g' is a constant to be evaluated from experiment. Q is Q_1Q_2, an operator which changes two neutrons into two protons. Ω_e and Ω_N are both linear combinations of the five operators occurring in beta-theory; Ω_e operates on the electron wave functions while Ω_N operates on the nuclear wave functions. Ω_N is to be interpreted as $\Omega_{N1}+\Omega_{N2}$. δ is a matrix making $\delta\Psi^*$ a spinor. Writing

$$\Psi=\Sigma_m\, a_m\psi_m$$

and using the Jordan-Wigner anticommutation relation

$$a_m{}^*a_n{}^*+a_n{}^*a_m{}^*=0,$$

one obtains, as the perturbing hamiltonian for the creation of one electron into state m and another into state n,

$$2^{-\frac{1}{2}}g'\{(\psi_m{}^\dagger\Omega_e\delta\psi_n{}^*-\psi_n{}^\dagger\Omega_e\delta\psi_m{}^*)\Omega_N Q\}.$$

For the tensor and pseudovector interactions,

$$\psi_m{}^\dagger\Omega_e\delta\psi_n{}^*-\psi_n{}^\dagger\Omega_e\delta\psi_m{}^*=0$$

while for the scalar, vector, and pseudoscalar interactions,

$$\psi_m{}^\dagger\Omega_e\delta\psi_n{}^*-\psi_n{}^\dagger\Omega_e\delta\psi_m{}^*=2\psi_m{}^\dagger\Omega_e\delta\psi_n{}^*.$$

For these last three interactions, in the "allowed" approximation for plane wave electrons, the transition probabilities are:

Scalar: $w_S(\epsilon_m,\theta)d\epsilon_m d\theta$

$$=(\epsilon_m\epsilon_n+1)\left\{1-\frac{\alpha\cos\theta}{\epsilon_m\epsilon_n+1}\right\}F.$$

Vector: $w_V(\epsilon_m,\theta)d\epsilon_m d\theta$

$$=(\epsilon_m\epsilon_n+1)\left\{1+\frac{\alpha\cos\theta}{\epsilon_m\epsilon_n+1}\right\}F.$$

Pseudoscalar: $w_{PS}(\epsilon_m,\theta)d\epsilon_m d\theta$

$$=(\epsilon_m\epsilon_n-1)\left\{1-\frac{\alpha\cos\theta}{\epsilon_m\epsilon_n-1}\right\}F.$$

Here

$$F=\frac{4\pi^3(mc^2)^5}{c^6\hbar^7}g'^2\alpha\sin\theta\frac{\epsilon_m\epsilon_n}{(1+\epsilon_m)(1+\epsilon_n)}d\epsilon_m d\theta,$$

$$\alpha=(\epsilon_m{}^2-1)^{\frac{1}{2}}(\epsilon_n{}^2-1)^{\frac{1}{2}},\quad \epsilon_n=\epsilon_0-\epsilon_m.$$

ϵ_m is the energy of an electron in the state m, and ϵ_0 is the nuclear mass difference, both in units of mc^2. θ is the angle between the electrons.

Integrating over ϵ_m and assuming that $\epsilon_0=5$, $|H^2|=1$, and the half-life $=10^{21}$ years, one finds that

$$G'\equiv g'(1/mc^2)(mc/\hbar)^3=10^{-24}.$$

It is interesting that the Fermi G is 10^{-11} for the lightest mirror nuclei; it may not be inconsistent to say that G' is G^2.

A fair approximation for the half-life as a function of ϵ_0 for $\epsilon_0>3$ is

$$\tau=2\times10^{24}\text{ years}/[\epsilon_0{}^2(\epsilon_0{}^2-4)(\epsilon_0-2)].$$

The term considered here will also make possible the emission of two electrons followed by the absorption of one of these electrons with the emission of a neutrino—that is, a second-order contribution to single beta-decay. However, in the integration over the momentum of the virtual electron, the integrand goes to zero rapidly when the wavelength of the electron becomes of the order of the nuclear diameter. It can then be seen that this contribution to single beta-decay is perhaps 10^{-30} as large as the first-order process.

I should like to acknowledge very helpful discussions with Professors L. Wolfenstein and J. Ashkin.

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The Interpretation of Isomeric Transitions

STEVEN A. MOSZKOWSKI
Argonne National Laboratory and University of Chicago, Chicago, Illinois
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THEORETICAL calculations.—The half-life of nuclear isomeric states as a function of gamma-ray energy, nuclear radius, and type and multipole order of radiation has been calculated. As a simple model, it is assumed that the radiation is due to a single proton moving in the potential well of the rest of the nucleus. Radial integrals which appear are replaced by the proper power of the nuclear radius R. Because of cancellation of nuclear wave functions, this is expected to give a considerable overestimate of the values of the radial integrals and thus an underestimate of the half-life.

The half-life τ_γ for change of nuclear spin I from $L+\frac{1}{2}$ to $\frac{1}{2}$ is calculated to be

$$\tau_\gamma{}^{EL}=0.693\left[\frac{2(L+1)}{L(1\cdot3\cdot5\cdots2L+1)^2}\frac{e^2}{\hbar c}\omega\left(\frac{\omega}{c}R\right)^{2L}\right]^{-1},$$

for electric 2^L-pole radiation (EL), (1)

$$\tau_\gamma{}^{ML}=\tau_\gamma{}^{EL}\left[\frac{\hbar}{mcR}\left(\mu L-\frac{L}{L+1}\right)\right]^{-2},$$

for magnetic 2^L-pole radiation (ML). (2)

This is the theoretical half-life for gamma-emission[1] not including the effect of internal conversion. Here μ denotes magnetic moment due to intrinsic spin. The angular frequency of the radiation is ω. The mass of the proton is m. Equations (1) and (2) are nearly unchanged for any other transition for which the nuclear spin decreases by L; if the spin increases by L, the above half-lives are multiplied by the factor $(2I_{\text{initial}}+1)/(2I_{\text{final}}+1)$. Figure 1 shows both the theoretical estimates and the experimental values of the half-life of odd-A nuclei, the latter corrected for internal

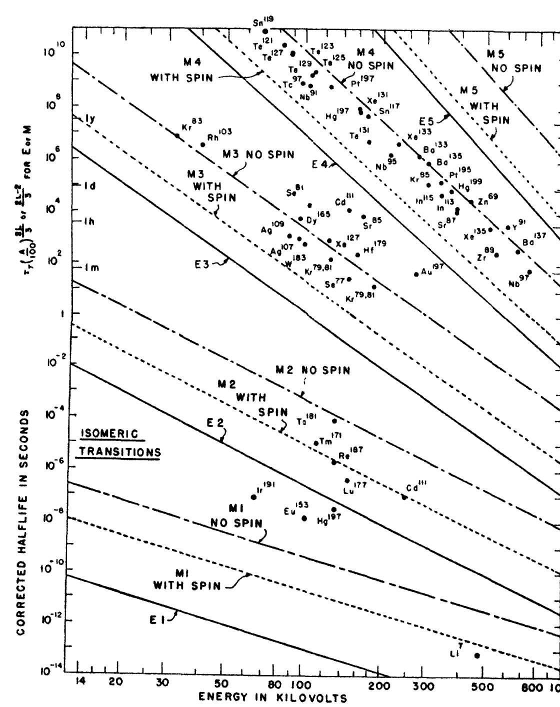

FIG. 1. Corrected half-life of odd-A isomeric transitions as function of energy. The points show experimental values of half-lives corrected for internal conversion (using theoretical values), and to $A=100$. The lines indicate theoretical predictions from the single-proton model, according to Eqs. (1) and (2), for $A=100$. (Nuclear radius, $R=1.5\times10^{-13}A^{\frac{1}{3}}$ cm.)

conversion (using theoretical values[2,3]) and to $A=100$. There are two lines for each magnetic multipole order. For the line labeled "no spin," only the contribution of orbital proton motion to transitions has been considered ($\mu=0$). The other line, indicated by "with spin," has been calculated on the assumption that in addition to orbital motion all of the magnetic moment due to intrinsic proton spin contributes to the interaction responsible for the transition ($\mu=2.79$).

Interpretation of Data.—There is a very well-defined group of transitions with half-lives and internal conversion data corresponding to magnetic 2^4-pole radiation, also[4] called $l=5$, involving a spin change of 4 and change of parity ($\Delta I=\pm 4$, yes). These transitions occur for odd nucleon numbers 39 to 49 and for 67 to 81, as predicted by the shell model.[5] Most of the transitions have half-lives which, when corrected for internal conversion, $A=100$, and energy, lie within a factor of 10 of each other. When the spins of initial and final state are considered, this agreement is improved to within a factor of 3. The half-lives fall near the "no spin" line and are larger by a factor of the order of 100 than for the "with spin" line. This is probably not due to the absence of a spin effect but happens because the spin effect is counterbalanced by cancellation of wave functions in the radial integral.

There is another not so well-defined group of shorter half-lives, also called $l=4$. The interpretation of experimental conversion coefficients and K/L ratios for the Kr^{83} and Ag^{107} transitions[2,3,6,7] favors an assignment of electric 2^3-pole radiation ($\Delta I=\pm 3$, yes) for both cases. It is likely that many of the other transitions in this group are also electric 2^3-pole. Figure 2 shows decay schemes for these cases. The single particle shell model is modified in that one postulates the existence of levels whose spins are determined by coupling spins of several odd nucleons. Thus 3, 5, or 7 nucleons in a $g_{9/2}$ orbit couple to give a state of 7/2 even, in addition to the expected state of 9/2 even. The electric 2^3-pole transitions occur for 43, 45, or 47 odd nucleons exactly where there are 3, 5, or 7 $g_{9/2}$ particles according to the shell model. They are not found for 39, 41, or 49 odd nucleons where there is one particle or one hole in the $g_{9/2}$ shell. A magnetic dipole gamma-ray should follow these transitions wherever the ground state is 9/2 even, as happens for Kr^{83} and Tc^{99}. The half-lives of these electric 2^3-pole transitions are 1000 or more times larger than the theoretical formula. For a many-particle transition to or from a $(g_{9/2})^3{}_{7/2}$ level, the effect of cancellation in the radial integral is probably large enough to account for a factor as large as this.

In regard to very short-lived states, a few transitions are well established as electric quadrupole ($\Delta I=\pm 2$, no). It is probably significant that those transitions with half-lives shorter than the theoretical predictions occur in the region of neutron and proton numbers where large quadrupole moments are prevalent.[8]

The half-life of only one magnetic dipole transition ($\Delta I=\pm 1$, no), that of Li^7, has been measured.[9] It agrees well with theory when the full value of proton magnetic moment is considered. No cancellation is expected for this case because the initial and final state probably have very similar radial wave functions.

Many of these conclusions have been reached independently by Goldhaber and Sunyar.[10] The author wishes to thank Dr. Maria G. Mayer for suggesting this problem and for valuable guidance throughout the course of the work.

[1] The problem of gamma-ray half-lives was previously considered in some detail by Weisskopf with resulting estimates of $\tau_\gamma{}^{EL}$ same as given in Eq. (1), $\tau_\gamma{}^{ML}=\tau_\gamma{}^{EL}(\hbar/mcR)^{-2}$. J. M. Blatt and V. F. Weisskopf, Chapter XII of a forthcoming book on nuclear structure (John Wiley and Sons, Inc., New York).

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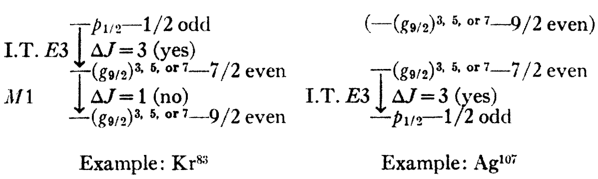

FIG. 2. Decay schemes of electric 2^3-pole isomeric transitions for nuclei with 43, 45, or 47 odd nucleons.

Variation of Dielectric Constants of Ionic Crystals with Pressure

SUMNER MAYBURG
Westinghouse Atomic Power Division, Pittsburgh, Pennsylvania
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RECENTLY, Rao[1] has disagreed with certain calculated results in my paper on the effect of pressure on the low frequency dielectric constant of ionic crystals.[2] In addition, Rao presents a Born-Madelung type theory by which he obtains good agreement with my experimental values.

It is the purpose of this letter to point out that the disagreement between Rao and myself can be completely explained by a difference in our definitions of compressibility. I define the compressibility of a material by

$$\chi=-\partial \ln V/\partial p. \tag{1}$$

The results of Rao's calculations indicate that he defines the compressibility as

$$\chi=-(1/V_0)\partial V/\partial p. \tag{2}$$

In (1) and (2), V is the volume at the pressure p and V_0 is the volume at zero pressure. The difference between (1) and (2) becomes important when one wants to determine the change of compressibility with pressure, $\partial\chi/\partial p$, from the data of Bridgman which are in the form

$$(V_0-V)/V_0=ap-bp^2, \tag{3}$$

where a and b are constants for a given material. Then, Eq. (1) leads to

$$(\partial\chi/\partial p)_{p=0}=-2b+a^2, \tag{1a}$$

while Eq. (2) yields

$$(\partial\chi/\partial p)_{p=0}=-2b. \tag{2a}$$

The difference between (1a) and (2a) is by no means negligible and may be as much as 25 percent.

Using the compressibility as defined by Eq. (1), I have recalculated the quantity $\partial \ln K/\partial p$ from the Born-Madelung equation used by Rao; the equation is

$$K-K_0=\text{constant } \rho^{4/3}\chi \tag{4}$$

TABLE I. Calculated and observed values of $-\partial \ln K/\partial p$.

	$-\partial \ln K/\partial p$ [from (4)]	$-\partial \ln K/\partial p$ [observed]
LiF	0.29×10^{-5} bars^{-1}	$0.448\pm0.028\times10^{-5}$ bars^{-1}
NaCl	0.72	0.98 ± 0.06
KCl	0.59	1.05 ± 0.08
KBr	0.66	1.17 ± 0.09
MgO	0.13	0.320 ± 0.019