Rev. **78**, 494(L) (1950). Steinberger, Panofsky, and Steller, Phys. Rev. **78**, 802 (1950). We wish to thank Drs. Steinberger and Panofsky for information on additional recent experimental results.

[2] We are indebted to Dr. Kaplon for communicating his results to us in advance of publication.

[3] W. Pauli, *Handbuch der Physik* **24**, 1 (233).

very short lifetime (of the order of the dimensions of the system divided by the velocity of light) into charged or neutral mesons together with a stable nucleon. The relative probability of production of the charged and neutral mesons would then depend only on the relative strength of coupling of the two fields to the nucleon.

After the completion of this work, it was brought to the attention of the authors that Fujimoto and Miyazawa of the University of Tokyo[4] had independently suggested that the large neutral cross section could be attributed to the existence of nucleon isobars. Their calculations were made with a strong coupling quantum-mechanical treatment of symmetrical pseudoscalar theory. We wish to develop these arguments in greater detail than has been done by Fujimoto and Miyazawa in their letter. We shall also show that a simple classical model gives predictions for this process which, although somewhat different from the strong coupling treatment, lead to essentially the same results that they have obtained. A more detailed comparison of the results of these methods will be given in Sec. II.

To investigate strongly nonlinear effects of this nature, a weak coupling approximation cannot be used. A strong coupling theory of the meson-nucleon interactions is necessary. Unfortunately, however, no rigorous methods have been devised for treating this problem. We therefore shall introduce an approximate model for the calculation which will allow a strong coupling treatment of the meson-nucleon interaction and of the effects of isobars on the phenomena in which we are interested. The method to be used is the classical treatment of the meson field and of the spin variables which was originally suggested by Heisenberg[5] and others.[6] We shall consider only the symmetrical pseudoscalar theory because of other evidence that the charged meson is described by a pseudoscalar field.[7] The application of this method is given in detail in Sec. II. In Sec. III some remarks will be made on the classical theory of meson scattering by nucleons and on the scattering of nucleons by nucleons.

II. CLASSICAL CALCULATION OF MESON PRODUCTION BY PHOTONS

The approximations of the classical method allow a simple solution of this problem without a weak coupling treatment of the meson nucleon interaction.[8] It is necessary to assume that the meson field is unquantized, i.e., that quantum fluctuations of the field are unimportant. This will apparently be so if the coupling is strong leading to many mesons about the nucleon. A severer restriction is the assumption that the nucleon spin and isotopic spin vectors can be treated as classical unit vectors which can take on continuously variable orientations, i.e., that they are also unquantized. It is also necessary to assume that the effects of nucleon recoil can be approximated by the introduction of a cut-off radius corresponding to the distance at which the nucleon recoil will appreciably modify the behavior of the system. Such a distance may reasonably be chosen to be approximately the nucleon Compton wavelength.

The hamiltonian for mesons interacting with one nucleon (at rest) and the electromagnetic field is

$$H=\int d^3x\{\tfrac{1}{2}(\boldsymbol{\pi}\cdot\boldsymbol{\pi}+\nabla\boldsymbol{\phi}\cdot\nabla\boldsymbol{\phi}+\mu^2\boldsymbol{\phi}\cdot\boldsymbol{\phi})$$
$$+(4\pi)^{\frac{1}{2}}(f/\mu)[\boldsymbol{\sigma}\cdot\nabla\boldsymbol{\phi}\cdot\boldsymbol{\tau}+(4\pi)^{\frac{1}{2}}e(\boldsymbol{\sigma}\cdot\mathbf{A})(\boldsymbol{\tau}\cdot\boldsymbol{\phi}\times\mathbf{k})]U$$
$$+(\mu\cdot\mathbf{H})U-\mathbf{A}\cdot\nabla(\boldsymbol{\phi}\times\mathbf{k})\cdot\boldsymbol{\phi}\}. \quad (4)$$

In this expression, the neutral fields φ_1 and φ_2 are the components of the charged field

$$\varphi=(\varphi_1-i\varphi_2)/\sqrt{2} \quad (5)$$

and φ_3 is the neutral meson field. The vector $\boldsymbol{\phi}$ in charge space then has components φ_1, φ_2, φ_3. Similarly $\boldsymbol{\tau}$ is the isotopic spin vector in charge space. We also define $\mathbf{k}$ as a unit vector in the direction of the 3-axis in charge space. $\mathbf{A}$ is the vector potential. U is the nucleon source density; for a point source, U would be a delta-function. The term $\boldsymbol{\mu}\cdot\mathbf{H}$ gives the electromagnetic interaction of the nucleon and is taken to be

$$(4\pi)^{\frac{1}{2}}e(a+b\boldsymbol{\tau}\cdot\mathbf{k})(\boldsymbol{\sigma}\cdot\mathbf{H}) \quad (6)$$

where a and b are determined by the values of the static moments, i.e.,

$$a+b=\gamma_N/2M, \quad a-b=\gamma_P/2M, \quad (7)$$

where

$$\gamma_N=-1.91, \quad \gamma_P=2.79. \quad (8)$$

This assumption, that the static moments represent the interaction of nucleons with photons of energies near the meson rest energy of 140 Mev, is of course valid only if the moments show a negligible energy variation over this energy range. This is true only if the currents about the nucleon which contribute to the moment are confined to regions small compared with the wavelength of the radiation, which (at threshold for meson production) is the meson Compton wavelength. This condition is in fact satisfied for the pseudoscalar theory,[9] which predicts a distribution of the dimensions of the nucleon Compton wavelength for both the meson and the nucleon currents which contribute to the magnetic moment. The ratio of wavelength to the dimensions of the contributing currents is therefore $M/\mu\simeq 7$ and the condition for energy inde-

[4] Y. Fujimoto and H. Miyazawa, Prog. Theor. Phys. **5**, 1052(L) (1950).
[5] W. Heisenberg, Z. Physik **113**, 61 (1939).
[6] J. R. Oppenheimer and J. Schwinger, Phys. Rev. **60**, 150 (1941). H. J. Bhabha and H. C. Corben, Proc. Roy. Soc. (London) **A178**, 273 (1941).
[7] S. Tamor and R. E. Marshak, Phys. Rev. **80**, 766(L) (1950). Brueckner, Serber, and Watson, Phys. Rev. **81**, 575 (1951).
[8] See W. Pauli, *Meson Theory* (Interscience Publishers, Inc., New York, 1946), for a detailed discussion of the method.
[9] K. M. Case, Phys. Rev. **76**, 1 (1949).

pendence is approximately satisfied. Aside from these considerations, however, a change in the magnitude of the moments which is not strongly energy dependent will not change the principal features of this calculation, which has to do with the resonance interaction of nucleon isobars. We shall therefore use the static values for the moments but realize that the quantitative values of our results are somewhat in doubt.

From this hamiltonian we obtain the equation of motion for the meson field by the relations,

$$\dot{\pi}_i = -\partial H/\partial \varphi_i, \quad \dot{\varphi}_i = \partial H/\partial \pi_i. \tag{9}$$

We shall treat the spin and isotopic spin as classical unit vectors with the properties

$$\{\sigma_x, \sigma_y\} = -2\sigma_z, \quad \{\tau_x, \tau_y\} = -2\tau_z, \quad \text{(and cyclic)} \tag{10}$$

where $\{\ \ \}$ is the classical poisson bracket. The equations of motion for the spins are then given by

$$d\boldsymbol{\sigma}/dt = \{H, \boldsymbol{\sigma}\}. \tag{11}$$

Thus we obtain

$$(\Box - \mu^2)\boldsymbol{\phi} = (4\pi)^{\frac{1}{2}}(f/\mu)\boldsymbol{\sigma}\cdot\nabla U\boldsymbol{\tau} - 4\pi(fe/\mu)\boldsymbol{\sigma}\cdot\mathbf{A}(\boldsymbol{\tau}\times\mathbf{k})U - 2(4\pi)^{\frac{1}{2}}e\mathbf{A}\cdot\nabla(\boldsymbol{\phi}\times\mathbf{k}), \tag{12}$$

$$d\boldsymbol{\sigma}/dt = \boldsymbol{\sigma}\times\mathbf{F}, \quad d\boldsymbol{\tau}/dt = \boldsymbol{\tau}\times\mathbf{G},$$

where

$$\begin{aligned}\mathbf{F} &= -2(4\pi)^{\frac{1}{2}}\int d^3xU\{(f/\mu)\nabla\boldsymbol{\phi}\cdot\boldsymbol{\tau} \\ &\qquad -(4\pi)^{\frac{1}{2}}(fe/\mu)\mathbf{A}(\boldsymbol{\phi}\times\boldsymbol{\tau})\cdot\mathbf{k} + e(a+b\boldsymbol{\tau}\cdot\mathbf{k})\mathbf{H}\} \\ \mathbf{G} &= -2(4\pi)^{\frac{1}{2}}\int d^3xU\{(f/\mu)\boldsymbol{\sigma}\cdot\nabla\boldsymbol{\phi} \\ &\qquad +(4\pi)^{\frac{1}{2}}(fe/\mu)\boldsymbol{\sigma}\cdot\mathbf{A}(\boldsymbol{\phi}\times\mathbf{k}) + eb\mathbf{k}\boldsymbol{\sigma}\cdot\mathbf{H}\}.\end{aligned} \tag{13}$$

We shall now introduce the perturbation aspect of the problem. We assume that we can separate the variables into an unperturbed part and a perturbed part, i.e.,

$$\boldsymbol{\phi}(r,t) = \boldsymbol{\phi}_0(r) + (2\pi)^{-\frac{1}{2}}\int d\omega \exp(-i\omega t)\boldsymbol{\phi}_1(\omega) \tag{14}$$

and similarly for $\boldsymbol{\sigma}$, $\boldsymbol{\tau}$, $\mathbf{F}$, $\mathbf{G}$. We then argue that, since the perturbation in the system is introduced by the electromagnetic field, $\boldsymbol{\phi}_1$ will be of order $\alpha(=1/137)$ compared with the unperturbed $\boldsymbol{\phi}_0$. The perturbation approximation will then consist of neglecting terms of order $\boldsymbol{\phi}_1\cdot\boldsymbol{\tau}_1$, etc., which are of order α^2 and retaining only terms of order α. We note that this approximation does not apply to the strong meson-nucleon coupling. We also introduce fourier representations for $\boldsymbol{\phi}$ and U.

$$\begin{aligned}\boldsymbol{\phi}(\mathbf{r}) &= (2\pi)^{-\frac{3}{2}}\int d^3l \exp(i\mathbf{l}\cdot\mathbf{r})\boldsymbol{\phi}_l \\ U(\mathbf{r}) &= (2\pi)^{-3}\int d^3l \exp(i\mathbf{l}\cdot\mathbf{r})U_l\end{aligned} \tag{15}$$

(we note that if $U(\mathbf{r})$ is a delta-function, corresponding to a point nucleon source, $U_l=1$). We also write

$$\mathbf{A}(\mathbf{r},t) = [\boldsymbol{\varepsilon}/(2\pi)^{\frac{1}{2}}]\exp[-i(\boldsymbol{\nu}\cdot\mathbf{r}-\nu t)] \tag{16}$$

and set

$$\int H(r)U(r)d^3r = H(0). \tag{17}$$

Inserting these definitions for $\boldsymbol{\phi}$ and U and eliminating φ_0 and φ_1 from the equations for F and G, we have

$$\begin{aligned}\mathbf{F}_0 &= -\frac{f^2}{\mu^2\pi^2}\int d^3l\frac{(U_l)^2\mathbf{l}(\boldsymbol{\sigma}_0\cdot\mathbf{l})}{l^2+\mu^2} \\ \mathbf{F}_1 &= \frac{f^2}{\mu^2\pi^2}\int d^3l(U_l)^2\mathbf{l}\left\{\frac{-(\boldsymbol{\tau}_0\cdot\boldsymbol{\tau}_1)(\boldsymbol{\sigma}_0\cdot\mathbf{l})}{l^2+\mu^2}+\frac{(\boldsymbol{\tau}_0\cdot\boldsymbol{\tau}_1)(\boldsymbol{\sigma}_0\cdot\mathbf{l})}{\omega^2-l^2-\mu^2}\right. \\ &\qquad \left.+\frac{\tau_0{}^2(\boldsymbol{\sigma}_1\cdot\mathbf{l})}{\omega^2-l^2-\mu^2}\right\} - 2(4\pi)^{\frac{1}{2}}e(a+b\boldsymbol{\tau}_0\cdot\mathbf{k})\mathbf{H}_0(0)\delta(\omega-\nu) \\ \mathbf{G}_0 &= \frac{-f^2\boldsymbol{\tau}_0}{\mu^2\pi^2}\int d^3l\frac{(U_l)^2(\boldsymbol{\sigma}_0\cdot\mathbf{l})^2}{l^2+\mu^2} \\ \mathbf{G}_1 &= \frac{f^2\boldsymbol{\tau}_1}{\mu^2\pi^2}\int d^3l\frac{(\boldsymbol{\sigma}_0\cdot\mathbf{l})^2(U_l)^2}{\omega^2-l^2-\mu^2} + (4\pi)^{\frac{1}{2}}\frac{eif^2}{\mu^2\pi^2}(\boldsymbol{\tau}_0\times\mathbf{k})(\boldsymbol{\sigma}_0\cdot\boldsymbol{\varepsilon}) \\ &\qquad \times\int d^3lU_lU_{l-\nu}\left(\frac{1}{\omega^2-l^2-\mu^2}+\frac{1}{l^2+\mu^2}\right) \\ &\qquad \times\boldsymbol{\sigma}_0\cdot\mathbf{l}\delta(\omega-\nu) - 2(4\pi)^{\frac{1}{2}}\frac{eif^2}{\mu^2\pi^2}(\boldsymbol{\tau}_0\times\mathbf{k}) \\ &\qquad \times\int d^3lU_lU_{l-\nu}\frac{(\boldsymbol{\sigma}_0\cdot\mathbf{l})\boldsymbol{\sigma}_0\cdot(\mathbf{l}-\boldsymbol{\nu})(\boldsymbol{\varepsilon}\cdot\mathbf{l})\delta(\omega-\nu)}{(\omega^2-l^2-\mu^2)[(l-\nu)^2+\mu^2]} \\ &\qquad - 2b(4\pi)^{\frac{1}{2}}e\mathbf{k}\boldsymbol{\sigma}_0\cdot\mathbf{H}_0(0)\delta(\omega-\nu).\end{aligned} \tag{18}$$

In evaluating the integrals over $\mathbf{l}$, we shall assume the convention that in convergent integrals, we can replace U_l by 1, its value for a point source. If an integral diverges, we shall set $U_l=1$ but change the limits of integration by replacing

$$\int_0^\infty dl \quad \text{by} \quad \int_0^{l_{\max}} dl. \tag{19}$$

This is equivalent to inserting a cut-off radius. In addition, in the terms of order α, we shall retain only the leading (and largest) term in the integrals. These methods for obtaining approximate values for the integrals are valid only in the region where $l_{\max}$ is much greater than the frequencies involved in the process. Since $l_{\max}$ will be assumed to be approximately the nucleon momentum, we will be restricted to photon energies which are small compared to the nucleon rest energy. We then have

$$\int d^3l\, l\frac{(\boldsymbol{\sigma}_0\cdot\mathbf{l})U_l^2}{\omega^2-\mu^2-l^2}=\frac{4\pi}{3}\boldsymbol{\sigma}_0\int_0^\infty\frac{l^4dlU_l^2}{\omega^2-\mu^2-l^2}$$

$$=\frac{4\pi}{3}\boldsymbol{\sigma}_0\left\{\int_0^{l_{\max}}dl(-l^2-\omega^2+\mu^2)+(\omega^2-\mu^2)^2\int_0^\infty\frac{dl}{\omega^2-\mu^2-l^2}\right\}. \quad (20)$$

In evaluating the last integral, we deform the contour about the poles at

$$l=\pm(\omega^2-\mu^2)^{\frac{1}{2}}\equiv\pm q$$

to give a result corresponding to outgoing waves, i.e.,

$$\lim_{\epsilon\to 0}\frac{1}{2}\int_{-\infty}^{\infty}\frac{dl}{(\omega^2-\mu^2-l^2-i\epsilon)}=\frac{\pi i}{2q}. \quad (21)$$

The total integral then is the sum of three contributions

$$(4\pi/3)\boldsymbol{\sigma}_0\{-\tfrac{1}{3}l_{\max}{}^3-(\omega^2-\mu^2)l_{\max}+\tfrac{1}{2}\pi iq^3\}. \quad (22)$$

The integral

$$\int d^3lU_lU_{l-\nu}\left(\frac{1}{\omega^2-l^2-\mu^2}+\frac{1}{l^2+\mu^2}\right)\boldsymbol{\sigma}_0\cdot\mathbf{l} \quad (23)$$

in this approximation is

$$\int_0^{l_{\max}}l^2dl\left(\frac{1}{\omega^2-\mu^2-l^2}+\frac{1}{l^2+\mu^2}\right)\int d\Omega(\boldsymbol{\sigma}_0\cdot\mathbf{l})=0. \quad (24)$$

The failure of this term to contribute can be explained on elementary grounds; the direct coupling term in the hamiltonian $fe\boldsymbol{\sigma}\cdot\mathbf{A}\phi$ leads to emission of mesons in s-states. Then, since the meson is coupled with the nucleon only when in P-states (through the $\boldsymbol{\sigma}\cdot\nabla\phi$ interaction), such s-state mesons cannot react on the spins.

To evaluate the integral

$$\int d^3l\frac{(\boldsymbol{\sigma}_0\cdot\mathbf{l})(\boldsymbol{\varepsilon}\cdot\mathbf{l})\boldsymbol{\sigma}_0\cdot(\mathbf{l}-\boldsymbol{\nu})U_lU_{l-\nu}}{(\omega^2-l^2-\mu^2)[(\mathbf{l}-\boldsymbol{\nu})^2+\mu^2]} \quad (25)$$

we note that it must be proportional to the scalar $(\boldsymbol{\sigma}_0\cdot\boldsymbol{\varepsilon})(\boldsymbol{\sigma}_0\cdot\boldsymbol{\nu})$ since other scalar products vanish, due to the orthogonality of $\boldsymbol{\varepsilon}$ and $\boldsymbol{\nu}$. We can, therefore, approximate the integral by

$$(\boldsymbol{\sigma}_0\cdot\boldsymbol{\nu})\int d^3l(\boldsymbol{\sigma}_0\cdot\mathbf{l})(\boldsymbol{\varepsilon}\cdot\mathbf{l})/l^4=\frac{4}{3}\pi(\boldsymbol{\sigma}_0\cdot\boldsymbol{\nu})(\boldsymbol{\sigma}_0\cdot\boldsymbol{\varepsilon})l_{\max}. \quad (26)$$

In collecting these results, we observe that since the equations of motion for the spins can be written

$$-i\omega\boldsymbol{\sigma}_1=\boldsymbol{\sigma}_0\times\mathbf{F}_1+\boldsymbol{\sigma}_1\times\mathbf{F}_0,\quad -i\omega\boldsymbol{\tau}_1=\boldsymbol{\tau}_0\times\mathbf{G}_1+\boldsymbol{\tau}_1\times\mathbf{G}_0, \quad (27)$$

we can drop terms in $\mathbf{F}_1$ and $\mathbf{G}_1$ proportional to $\boldsymbol{\sigma}_0$ and $\boldsymbol{\tau}_0$ respectively. We introduce the new constant,

$$a=\tfrac{1}{2}\pi l_{\max}, \quad (28)$$

where a is the cut-off radius and define

$$N=\tfrac{2}{3}(f^2/\mu^2a)(\omega^2-a\mu^3+iaq^3). \quad (29)$$

The equations for the spin motion then are

$$\begin{aligned}i\omega\boldsymbol{\sigma}_1&=N\boldsymbol{\sigma}_0\times\boldsymbol{\sigma}_1+2e(4\pi)^{\frac{1}{2}}(a+b\boldsymbol{\tau}_0\cdot\mathbf{k})\boldsymbol{\sigma}_0\times\mathbf{H}_0\delta(\omega-\nu)\\ i\omega\boldsymbol{\tau}_1&=N\boldsymbol{\tau}_0\times\boldsymbol{\tau}_1+2e(4\pi)^{\frac{1}{2}}b\boldsymbol{\tau}_0\times\mathbf{k}\boldsymbol{\sigma}_0\cdot\mathbf{H}_0\delta(\omega-\nu)\\ &\quad+\frac{4}{3}e\frac{f^2(4\pi)^{\frac{1}{2}}}{\mu^2a}\boldsymbol{\tau}_0\times(\boldsymbol{\tau}_0\times\mathbf{k})\boldsymbol{\sigma}_0\cdot\boldsymbol{\nu}\boldsymbol{\sigma}_0\cdot\boldsymbol{\varepsilon}\delta(\omega-\nu).\end{aligned} \quad (30)$$

At this point we can introduce some simplification of these expressions. An examination of the derivation of these equations shows that the term in the equation for $\boldsymbol{\tau}_1$ proportional to $\boldsymbol{\tau}_0\times(\boldsymbol{\tau}_0\times\mathbf{k})$ is the result of the virtual emission and reabsorption of mesons which, while virtual, interact with the electromagnetic field. Since we have taken into account of such processes by the introduction of a moment interaction proportional to $\boldsymbol{\mu}\cdot\mathbf{H}$ (the anomalous moment part of which is due to such virtual meson interactions with the electromagnetic field), we can consistently neglect the contribution of this term, assuming that it has been properly represented by the anomalous moment term. An explicit calculation with this term included shows that the contribution in the weak coupling limit although of the same form as that due to the anomalous moments is actually considerably smaller. This is the result of the classical average over the unit vector $\boldsymbol{\tau}_0$ which appears to a high power in this term. Rigorously some sort of expectation value should replace the averaging process. Hence, one might conclude that dropping this term and using the anomalous moments may indeed give a better approximation to the correct cross sections.

We further simplify the equations if we neglect $a/b=(\gamma_P+\gamma_N)/(\gamma_P-\gamma_N)=+0.19$. We can solve for $\boldsymbol{\sigma}_1$ by making the obvious substitution,

$$\boldsymbol{\sigma}_1=A\boldsymbol{\sigma}_0\times\mathbf{H}_0+B\boldsymbol{\sigma}_0\times(\boldsymbol{\sigma}_0\times\mathbf{H}_0), \quad (31)$$

i.e., expressing $\boldsymbol{\sigma}_1$ which is orthogonal to $\boldsymbol{\sigma}_0$ by a linear combination of two mutually orthogonal vectors which are also orthogonal to $\boldsymbol{\sigma}_0$. We find

$$\boldsymbol{\sigma}_1=4be\frac{\pi^{\frac{1}{2}}\boldsymbol{\tau}_0\cdot\mathbf{k}}{N^2-\nu^2}i\nu\left[\boldsymbol{\sigma}_0\times\mathbf{H}_0+\frac{N}{i\nu}\boldsymbol{\sigma}_0\times(\boldsymbol{\sigma}_0\times\mathbf{H}_0)\right]. \quad (32)$$

Similarly

$$\boldsymbol{\tau}_1=4be\frac{\pi^{\frac{1}{2}}\boldsymbol{\sigma}_0\cdot\mathbf{H}_0}{N^2-\nu^2}i\nu\left[\boldsymbol{\tau}_0\times\mathbf{k}+\frac{N}{i\nu}\boldsymbol{\tau}_0\times(\boldsymbol{\tau}_0\times\mathbf{k})\right]. \quad (33)$$

To find the outgoing amplitude of the meson wave, we have at large distances

$$\phi_1(r)=\frac{1}{(2\pi)^{\frac{3}{2}}}\int d^3l\phi_{1l}\exp(i\mathbf{l}\cdot\mathbf{r})$$
$$=\frac{e^{iqr}}{4\pi r}(4\pi)^{\frac{1}{2}}\frac{f}{\mu}\bigg[i(\boldsymbol{\sigma}_0\cdot\mathbf{q}\boldsymbol{\tau}_1+\boldsymbol{\sigma}_1\cdot\mathbf{q}\boldsymbol{\tau}_0)-(4\pi)^{\frac{1}{2}}e\boldsymbol{\sigma}_0\cdot\boldsymbol{\varepsilon}\boldsymbol{\tau}_0\times\mathbf{k}$$
$$+2e(4\pi)^{\frac{1}{2}}\frac{\boldsymbol{\varepsilon}\cdot\mathbf{q}\boldsymbol{\sigma}_0\cdot(\mathbf{q}-\boldsymbol{\nu})}{(\mathbf{q}-\boldsymbol{\nu})^2+\mu^2}\boldsymbol{\tau}_0\times\mathbf{k}\bigg] \tag{34}$$

where $q=(\nu^2-\mu^2)^{\frac{1}{2}}$ is the meson momentum.

The differential cross section then is given by

$$d\sigma/d\Omega=\langle|A_s|^2\rangle q/\nu\epsilon^2 \tag{35}$$

where A_s is the coefficient of e^{iqr}/r in Eq. (34) and $\langle\ \rangle$ denotes an average over spin directions and over the polarization of the gamma-ray. In carrying out this average, we note that in general

$$\begin{aligned}\langle(\boldsymbol{\varepsilon}\cdot\mathbf{A})^2\rangle&=(A^2-(\mathbf{A}\cdot\boldsymbol{\nu})^2/\nu^2)\epsilon^2/2\\ \langle\boldsymbol{\sigma}_0\cdot\mathbf{A}\boldsymbol{\sigma}_0\cdot\mathbf{B}\rangle&=\tfrac{1}{3}\mathbf{A}\cdot\mathbf{B}\\ \langle\boldsymbol{\sigma}_0\cdot\mathbf{A}\boldsymbol{\sigma}_0\cdot\mathbf{B}\boldsymbol{\sigma}_0\cdot\mathbf{C}\boldsymbol{\sigma}_0\cdot\mathbf{D}\rangle&\\ =(\mathbf{A}\cdot\mathbf{B}\mathbf{C}\cdot\mathbf{D}&+\mathbf{A}\cdot\mathbf{C}\mathbf{B}\cdot\mathbf{D}+\mathbf{A}\cdot\mathbf{D}\mathbf{B}\cdot\mathbf{C})/15\end{aligned} \tag{36}$$

and that the average of expressions containing an odd number of factors of $\boldsymbol{\sigma}_0$ or $\boldsymbol{\tau}_0$ is zero. Carrying out the indicated averages, we find

$$\begin{aligned}&(d\sigma/d\Omega)[(1/9)(q/\nu)(f^2e^2/\mu^2)]^{-1}\\ &=\left|\frac{2b\nu q}{N^2-\nu^2}\right|^2\bigg\{[2(1+\sin^2\theta)\sin^2\varphi\\ &+(1+2\cos^2\varphi)(1+\cos^2\theta)]\nu^2/10\\ &+[2(1+\sin^2\theta)(1+7\cos^2\varphi)\\ &+2(2-3\sin^2\theta)(1-3\cos^2\varphi)\\ &+(1+2\cos^2\varphi)(2+7\sin^2\theta)]|N|^2/50\\ &+\sin^2\varphi\left[1-\frac{2q^2\sin^2\theta}{((\mathbf{q}-\boldsymbol{\nu})^2+\mu^2)^2}\right]\bigg\}\end{aligned} \tag{37}$$

where θ is the angle between the photon and meson directions and φ is the angle in charge space between the meson charge vector and the 3-axis (φ is equal to 0 degrees for neutral mesons and to 90 degrees for charged).

Reinserting the expression for N, and setting[10]

$$\tfrac{2}{3}f^2/\mu^2a=1/\nu_0$$

we have approximately

$$|N^2-\nu^2|^2=(1-\nu_0^2/\nu^2)^2+4(aq^3/\nu_0^2)^2$$

where we have dropped some small terms depending on the cut-off radius. We also insert the definition of b. The result for neutrals is

$$\frac{d\sigma}{d\Omega}\left[\frac{9\nu\mu^2}{qf^2e^2}\right]^{-1}=\frac{q^2}{\mu^2}\left(\frac{\gamma_N-\gamma_P}{2}\right)^2\cdot\frac{3/10(1+\cos^2\theta)+\nu^2/\nu_0^2(14+49\sin^2\theta)/50}{(1-\nu_0^2/\nu^2)^2+4(aq^3/\nu_0^2)^2} \tag{38}$$

and for charged (adding two identical contributions corresponding to the meson charge vector lying in the x- or y-direction)

$$\frac{d\sigma}{d\Omega}\left[\frac{9\nu\mu^2}{2qf^2e^2}\right]^{-1}=\frac{q^2}{\mu^2}\left(\frac{\gamma_N-\gamma_P}{2}\right)^2\cdot\frac{(5-\cos^2\theta)/10+(\nu^2/\nu_0^2)(8+3\sin^2\theta)/50}{(1-\nu_0^2/\nu^2)^2+4(aq^3/\nu_0^2)^2}+1-\frac{2q^2\sin^2\theta}{[(\mathbf{q}-\boldsymbol{\nu})^2+\mu^2]^2}. \tag{39}$$

If, in the expression for charged mesons, we neglect the first term, we have

$$\frac{d\sigma}{d\Omega}=\frac{2}{9}\frac{f^2}{\mu^2}e^2\left[1-\frac{2q^2\sin^2\theta}{[(\mathbf{q}-\boldsymbol{\nu})^2+\mu^2]^2}\right]\frac{q}{\nu}. \tag{40}$$

This is equivalent to the perturbation result[11] except for a factor 2/9 which results from the classical averaging over the spin variables.

We shall in addition at this point compare this result with that of Fujimoto and Miyazawa.[4] They find for the largest contribution to the charged production (from the meson charge interaction) a result identical with ours. For neutral mesons, they give

$$\frac{d\sigma}{d\Omega}=\frac{1}{144}\frac{e^2f^2}{\mu^2}\frac{q^3}{4\nu\nu_0^2}\frac{2+3\sin^2\theta}{(1-\nu/\nu_0)^2+0.114(aq_0^3/\nu_0^2)^2}. \tag{41}$$

We can put this in the form of our result if we make the approximation

$$\begin{aligned}(1-\nu/\nu_0)^2&=\tfrac{1}{4}(1-\nu/\nu_0)^2(1+\nu_0/\nu_0)^2\\ &\cong\tfrac{1}{4}[1-(\nu/\nu_0)^2]^2\end{aligned} \tag{42}$$

for ν/ν_0 not far from one. Since most of the contribution to the cross section comes from this region, the approximation is a fairly good one. Then we have

$$\frac{d\sigma}{d\Omega}=\frac{1}{144}\frac{e^2f^2}{\mu^2}\frac{q^3}{\nu\nu_0^2}\frac{2+3\sin^2\theta}{[1-(\nu/\nu_0)^2]^2+0.456(aq_0^3/\nu_0^2)^2} \tag{43}$$

which is to be compared with our result

$$\frac{d\sigma}{d\Omega}=\frac{0.459}{144}\frac{e^2f^2}{\mu^2}\frac{q^3}{\nu\mu^2}\frac{2+(7/6)\sin^2\theta(\nu/\nu_0)^2}{[1-(\nu/\nu_0)^2]^2+4(aq_0^3/\nu_0^2)^2} \tag{44}$$

[10] The energy ν_0 is the usual expression for the isobar spacing.

[11] Leslie L. Foldy, Phys. Rev. 76, 1 (1949).

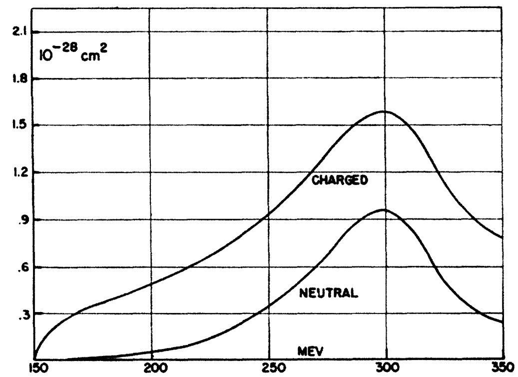

FIG. 1. Variation with photon energy of the total cross section for meson production. The isobar energy is 250 Mev and the cut-off radius is taken to be $(4/3)(\hbar/Mc)$, corresponding to the coupling constant $\frac{1}{6}$. The energies have been related to those in the center-of-momentum system by the proper Lorentz transformation.

where we have inserted the numerical values of γ_N and γ_P. These two results, Eq. (43) and Eq. (44), are then clearly of the same form and of the same magnitude. The difference in the coefficients of the damping term $(aq_0^3/\nu_0^2)^2$ appears to be due to a different approximation in the two calculations in the introduction of the cut-off radius, and not to an essential difference in the methods.

To examine the physical consequences of our results, we first simplify them by averaging them over angles and inserting the numerical values of the various constants, taking the cut-off radius as the nucleon compton wavelength. The cross sections then are

$$\sigma[2.49\times10^{-29}\text{ cm}^2\, qf^2/\nu]^{-1}$$
$$=\frac{(0.400+0.933\nu^2/\nu_0^2)q^2/\mu^2}{(1-\nu^2/\nu_0^2)^2+0.0816(q^3/\mu\nu_0^2)^2},\text{ neutral}$$
$$=\frac{(0.933+0.400\nu^2/\nu_0^2)q^2/\mu^2}{(1-\nu^2/\nu_0^2)^2+0.0816(q^3/\mu\nu_0^2)^2}+16.1,\text{ charged.}\quad(45)$$

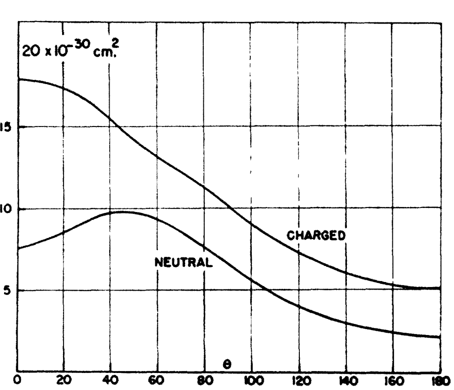

FIG. 2. Differential cross section in the laboratory system at 300-Mev photon energy. The peaking forward is due to the transformation from the center-of-momentum system to the laboratory system.

We can easily obtain the cross sections in the weak coupling $(\nu^2/\nu_0^2\ll1)$ and the strong coupling $(\nu^2/\nu_0^2\gg1)$ limit. We find

$$\sigma(2.49\times10^{-29}qf^2/\nu\text{ cm}^2)^{-1}$$
$$\left.\begin{aligned}&=0.400q^2/\mu^2,\text{ neutral}\\&\quad 0.933q^2/\mu^2+16.1,\text{ charged}\end{aligned}\right\}\text{weak coupling}$$
$$\left.\begin{aligned}&=0.933q^2\nu_0^2/\mu^2\nu^2,\text{ neutral}\\&\quad 16.1,\text{ charged}\end{aligned}\right\}\text{strong coupling.}\quad(46)$$

To investigate the validity of these limits, we can determine the numerical consequences of the two approximations at an energy at which the cross sections have been determined experimentally (for example, at 250 Mev in the c.m. system). The predicted results are

$$\left.\begin{aligned}&\sigma(\text{charged})\ll0.451\times10^{-28}\text{ cm}^2\\&\sigma(\text{neutral})\ll0.212\times10^{-30}\text{ cm}^2\end{aligned}\right\}\text{weak coupling}$$
$$\left.\begin{aligned}&\sigma(\text{charged})\gg0.402\times10^{-28}\text{ cm}^2\\&\sigma(\text{neutral})\ll1.47\times10^{-30}\text{ cm}^2\end{aligned}\right\}\text{strong coupling.}\quad(47)$$

These are to be compared with the observed cross sections of approximately 10^{-28} cm² for both charged and neutral mesons.[1] The charged cross section indicates that the coupling certainly cannot be considered as weak and that actually we are somewhat into the strong coupling domain. The smallness of the predicted neutral cross section in the two limits, in contradiction with experiment, is, in addition, a strong indication that the coupling is neither strong nor weak.

We can see, however, from the expressions for the cross sections that the neutral cross section can become large in the vicinity of

$$1-\nu^2/\nu_0^2=0$$

or, in terms of the original definition of ν_0,

$$\tfrac{2}{3}f^2/\mu^2a=1/\nu\quad(48)$$

where ν is the frequency of the radiation. If this condition is satisfied, the spin of the nucleon resonates to the imposed frequency and we have a "nucleon isobar" which is unstable against the decay into neutral or charged mesons. For example, if we assume that this condition is exactly satisfied at the energy of 250 Mev, then the cross sections are

$$\begin{aligned}\sigma&=0.869\times10^{-28}\text{ cm}^2\quad(\text{neutral})\\&=1.271\times10^{-28}\text{ cm}^2\quad(\text{charged}).\end{aligned}\quad(49)$$

These cross sections are, therefore, roughly equal at this energy and approximately equal to the observed cross sections at 10^{-28} cm². Characteristic energy and angular spectra are given in Figs. 1 and 2 on the basis of these results. The rapid drop of the neutral cross section on both sides of the resonance should be easily detected experimentally. The rapid rise of the neutral cross section with energy at energies below the "isobar frequency" has indeed been observed by Panofsky and Steinberger.[1]

III. CONSEQUENCES FOR OTHER PROCESSES

The existence of an unstable nucleon isobar will lead to anomalies in the scattering of mesons by nucleons. Pauli[8] gives for the scattering of neutral pseudoscalar mesons by nucleons the expression for the total cross section

$$\sigma=\frac{8}{9}\frac{f^4}{\mu^4}\frac{q^4}{\nu^2}\frac{1+(\nu/\nu_0)^2}{[1-(\nu/\nu_0)^2]^2+4(aq^3/\nu_0\nu)^2} \tag{50}$$

where ν_0 has been defined as above. With the values of the cut-off radius and ν_0 used in Sec. II, the variation of this cross section with energy is given in Fig. 3. The scattering of charged mesons will also have the same general behavior.

Nucleon-nucleon scattering will also show an anomaly at energies in the laboratory system of twice the resonance frequency, i.e., about 560 Mev. Since the resonance is very broad, nonstatic effects of some importance might also be expected to occur at considerably lower energies, although a quantitative estimate of the effect is difficult to make.

IV. CONCLUSIONS

We have made an application of classical theory of the problem of pseudoscalar meson production by photons. A comparison of the results with the experiments of Steinberger and Panofsky[4] suggests an explanation of the large cross section for neutral mesons in terms of the existence of unstable nucleon isobars with an excitation of approximately 200 to 300 Mev. We find, however, that this condition for the isobar energy implies that the strong coupling condition[8]

$$\tfrac{2}{3}(f^2/\mu a)\gg 1 \tag{51}$$

is not satisfied. It also follows therefore, that the classical approximations are only of very approximate validity since the classical treatment of the spin variables is valid only for large quantum numbers, i.e., in the region of high isobar excitation. In addition, certain other ambiguities due to the classical treatment of the

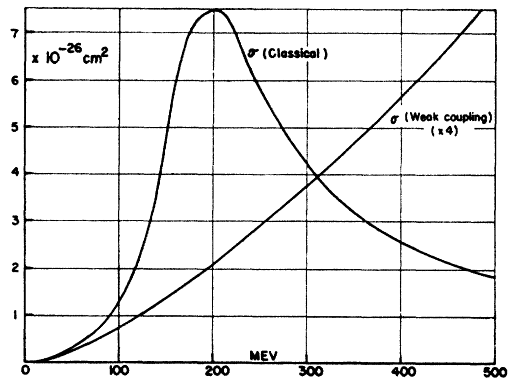

FIG. 3. Total cross section for the scattering of neutral mesons by nucleons. The meson energy in the laboratory system is indicated. The parameters are the same as in Figs. 1 and 2. The weak coupling approximation to the cross section has been multiplied by four.

isotopic spin are present, resulting from the average over initial isotopic spin directions. The charged cross section is thus some kind of an average for the production of π^+ and π^- mesons from protons and neutrons, respectively.

In addition we wish to emphasize that for pseudoscalar theory a fairly unambiguous nonrelativistic approximation exists only for pseudovector coupling, i.e.,

$$f\psi^*(\boldsymbol{\sigma}\cdot\nabla\varphi-\gamma_5\partial\pi/\partial t)\psi\rightarrow fU(\mathbf{r})\boldsymbol{\sigma}\cdot\nabla\varphi \tag{52}$$

where $U(\mathbf{r})$ is the nucleon source density. It is difficult to obtain a nonrelativistic approximation to the pseudoscalar coupling

$$f\psi^*\gamma_4\gamma_5\psi \tag{53}$$

when the coupling is strong. Our conclusions therefore relate only to the pseudovector coupling form of the theory.

Finally we wish to re-emphasize the crudity of the classical model for the treatment of this problem; the exact results are not to be taken too seriously. We feel that the importance of these results is only in the qualitative ideas introduced and the possibility of using them to get a qualitative agreement with experiment.

We are indebted to Professor J. R. Oppenheimer and to Dr. Abraham Pais for several interesting discussions and helpful comments on this subject.