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Neutron Thresholds from the Reactions $T^3(p,n)He^3$, $Li^7(p,n)Be^7$, $Be^9(d,n)^*B^{10}$, and $O^{16}(d,n)F^{17}$

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Techniques are described for observing low energy (15–60 kev) "threshold" neutrons in the presence of fast neutrons. The threshold of the $T^3(p,n)He^3$ reaction was found to be 1020.3 kev, which gives a Q-value of the reaction equal to -764.7 kev and a neutron-hydrogen mass difference of 783.2 kev. Interpretation of data near the threshold of the $Li^7(p,n)Be^7$ reaction gives indication of a level in Be^8 near 18.90 Mev. Three neutron thresholds have been observed in the reaction $Be^9(d,n)^*B^{10}$ at deuteron energies of 920, 985, and 1916 kev. The corresponding Q-values are -752, -805, and -1566 kev; these give excited states in B^{10} at 5.110, 5.163, and 5.924 Mev. The threshold of the reaction $O^{16}(d,n)F^{17}$ was determined to be 1836 kev, giving a Q value of -1631 kev and a mass of F^{17} equal to 17.007506.

INTRODUCTION

NEUTRON thresholds for endothermic reactions have been investigated in the case of a number of nuclear reactions.[1] The absolute determination of these threshold energies give the Q-values of the reactions and, therefore, the mass differences between the initial and final systems. The accuracy of establishing the position of the threshold is limited only by the absolute bombarding energy scale, which is known to about a tenth of one percent. This is in contrast to an accuracy of the order of one percent in measuring the Q-values of exothermic neutron emitting reactions.

The experiments reported herein concern the neutron thresholds from proton bombardment of T^3 and Li^7 and deuteron bombardment of Be^9 and O^{16}. All of these reactions except deuterons on Be^9 are endothermic and thus cannot occur at all below a certain reaction threshold. The deuteron disintegration of Be^9 is highly exothermic, but the residual nucleus may be left in a higher excited state than would be possible from the energy available from the reaction energy. One would expect such "threshold neutrons" (15–60 kev) to exhibit threshold characteristics similar to the endothermic reaction thresholds except that the sharpness of the threshold would depend on the width of the excited state of the residual nucleus.

The experimental problem of observing new "thresholds" is complicated somewhat by the presence of the faster neutrons from lower energy states and the ground state. These faster neutrons are essentially background as far as the new group of threshold neutrons is concerned, and in most cases outnumber them. If one is to establish accurately the position of the threshold, he must discriminate strongly against the fast neutrons in favor of the slower ones.

The application of threshold techniques to these new groups of slower neutrons presents a method capable of resolving closely spaced energy levels in the residual nucleus because the resolution is limited primarily by the target thickness, which for the very light elements can be reduced to a few kev and still give satisfactory intensity. Essentially, the same accuracy of establishing the position of the threshold can be attained as in the case of the endothermic reaction threshold.

The usefulness of the method of neutron thresholds is enhanced by the cone effect of the neutron emission which gives a very favorable factor in the forward direction just above threshold.[1] This cone effect is discussed in the Appendix.

According to theory[2] the cross section for emission of a neutron near threshold is proportional to $E^{l+\frac{1}{2}}$, where E is the neutron energy in the center-of-mass system and l is the angular momentum of the neutron, and so one might expect to be able to determine the angular momenta of the neutrons by examination of the experimental yield curve. However, because of the very high centrifugal barrier[3] in the region just above threshold for neutrons with nonzero angular momentum, there will be at most a small contribution from such neutrons unless neutrons with zero angular momentum were forbidden or made improbable by selection rules. If a group of neutrons, known by other methods, fails to show a threshold, it might be taken as an indication that neutrons with zero angular momentum are forbidden by the selection rules.

APPARATUS

The bombarding particles used in these experiments were accelerated by the Rice Institute pressure Van de Graaff generator, which was stabilized by a magnetically focused beam of electrons going down the vacuum tube in the opposite direction to the positive ions.[4]

For all the absolute voltage determinations and for most of the excitation curves, the slit width of the magnetic beam analyzer was 0.005 inch which allowed

* A preliminary report of these results has been given by Bonner, Butler, and Risser, Phys. Rev. 79, 240(A) (1950), and in Proceedings of the Harwell Nuclear Physics Conference (September, 1950), unpublished.

[1] R. F. Taschek and A. Hemmendinger, Phys. Rev. 74, 383 (1948); Hanson, Taschek, and Williams, Revs. Modern Phys. 21, 635 (1949); Bonner, Evans, and Hill, Phys. Rev. 75, 1398 (1949); Richards, Smith, and Browne, Phys. Rev. 80, 524 (1950); H. B. Willard and W. M. Preston, Phys. Rev. 81, 480 (1951).

[2] E. P. Wigner, Phys. Rev. 73, 1002 (1948).

[3] Feshbach, Peaslee, and Weisskopf, Phys. Rev. 71, 145 (1947).

[4] S. J. Bame and L. M. Baggett, Rev. Sci. Instr. 20, 839 (1949).

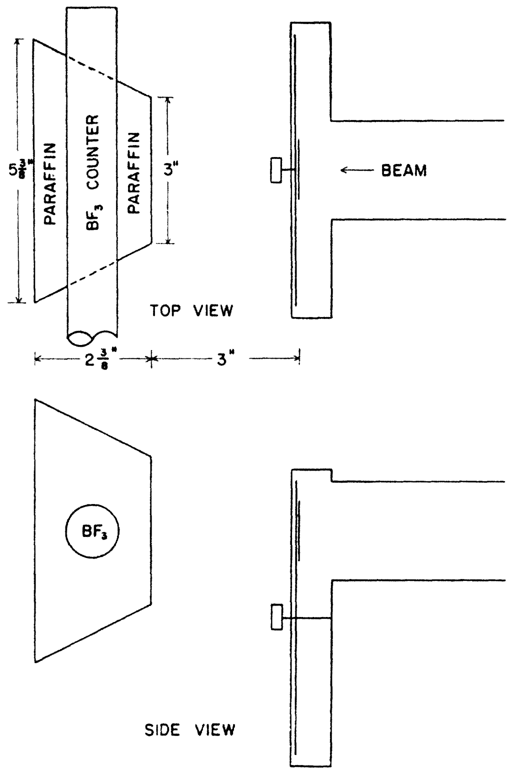

FIG. 1. Target-counter arrangement for 27° geometry.

an energy spread of 0.08 percent in the incident beam. For the Be^9 excitation curve below the 920-kev threshold and above the 985-kev threshold, the slit width was 0.012 inch.

In order to obtain accurate data from thin targets which are bombarded by deuterons, it was necessary to take steps to reduce the background. A deflecting magnet was placed across the vacuum tube about one meter before the analyzing magnet. This magnet separated the beam into its various mass components, while adjustable quartz plates on each side of the beam were moved in to stop the unwanted portions of the beam and to permit only the desired mass component to pass through to the analyzing magnet.

The ion source of the electrostatic accelerator was supplied with either hydrogen or deuterium by means of a $\frac{1}{8}$-in. Saran tube going from the storage bottle outside the accelerator tank up to the palladium leak inside the central electrode. When the absolute value of the deuteron-neutron thresholds in terms of the standard calibration reactions using protons as bombarding particles was determined, it was necessary to switch from protons to deuterons, or vice versa, with as little delay as possible.

The target holder was a brass disk capable of holding five targets simultaneously and was contained in a short brass cylinder as shown in Fig. 1. Target positions could be changed by rotating the target holder from outside the vacuum system by means of a knob connected to the shaft passed through a Teflon seal to the vacuum system.

The background counting rate was determined by rotating into the path of the beam a silver blank and recording the counting rate in the same manner as with a target.

COUNTING TECHNIQUE

Various materials, shapes, sizes, and orientations of moderators around a BF_3 counter were tried in an effort to attain an optimum geometry, the optimum conditions being (1) a good counting rate and (2) the highest ratio of counts from threshold neutrons (15–60 kev) to that from fast neutrons. A bare BF_3 counter satisfies the second condition, but gives a very low counting rate. Cylindrical paraffin moderators coaxial with the counter tube were tried, but were abandoned in favor of a truncated cone of paraffin because of better defined boundary of the "counting cone" in the case of a truncated cone. A well-defined counting cone is important when we try to compare the experimental curve with theory. The "cone effect" near threshold caused by the moving center of mass, is discussed in the Appendix. The final version of the counting arrangement is shown in Fig. 1; thin cadmium sheets completely cover the paraffin moderator.

A graphite moderator was tried with the hope that it would give a very high ratio of threshold neutron counts to fast neutron counts; but it actually gave about the same ratio as the paraffin moderator, and the total counting rate was so low as to make the graphite moderator of little value. A heavy water moderator was tried in order to avoid neutron capture by the moderator as in the case of paraffin, but it too gave a lower counting rate than the paraffin and gave no higher ratio of "threshold" to fast neutrons than did the paraffin. The arrangement in Fig. 1 is hereinafter referred to as the "27° geometry."

$T^3(p,n)He^3$

It was thought that this reaction would be a particularly good one for testing the counting technique, because the high center-of-mass velocity confines the total neutron yield to a cone of half-angle 27° until a bombarding energy of 24 kev above threshold is reached. This reaction provided a region (large compared with target thickness) over which the shape of the rising slope of the curve could be critically compared with a calculated curve. This is important to determine whether or not the effective cone of the counter is the same as the geometric cone. Furthermore, the reaction is almost ideal as regards certain assumptions which are made in calculating a theoretical curve. Because of the low coulomb barrier (about 400 kev) and because no excited states of the intermediate nucleus He^4 have been reported in the energy range involved here, one would expect the cross section for formation of the intermediate nucleus to remain practically constant over a small range of bombarding energies. In calculating the geometrical effect of the neutron cone on the counting rate, it was assumed that the neutrons were emitted isotropically in the center-of-mass system. Taschek *et al.*[5] have shown that the neutrons from this reaction show approximate symmetry in the center-of-mass system near the threshold.

The tritium targets were made by evaporating thin deposits of zirconium onto $\frac{1}{2}$-in. diameter polished tungsten disks and allowing the zirconium to take up tritium by first heating it, then cooling it in an atmosphere of the gas.[6]

The circles in Fig. 2 give the experimental neutron counting rates for a target which was 40 micrograms of Zr per cm^2 thick (about 4 kev). The background counting rate was negligible. The counting rate rises smoothly until a bombarding energy of 15 kev about threshold is reached, after which the points fall rather slowly until 24 kev above threshold is reached. At this energy the neutron cone has opened up to 27°, and therefore the neutrons start missing the cone of the counter entirely at higher energies, and the yield falls more rapidly. Apparently then, the outer layer of paraffin between 21° and 27° (at 15 kev above threshold, the cone has opened up to 21°) is not so effective in contributing to the counting rate as the inner cone (less than 21°).

The curve in Fig. 2 represents the expected theoretical counting rate, assuming a counter cone of half-angle 21°, which appears to be approximately the effective cone of the counter. The curve was calculated by multiplying the geometrical effect by the square root of the neutron energy in the center-of-mass system and integrating the product curve over a 4-kev target thickness. Since both the solid angle (as a function of cone half angle) and the neutrons per unit solid angle are concentrated toward the outer edge of the cone near threshold, one would expect the experimental counting rate above 1035 kev to be somewhat higher than the calculated curve based on an effective cone of 21°.

Above 1060 kev the experimental points drop faster than the calculated curve; this can be explained qualitatively on the basis of the counter efficiency as a function of neutron energy. By making a rough extrapolation of the curves of Hanson and McKibben[7] for the counting efficiency of various sizes of paraffin cylinders as a function of neutron energy, it is estimated that the efficiency of the counter is almost flat up to an energy of about 100–200 kev and decreases for higher energies. The neutrons emitted just above threshold have an energy of about 60 kev in the laboratory system. At a bombarding energy of 1080 kev, the neutrons emitted in the forward direction have an energy of 200 kev in the laboratory system; and for a bombarding energy of 1120 kev, the maximum neutron energy is about 250 kev.

Experiments to determine accurately the threshold of this reaction were made with thin tritium targets. The absolute value of the threshold energy is based on the accurate work of Herb, Snowdon, and Sala,[8] who determined the voltages of the 873.5-kev resonance in the reaction $F^{19}(p,\alpha,\gamma)O^{16}$, the 993.3-kev resonance in $Al^{27}(p,\gamma)Si^{28}$, and the threshold for the reaction $Li^7(p,n)Be^7$ at 1882.2 kev to an over-all accuracy of about 0.1 percent.

To determine the value of the $T^3(p,n)He^3$ threshold, the beam analyzing magnet currents required for the fluorine and aluminum resonances were determined by bombarding a target of LiF about 0.4 microgram per cm^2 thick (about 100 volts thick for 900-kev protons) and a freshly cleaned disk of commercial 2*S* aluminum. The tritium threshold was then studied using the geometry of Fig. 1. For the range of magnet currents which were used in the calibration, the bombarding energy is given by the following equation:

$$E = AI^2 + B,$$

where E is the bombarding energy, I the magnet current, and A and B are constants determined by the two resonance calibration points. A typical run is shown in Fig. 3.

The half-width of the F^{19} resonance was about 4.3 kev; and the total rise of the thick target Al^{27} resonance took place in about 1.5 kev, thus confirming the value of the beam energy resolution determined by the setting of the resolution slit. The half-width of the Al^{27} resonance is known to be about 0.4 kev or less.

For a thick target yield above threshold, the (neutron yield)$^{\frac{2}{3}}$ is a linear function of bombarding energy. The position of the tritium threshold was determined by plotting the (neutron yield)$^{\frac{2}{3}}$ and extrapolating to the abscissa. This relation applies to the threshold data, since all points taken were less than 4 kev (target thickness) above the threshold. The values of the tritium threshold thus determined for five different

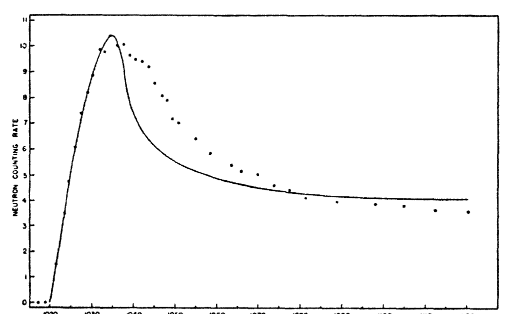

FIG. 2. Neutrons from the reaction $T^3(p,n)He^3$. Circles give experimental points using a counter half-angle of 27° and a 4-kev target. The curve gives the calculated values for an effective counter half-angle of 21°.

[5] Taschek, Jarvis, Hemmendinger, Everhart, and Gittings, Phys. Rev. **75**, 1361 (1949).
[6] A. B. Lillie and J. P. Conner, Rev. Sci. Instr. **22**, 210 (1951).
[7] A. O. Hanson and J. L. McKibben, Phys. Rev. **72**, 673 (1947).
[8] Herb, Snowdon, and Sala, Phys. Rev. **74**, 246 (1949).

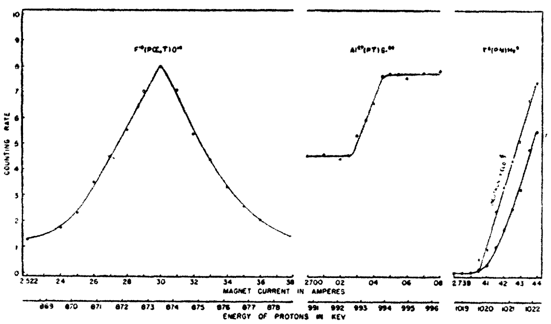

FIG. 3. Typical run on absolute determination of threshold energy for the reaction $T^3(p,n)He^3$. Targets used on this run: LiF (about 0.1 kev thick), thick Al disk, and a 4-kev tritium target.

runs are 1019.9, 1019.7, 1020.6, 1021.1, and 1020.3 kev. The average value is 1020.3 kev, with an estimated limit of error of 1.5 kev.

This value of the threshold agrees very well with that obtained by Taschek *et al.*[9] (1019 kev). When the average threshold value of 1020.3 kev is used, the Q-value of the reaction is -764.7 kev; and the neutron-hydrogen mass difference is found to be 783.2 ± 2 kev when 18.5 kev[10] is used as the end point of the beta-spectrum from T^3 and the neutrino rest mass is neglected.

$Li^7(p,n)Be^7$

This reaction has been investigated rather thoroughly by Taschek and Hemmendinger[1] and others. Breit and Bloch[11] have discussed the possibility of a resonance below threshold on theoretical considerations and on the basis of the cross section for the reaction obtained by Taschek and Hemmendinger. Their values of total cross section were constant from about 50 kev about threshold to about 190 kev above, indicating that either the probability of neutron emission is independent of bombarding energy in this range, or that the cross section for formation of the compound nucleus is a decreasing function of energy. Since the behavior of the cross-section curve below 23 kev above threshold was unknown, it was felt that by determining the trend of the cross-section curve closer to threshold, one might draw more definite conclusions as to the possibility of existence of an excited state in the Be^8 nucleus corresponding to a bombarding energy in the neighborhood of the threshold.

A target of LiF, 4 micrograms per cm^2 thick (about 600 volts for 1.9-Mev protons), was bombarded with protons from the threshold to 1940 kev. The geometry was the same as in Fig. 1 except the counter was moved back until the average solid angle subtended by the paraffin was a cone of half-angle 5°. Figure 4 shows the experimental data.

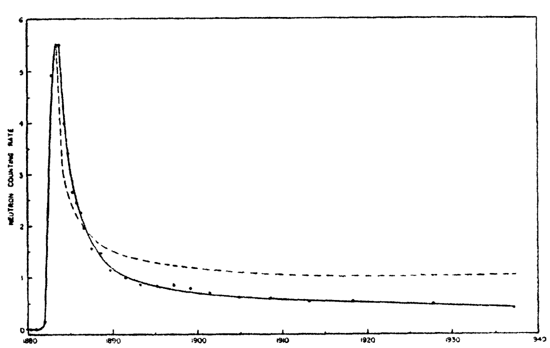

FIG. 4. Neutrons from the reaction $Li^7(p,n)Be^7$. Circles give experimental points using a counter half-angle of 5° (effective angle: 4°) and a 0.6-kev target of LiF. The broken curve represents the theoretical counting rate assuming an isotropic distribution of neutrons in the center-of-mass system and assuming that the cross section increases as the neutron velocity.

Since the outside layer of paraffin seemed to be rather ineffective in contributing to the counting rate in the $T^3(p,n)He^3$ reaction, the theoretical curve for the reaction was calculated using an effective angle of 4° (that is, the same "effective volume") instead of the geometrical angle of 5°. The calculated results using a 5° cone were found to be not appreciably different from the 4° curve, which is shown in Fig. 4.

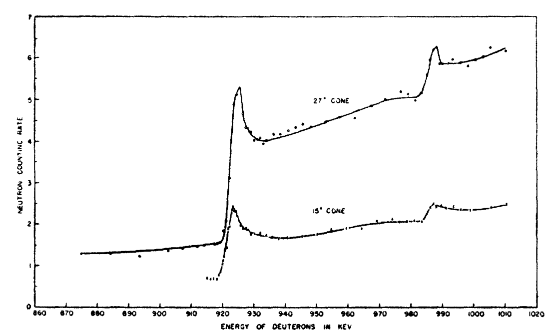

FIG. 5. Neutrons from the reaction $Be^9(d,n)^*B^{10}$ using counter half-angles of 27° and 15° and a 4-kev target.

Since the experimental points above 1890 kev drop faster than the theoretical curve, there is an indication that the cross section for formation of the compound nucleus is a decreasing function of energy in this interval. This is consistent with the results of Taschek and Hemmendinger at higher energies, and it lends support to the existence of a level in Be^8 near 18.90 Mev.[12]

$Be^9(d,n)^*B^{10}$

The neutrons from this reaction have been previously investigated by other methods,[13,14] and the group of

[9] Taschek, Argo, Hemmendinger, and Jarvis, Phys. Rev. **76**, 325 (1949).
[10] Hornyak, Lauritsen, Morrison, and Fowler, Revs. Modern Phys. **22**, 299 (1950).
[11] G. Breit and I. Bloch, Phys. Rev. **74**, 397 (1948).
[12] Calculated with Bainbridge's 1951 *Table of Masses*.
[13] T. W. Bonner and W. M. Brubaker, Phys. Rev. **50**, 308 (1936).
[14] Evans, Malich, and Risser, Phys. Rev. **75**, 1161 (1949).

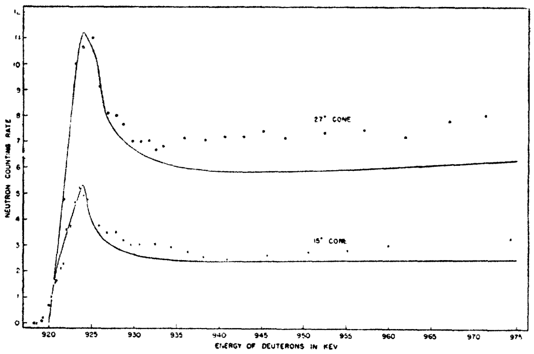

FIG. 6. "Threshold" neutrons from the reaction $Be^9(d,n)^*B^{10}$. Circles and crosses give data for counter cones of 27° and 15°. Experimental points show only "threshold" neutrons (fast neutrons have been subtracted) corrected for the cross section for formation of the compound nucleus. The curves represent the theoretical counting rates.

neutrons having a threshold at about 0.9 Mev reported.[14]

The targets were made by evaporating metallic beryllium onto silver disks. Targets which were used varied in weight from 8 to 50 micrograms per cm^2.

Figure 5 gives results obtained with an 8-microgram per cm^2 target, which is about 4 kev thick for 900-kev deuterons. Curves are given for 15° and 27° cones. The data at 15° were obtained by moving the counter and moderator back from the target until the average solid angle subtended by the paraffin was a cone of half-angle 15°.

Two thresholds are indicated—one at 920-kev bombarding energy, the other at 985 kev. The calculated Q values are -752 kev and -805 kev, respectively. These thresholds indicate a doublet energy level in the residual nucleus B^{10} at 5.110 Mev an d 5.163 Mev, calculated on the basis of a reaction Q-value of 4.358 Mev.[12] Since the curves exhibit sharp breaks at the thresholds, these states in B^{10} are quite narrow, probably 3 kev or less, although they usually break up into Li^6 and an alpha-particle.[15]

In order to determine how well a theoretical curve will fit the experimental data, it is necessary to correct the experimental data for the fast neutron counts and for the cross section for formation of the intermediate nucleus. The total yield was determined by the use of a flat energy response neutron detector[7] at 90° to the deuteron beam. The corrected data is given in Fig. 6.

The theoretical values shown in Fig. 6 are equal to the integrated effects over a 4-kev target of the product of the geometrical factors and the square root of the neutron energy in the center-of-mass system.

The calculated curves fall below the experimental points. This discrepancy can be explained in a number of ways. If one assumes a small percentage of p-neutrons, a very good fit can be made with the experimental points. However, the high centrifugal barrier for p-neutrons with such low energies makes it very unlikely that these neutrons make any appreciable contribution to the yield. A more likely explanation of this relatively small deviation is the difficulty in subtracting out accurately the effects of the fast neutrons.

Figure 7 gives data obtained at bombarding energies from 480 to 2130 kev. No thresholds were observed below 920 kev, although Ajzenberg,[16] using a bombarding energy of 3.4 Mev, found a group of neutrons corresponding to a level in B^{10} at 4.79 Mev. This group should have a threshold at about 525 kev. Since our data showed no threshold at this energy, a reasonable conclusion is that s-neutrons are forbidden at such low bombarding energies. s-deuterons are expected to be predominant, leaving the excited B^{11} intermediate nucleus with odd parity. If the 4.79-Mev state of B^{10} has even parity, then s-neutron emission is forbidden near threshold. Figure 7 shows that there is another

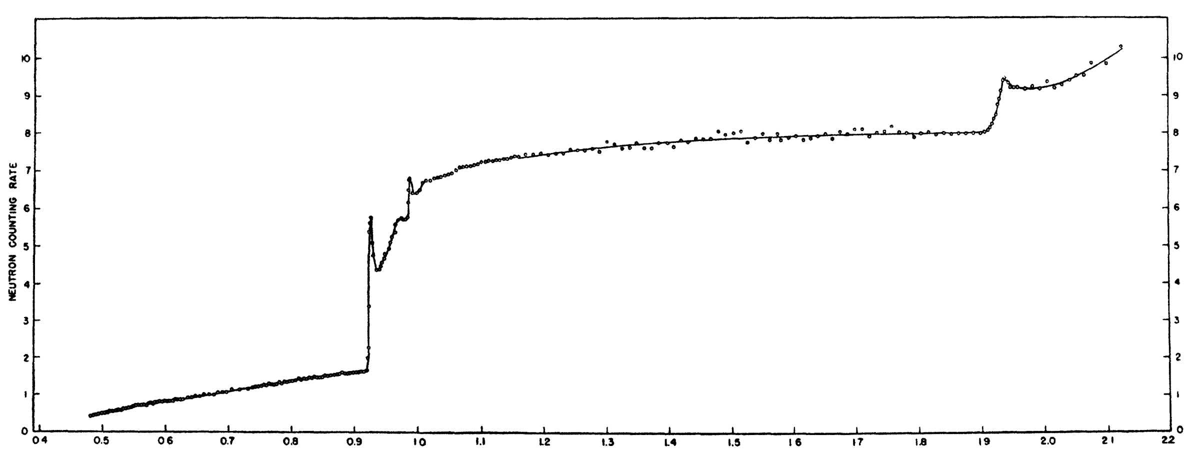

FIG. 7. Neutrons from the reaction $Be^9(d,n)^*B^{10}$ over a wide range of bombarding energies. Target thicknesses ranged from 8 to 50 micrograms per cm^2.

[15] Chao, Lauritsen, and Rasmussen, Phys. Rev. **76**, 582 (1949).
[16] Fay Ajzenberg, Phys. Rev. **82**, 45 (1951).

TABLE I. Data on the neutron "thresholds" in the reaction $Be^9(d,n)^*B^{10}$.

Be^9 threshold (kev)	Q-values (kev)	B^{10} energy levels (Mev)	Level spacing (Mev)
920±2	−752±2	5.110	
			0.053
985±2	−805±2	5.163	
			0.761
1916±4	−1566±4	5.924	

threshold at a bombarding energy of 1916 kev, giving a Q-value of −1566 kev and an energy level of 5.924 Mev in the B^{10} nucleus.

The 920-kev threshold was calibrated in terms of the 1882.2-kev threshold of the $Li^7(p,n)Be^7$ reaction using the mass-one beam, the 446-kev resonance in the $Li^7(p,\gamma)Be^8$ reaction, and the 485-kev resonance in the reaction $F^{19}(p,\alpha,\gamma)O^{16}$, using the mass-two beams. The respective results were 920, 919, and 920 kev.

The value of the 1916-kev threshold was measured by comparing it with the 873.5 kev $F^{19}(p,\alpha,\gamma)O^{16}$ resonance using the mass-two beam. The values of all three thresholds, their probable errors, Q-values, and corresponding B^{10} energy levels are given in Table I.

$O^{16}(d,n)F^{17}$

This reaction has been observed by Heydenburg and Inglis[17] and others, but the thin target excitation curve in the region just above threshold has not previously been investigated. A target of $Ca(OH)_2$, 85 micrograms per cm^2 thick (about 20 kev for 1.8-Mev deuterons), was prepared by evaporating metallic calcium onto a silver disk and allowing the calcium deposit to become oxidized and slaked by exposure to the air. The target was then bombarded with deuterons with energies from 1800 kev to 2077 kev and the counting rates recorded with the 27° geometry previously described. The background was relatively large because of the low cross section for this reaction.

The circles in Fig. 8 indicate the experimental values which were obtained using the 27° geometry, and the broken curve represents the values calculated by multiplication of the geometrical correction by the square root of the neutron energy in the center-of-mass system, and integration over a 20-kev target. The curves are quite dissimilar; their differences can be explained at least qualitatively. The reason that the experimental curve does not show the characteristic dip beyond the peak is believed to be incomplete oxidation and/or slaking of the layer of calcium next to the silver disk. This would cause the target density to taper off instead of having a sharp boundary. The reason for the major difference between the experimental and calculated curves lies in the variation of the cross section for the formation of the compound nucleus. The data of Heydenburg and Inglis[17] for the short-range proton yield from deuterons on O^{16} show a resonance at about 1.7 Mev, a minimum at 1.95 Mev,

[17] N. P. Heydenburg and D. R. Inglis, Phys. Rev. 73, 230 (1948).

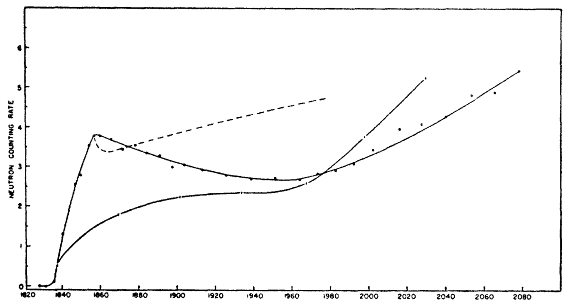

FIG. 8. Neutrons from the reaction $O^{16}(d,n)F^{17}$. Circles give experimental points using the 27° geometry and a 20-kev target of $Ca(OH)_2$. The broken curve represents the calculated counting rate assuming constant cross section for formation of the compound nucleus. The crosses give the total cross section for the reaction by measurement of positron activity of F^{17}. Units are arbitrary.

and another resonance at about 2.2 Mev. Similar resonance effects are evident in the neutron curve.

The absolute value of the threshold was determined by calibration of the beam analyzing magnet with the 873.5-kev resonance in F^{19} by means of the molecular hydrogen beam. Two runs were made, both giving a threshold value of 1836 kev. The error is estimated to be 3 kev.

When the value of 1836 kev is used for the reaction threshold, the Q-value is −1631 kev, which gives the mass of F^{17} as 17.007506±0.000018, when the values of deuteron and neutron masses given by Bainbridge[12] are used. The probable error in this mass is mostly due to the uncertainty in the masses of the neutron and the deuteron.

We are indebted to the Research Corporation and the joint ONR-AEC program for financial aid.

APPENDIX

Because of the center-of-mass velocity, the neutrons emitted just above threshold will be concentrated in a small cone in the forward direction in the laboratory system of coordinates. As the energy of the bombarding particle is increased above threshold, the cone of neutrons opens up until it reached 2π steradians. Any further increase in bombarding energy causes neutrons to be emitted throughout the entire solid angle of 4π steradians.

To derive a quantitative expression for the effect of this changing cone on the counting rate, we determine the fraction F of the neutrons that lie within the constant cone subtended by the counter as the total cone of neutrons opens up. If an isotropic distribution of neutrons in the center-of-mass system is assumed and the laws of conservation of energy and momentum are applied, the fraction F for the case when the center-of-mass velocity is greater than the neutron velocity in the center-of-mass system is given by

$$F=1-\cos\theta[1-(M_1M_4/M_2M_3)(E_1/\Delta E_1)\sin^2\theta]^{\frac{1}{2}},$$

where θ is the half-angle of the counter cone and the subscripts 1, 2, 3, 4 refer to the bombarding particle, bombarded nucleus, residual nucleus, and emitted particle.

When the center-of-mass velocity is less than the neutron velocity in the center-of-mass system, F is given by

$$F=\frac{1}{2}\left[1+\left(\frac{M_1M_4}{M_2M_3}\frac{E_1}{\Delta E_1}\right)^{\frac{1}{2}}\sin^2\theta-\cos\theta\left(1-\frac{M_1M_4}{M_2M_3}\frac{E_1}{\Delta E_1}\sin^2\theta\right)^{\frac{1}{2}}\right].$$

When the two velocities are equal, both forms reduce to

$$F=\sin^2\theta.$$