potassium content of the crystal. The crystal, 5.26 g of optically perfect Harshaw KI (0.5 percent Tl added to the melt) gave a total K^{40} beta-count of about 2000 counts/minute. The crystal gave pulses about one-fifth as large as NaI-Tl pulses of the same energy. The energy resolution width was greater than that of NaI-Tl by the square root of the pulse size ratio.

The Jordan and Bell amplifier was used, modified to employ delay line pulse shaping. At the pulse width used (3 μsec) the slow components of the light decay in KI-Tl[1] yielded delayed pulses of about the same size as the thermionic emission background of the photomultiplier tube. This indicated that the mean time interval between photoelectrons of the slow components was greater than the amplifier pulse width, so that the delayed pulses were essentially of single-electron size. These slow component pulses were too small to affect the experiment. The fast component, on the other hand, decayed in much less than the amplifier rise time used (1 μsec).

The energy calibration and resolution width were determined from the photopeak of the annihilation radiation of Cu^{64}. Background (≈ 1 percent) was measured with the aid of a NaI-Tl crystal. The small (≈ 2 percent) effect of the detection by the crystal of its own K^{40} gamma-radiation (1.46 Mev) was calculated, using the known gamma-ray rate,[2] the geometry, and a pulse-height distribution obtained with an external gamma-ray source (K^{42}, 1.51 Mev).[3] The beta-ray distribution was corrected for resolution.[4] A further distortion of the spectrum exists, caused by those beta-particles which emerge from the surface of the crystal. A method was developed whereby this distortion could be approximately calculated, using an empirical electron range-energy curve. The energy distribution was then corrected for this effect, and a Kurie plot analysis made.

An "allowed" plot of the data, not corrected for resolution or the above-mentioned surface effect, is shown in Fig. 1. The convexity of this plot is a good indication that the deviation from an allowed shape is real. The "allowed" plot of the corrected data Fig. 2 ($c=1$), is quite nonlinear. The result of applying the unique axial vector or tensor theoretical correction factor corresponding to the known spin change of four, and assuming a parity change, is shown in Fig. 2 ($c=c$). The fit to a straight line is quite good, in agreement with recent measurements.[5-7] However, the deviations from linearity are much smaller in the present experiment. This confirmation, by a different method, of agreement with theory for a third-forbidden transition, further strengthens the evidence for the correctness of either the axial vector or tensor interaction in beta-decay.

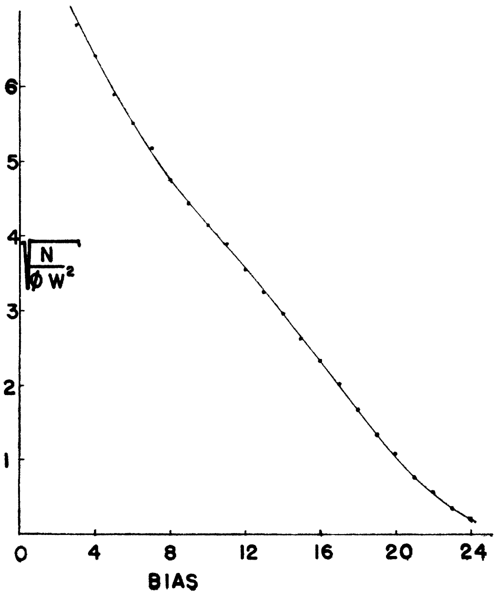

FIG. 1. "Allowed" Kurie plot of uncorrected data.

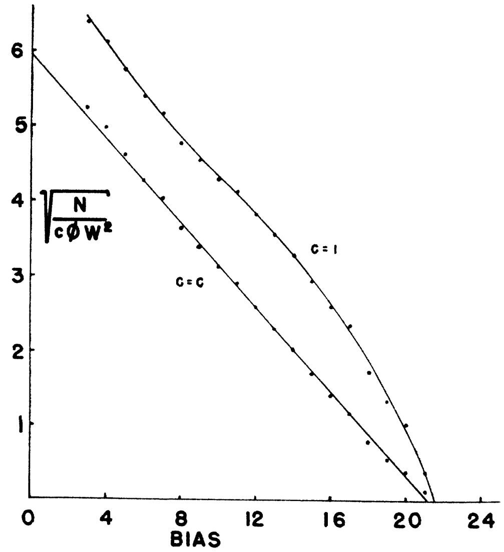

FIG. 2. Kurie plot analysis of corrected data.

The end point energy obtained is 1.28±0.03 Mev, somewhat lower than previous measurements.[5-7]

The linearity of the Kurie plot allows an extrapolation of the total counting rate to zero energy to be made easily. In this way the beta-decay rate was measured as 27.1±0.6 betas/sec/gram K, or $\lambda_\beta = 4.67 \pm 0.15 \times 10^{-10}$ yr^{-1} (using the relative abundance K^{40}/K of $1.19 \pm 0.01 \times 10^{-4}$ given by Nier[8]). This has been corrected for all the effects mentioned above, using integral forms of the corrections. The value is in agreement with that of Sawyer and Wiedenbeck,[9] who also used a 4π geometry. They obtained 28.3±1.0 betas/sec/gram K.

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Contribution to the Theory of Impurity Centers in Silicon

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An electron bound to an impurity is usually treated like a hydrogen atom in which the electrostatic force is reduced by the dielectric constant ϵ of the crystal.[1] We shall use a more detailed treatment for a phosphorus impurity in silicon.

The electron moves in the field of the ion, in the dipole field of the polarized atoms, and in the periodic field of the crystal. The atoms are polarized by the ion and electron charge and by all other dipoles. The polarizability of a silicon atom $\alpha = 3.81A^3$ is obtained from the equation of Clausius-Mosotti with $\epsilon = 13$. The

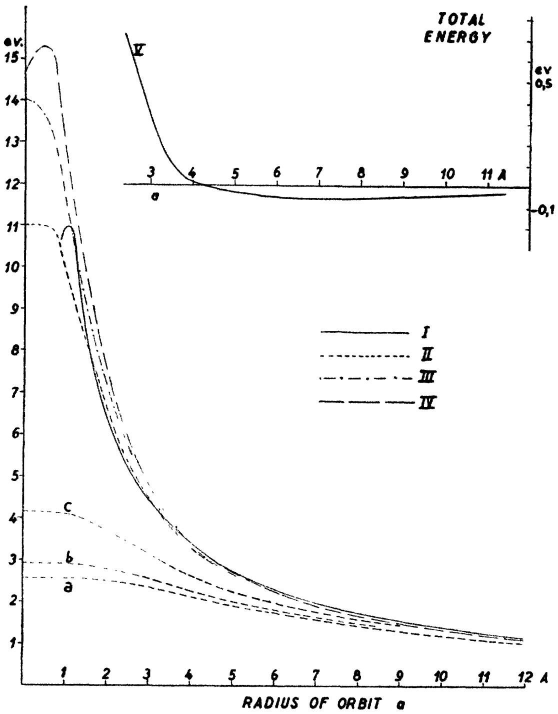

FIG. 1. (I) Negative energy of the electron bound to a free phosphorus atom. (II) Energy of the electron in the dipole field arising from the ion charge. There are contributions from the continuum of distant (>5.16A) dipoles (a), from 12 dipoles at 4.49A (b−a), from 12 dipoles at 3.38A (c−b), and from 4 dipoles at 2.27A (II−c). (III−II) Energy arising from the difference of polarizability of a P-ion with respect to a Si-atom. (IV−III) Energy in the periodic potential of the crystal. (V) Total energy = IV−I. The minimum is −0.065 ev at $a=7$A.

28 nearest silicon atoms are considered explicitly; those farther away are treated as a continuum. The dipole moments due to the ion and the inner field are calculated making use of symmetry. For the electron-dipole field we take an approximation of the radial component of the ion-dipole field, namely,

$$F(d)=(e/d^2)\beta(d);\quad \beta(d)=\begin{cases}1-(d/C)^2 & \text{for } d<5.16\text{A};\ C=5.37\text{A}\\ 1/\epsilon=0.077 & \text{for } d>5.16\text{A}\end{cases}$$

and then assume that the electron-dipole field is the same expression, where d is the distance from the electron.

The method consists in minimizing the energy for the hydrogen-like wave function $\psi(r)=(\pi a^3)^{-1/2}e^{-r/a}$ with respect to the radius a.

The sum of kinetic energy $\hbar^2/2ma^2$ and potential energy $-e^2/a$ of the electron in the field of the ion must be slightly altered to give an ionization potential of 10.9 ev for free phosphorus (curve *I* of Fig. I). This first energy is negative. The other energy terms are all positive. Second term: the energy of the electron in the dipole field arising from the ion charge (curve *II*) leaves a weak bond for $a=7$A and a stronger one for $a=1$A. Third term: the energy of the electron in the dipole field arising from the electron itself is different in the bound and conducting states. Namely, the bound electron acts on the phosphorus ion, whose polarizability is taken to be smaller than that of a silicon atom by a factor $(14/15)^3$. For calculation of the resulting repulsion from the ion (*III-II*) cut-off radii $R_{P+}=1.10$A for a phosphorus ion and $R_{Si}=1.17$A for a silicon atom are assumed, within which there are no forces between the electron and the dipole. Incidentally, the potential energy of a conduction electron in the field of the dipoles which it induces becomes −10.0 ev, corresponding to an image-force barrier. Fourth term: it is found that a periodic potential in the crystal changes the energy only for small a (*IV-III*).

In the above model the minimum of the total energy is −0.065 ev for $a=7$A, in essential agreement with experiment[2] and with the simple theory. It is only by taking all effects together that a bond with small a is excluded; but once the orbit is large, the phenomenological theory holds.

I wish to express my gratitude to Professor W. Pauli for his kind support, to Professor G. Busch for suggesting the problem, and to Dr. R. Schafroth for much helpful advice.

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A Scattering Approximation

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SCATTERING of a plane wave by a square well potential has been treated in an approximation comparable in simplicity to the Born approximation but with the added feature of being exact at low energies as well as high.

One proceeds from the integral formula for the scattering amplitude:

$$f(\theta)=(mV_0/2\pi\hbar^2)\int \exp(-i\mathbf{k}'\cdot\mathbf{r})\psi(\mathbf{r})d\mathbf{r};\tag{1}$$

V_0 is the depth of the well, m is the reduced mass, and the integral is over the domain $r<a$, where a is the radius of the well; and $\psi(\mathbf{r})$ is the exact wave function inside the well, i.e.,

$$\psi(\mathbf{r})=\Sigma_l\, a_l(2l+1)i^l j_l(\alpha r)P_l(\cos\theta),\tag{2}$$

where α is the interior wave number and the coefficients a_l are found as usual by matching logarithmic derivatives:

$$a_l^{-1}=x^2\{[j_l(y)n_l'(x)-(y/x)j_l'(y)n_l(x)]-i[j_l(y)j_l'(x)-(y/x)j_l'(y)j_l(x)]\},\tag{3}$$

where $x=ka$; $y=\alpha a$.

It is now observed that the series (2) is summable if one substitutes for the spherical bessel functions in Eq. (3) their asym-

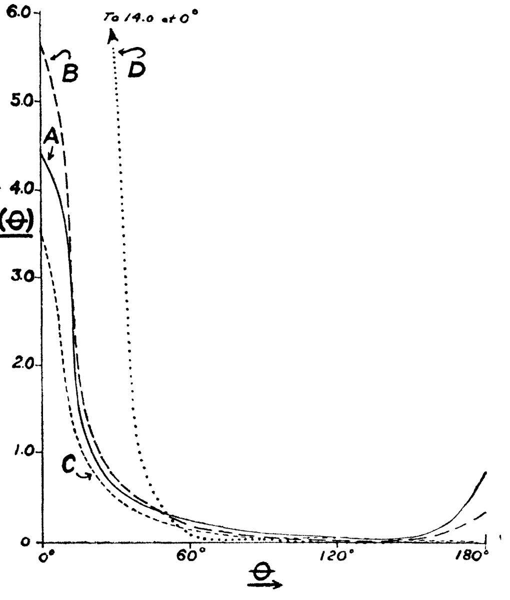

FIG. 1. Differential cross section in barns per steradian: A. exact; B. approximation based on Eq. (5); C. approximation based on Eq. (5) with $a_1=a_0$; D. Born approximation.