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The Analysis of π-Meson Production in Nucleon-Nucleon Collisions*†

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A phenomenological analysis of meson production in nucleon-nucleon collisions is proposed. On the basis of an hypothesis that the production takes place for collisions whose impact parameters tend to be less than the range of nuclear forces, a partial wave analysis of the scattering matrix is given. The theory seems capable of describing the experimental results in a simple manner. It is shown, on the assumption that the π-meson is pseudoscalar, that angular momentum and parity considerations play an important role in interpreting the experimental results. If processes involving mesons are related to nuclear forces, then the hypothesis of charge symmetry in nuclear phenomena should receive a crucial test in experiments concerning meson production.

I. INTRODUCTION

THE data which has at present been obtained on the production of π-mesons in the collisions of two nucleons is very incomplete, yet it is sufficient to establish a number of interesting features of these processes. Indeed, there seems to be enough quantitative information to warrant the development of a systematic and unified interpretation of the phenomena of meson production in nucleon collisions, and it is the purpose of the present paper to sketch the outline of such a means of interpretation on the basis of a phenomenological theory. Although this type of analysis is less satisfying than one based on a fundamental theory of elementary particles, the lack of any satisfactory form of a basic theory makes it necessary to fall back on a phenomenological approach in the hope of obtaining a unified picture of the processes under consideration. The theory developed here should also be of assistance in the study of meson production in complex nuclei (which is not considered in the present paper) and in the comparison with the inverse processes of meson absorption.

Some of the qualitative experimental information on meson production which has been obtained at this laboratory is given in Table I. From these results, with the corresponding cross sections, it is possible to deduce approximately the nucleon-nucleon cross sections for meson production; see Table II.

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We shall present in Sec. II a formalism for giving a partial wave analysis of the experiments on meson production in terms of a partial wave analysis of the scattering matrix. Such a study establishes naturally relationships between the energy spectrum of the mesons, their angular distribution, and the excitation function for the cross section. It is particularly useful, as shown by Brueckner, Serber, and Watson,[1] in the study of the inverse processes of meson absorption. In this section the requirements of conservation of parity and angular momentum and of charge symmetry are also considered and are shown to be very effective in introducing simplifications into the analysis.

It has been pointed out by Brueckner, Chew, and

TABLE I. Qualitative experimental results for processes involving production of charged and neutral mesons by nucleons bombarding free nucleons or complex nuclei.

A. Free nucleons	B. Complex nuclei
(1) $P+P\rightarrow\pi^+$ (allowed)[a]	(1) $P+(P, N)\rightarrow\pi^+$ (allowed)[c]
(2) $P+N\rightarrow\pi^+$ (unobserved)	(2) $P+(P, N)\rightarrow\pi^-$ (allowed)[c]
(3) $N+N\rightarrow\pi^-$ (unobserved)	(3) $P+(P, N)\rightarrow\pi^0$ (allowed)[b]
(4) $N+P\rightarrow\pi^-$ (unobserved)	(4) $N+(P, N)\rightarrow\pi^+$ (allowed)[d]
(5) $N+N\rightarrow\pi^0$ (unobserved)	(5) $N+(P, N)\rightarrow\pi^-$ (allowed)[d]
(6) $N+P\rightarrow\pi^0$ (unobserved)	(6) $N+(P, N)\rightarrow\pi^0$ (unobserved)
(7) $P+P\rightarrow\pi^0$ (forbidden)[b]	

[a] Cartwright, Richman, Whitehead, and Wilcox, Phys. Rev. 78, 823 (1950).
[b] Bjorkland, Crandall, Moyer, and York, Phys. Rev. 77, 213 (1950).
[c] C. Richman and H. A. Wilcox, Phys. Rev. 78, 496 (1950).
[d] Bradner, O'Connell, and Rankin, private communication.

[1] Brueckner, Serber and Watson, Phys. Rev. 81, 575 (1951).

TABLE II. Nucleon-nucleon cross sections for production of charged and neutral mesons. The experimental results given in Table I, from which the cross sections are deduced, are indicated in column II.

I. Process		II. Experiment	III. Total cross sections ($\times 10^{28}$ cm^{+2}) Complex nuclei	Free
(1) $P+P\to\pi^+$	allowed	$A(1)$, $B(1)$	3.3 ± 1.0	3.0 ± 1.0[a]
(3) $P+P\to\pi^0$	forbidden	$A(7)$	?	0.1 ± 0.1
(3) $N+P\to\pi^+$	allowed	$B(4)$, $B(1)$	0.8 ± 0.4	?
(4) $N+P\to\pi^-$	allowed	$B(2)$	0.8 ± 0.4	?
(5) $N+P\to\pi^0$	allowed	$A(7)$, $B(3)$	1.7 ± 0.9	?
(6) $N+N\to\pi^-$	possible	$B(5)$	?	?
(7) $N+N\to\pi^0$	unobserved		?	?

[a] Deduced from the cross section in the forward direction with the angular distribution found in Sec. IV.

Hart[2] that the production of mesons in nucleon-nucleon collisions is strongly dependent on the interactions of the nucleons in the final state. In the course of applying the theory of the present paper, the calculations of these authors have been extended. These considerations are discussed in Sec. III.

Cross sections deduced from the preceding development are given in Sec. III, and in Sec. IV these are compared with and fitted to the experimental results on the production of π^+ mesons in proton-proton collisions. A summary of the results obtained in the present paper is presented in the final section.

II. FORMAL CONSIDERATIONS

A. Introduction of General Transition Matrix

Above the threshold for meson production, the study of nuclear collisions is complicated by the interdependence of the elastic and the inelastic (meson production) scattering. This fact leads to characteristic difficulties in the use of the Schrödinger equation to describe high enery nuclear scattering events. To escape these difficulties, we propose the application of the scattering matrix in the analysis of meson production. In so doing we lose some insight into the mechanism of these phenomena, but are left on a sounder theoretical footing for the description of the rather complex experimental results.

The considerations are most easily carried out in the center-of-mass coordinate system. We may then partially characterize the collision by the relative momentum $\mathbf{p}$ of the incoming nucleons, which collide to produce a meson with momentum $\mathbf{q}$, and the final relative momentum $\mathbf{p}'$ of the nucleons after the meson is produced. In particular, we introduce a transition matrix

$$R=(\mathbf{q},\mathbf{p}'|R|\mathbf{p}) \tag{1}$$

(which also depends upon spin and isotopic spin variables of the particles in question) describing the transition from the initial to the final state accompanied by the production (or absorption) of a meson.

We suppose the two incoming nucleons in the initial state to have a wave function ψ_I, the two final nucleons to be in a state ψ_F, and the produced meson to be in a state ϕ_q. The probability amplitude for the transition is described by

$$(F|R|I)=(\psi_F\phi_q, R\psi_I). \tag{2}$$

We can expect that to a fairly good approximation ψ_I can be represented by a plane wave, but this approximation is not necessary in our analysis, since we can define a new R' as $R'=R\psi_I$. Using R' in Eq. (2), we must consider the new ψ_I to be a plane wave. We assume that ϕ_q represents a plane wave for the outgoing meson. However, considerable physical insight is gained by calculating ψ_F explicitly, so we do this in the next section.

The cross section is then given as (we use as units $\hbar=c=1$)

$$d\sigma=(2\pi)^4(dJ/v_R)\sum(F|R|I)^2, \tag{3}$$

where v_R is the relative velocity of the incoming nucleons, dJ is the volume in phase space accessible to the particles in the final state, and $\sum$ means a sum and average over the final and initial spin states, respectively.

B. Partial Wave Analysis

A few physical considerations can be employed to introduce considerable simplifications of the general expression (2). For energies available from present accelerators, the greater part of the center-of-mass energy of the incident nucleons goes into the rest mass of the meson, leaving relatively little for the kinetic energies of the three final particles (a total of about 20 Mev for the Berkeley cyclotron). This loss of kinetic energy is necessarily accompanied by large momentum transfers, which in turn suggests that the collision producing a meson takes place at close distances of approach for the two nucleons. This can be interpreted as representing a short range of interaction for producing mesons—roughly of order $\hbar/P$ (where P is the momentum transfer), which is considerably less than the range of nuclear forces. In view of this evidence of a short range of interaction and the relatively small kinetic energies of the final particles, it seems natural to attempt a partial wave analysis of the final state. Indeed, we may very well expect the process to lead predominantly to one angular momentum state for the relative motion of the two nucleons (s-state) and to another for the angular momentum of the meson relative to the two nucleons. We must, however, expect small admixtures of other angular momentum states.

The pronounced peak of high energy mesons which is observed experimentally is rather conclusive evidence that the final nucleon state is predominantly an s-state (zero relative angular momentum), since this peak corresponds to small relative energies for the two nucleons

[2] K. A. Brueckner, Phys. Rev. 82, 598 (1951). (In particular, see Appendix.)

(this point is developed in greater detail in Secs. III and IV). On the other hand, calculations of the meson energy spectrum, as described in the next section, indicate that most of the mesons have an angular momentum of $l=0$ or 1.

We now make more explicit use of our hypothesis of a short range interaction. Since the de Broglie wavelengths of the particles in the final state are appreciably greater that $\hbar/P$, our deduced range of interaction, we can with little error introduce a zero range approximation. In particular, this means that for S-states we can replace $\psi(\mathbf{r})$ by $\psi(0)$, and for P-states, which vanish at the origin, we can replace $\psi(\mathbf{r})$ by grad$\psi(0)$. If in addition we assume that for P-states we can use a plane wave approximation, the gradient operator can be replaced by $i\mathbf{P}$, where $\mathbf{P}$ is the momentum of the particle. This means, for instance, that for final nucleon S-states the dependence of Eq. (2) on $\psi_F(r)$ will be given by the multiplicative factor $(2\pi)^{\frac{3}{2}}\psi_F(0)$.

Then, in the approximation that we need consider only final nucleon S-states, the quantity $\sum|(F|R|I)|^2$ in Eq. (3) can be decomposed into the product of two factors, one of which is just

$$(2\pi)^3|\psi_F(0)|^2. \tag{4}$$

The other depends upon the meson angular momentum state. For meson S- and P-states (taken as plane waves),[3] we have for this factor

$$s: \quad \Gamma^2(p)g_{FI}, \tag{5}$$

$$p(\mathrm{sym}): \quad \Gamma^2(p)g_{FI}q^2, \tag{6}$$

$$p(\cos^2\theta): \quad \Gamma^2(p)g_{FI}q^2\cos^2\theta, \tag{7}$$

where θ is the angle between the vectors $\mathbf{q}$ and $\mathbf{p}$. The symbol "s" means a meson S-state; p(sym) means a meson P-state with a symmetrical angular distribution; $p(\cos^2\theta)$ means a meson P-state with a $\cos^2\theta$ angular distribution. The quantities $\Gamma^2(p)$ are numerical functions of the initial momentum, p, of the nucleons and are constant for a given beam energy. The g_{FI} are numerical quantities depending upon initial and final spin and isotopic spin states.

To include the possibility of some nucleon P-state admixture, we have two more permissible terms (since for nucleon P-states, the plane wave approximation for final states is satisfactory, we replace the factor $(2\pi)^3\psi_F{}^2(0)$ used above by p'^2 in this case),

$$(p,s): \quad \Gamma^2(p)g_{FI}p'^2, \tag{8}$$

$$(p,p): \quad \Gamma^2(p)g_{FI}p'^2q^2, \tag{9}$$

referring, respectively, to meson S- and p-states for a final nucleon p-state. We might also include a $\cos^2\theta$ factor in the last term. There will be no terms linear in p', as we are interested only in cross sections for which an integration has been performed over the angular distribution of the final nucleons. We also need consider no terms linear in $\cos\theta$, if we restrict ourselves to the consideration of proton-proton collisions.

The choice of any one of these factors [expressions (5) to (9)] permits us to calculate the cross section for meson production uniquely, except for a numerical factor to be used in fitting the results to the experimental magnitude of the cross section. In general we must expect to use a suitably chosen linear combination of these terms in obtaining the experimental cross section.

A more formal development of the ideas developed in this section is given in the Appendix.

TABLE III. Transitions allowed by Pauli principle, conservation of angular momentum, and conservation of parity for production of pseudoscalar mesons in nucleon-nucleon collisions. The final nucleons are assumed to be in S-states. The columns (a) indicate the corresponding matrices required by charge symmetry as calculated in Sec. II D.

	Meson in S-state	(a)	Meson in P-state	(a)
PP, π^+ / NN, π^-	${}^3P_0\to{}^1S_0$	M_1	${}^1S_0$, ${}^1D_2$ $\to{}^3S_1$	M_3
	${}^3P_1\to{}^3S_1$	M_3		
PP, π^0 / NN, π^0	${}^3P_0\to{}^1S_0$	M_1	forbidden	M_1
NP, π^+ / NP, π^-	${}^3P_0\to{}^1S_0$	M_1	${}^3S_1$, ${}^3D_1$ $\to{}^1S_0$	M_2
NP, π^0	${}^3P_0\to{}^1S_0$	0	${}^3S_1$, ${}^3D_{1,2}$ $\to{}^3S_1$	0
	${}^3P_1\to{}^3S_1$	M_3	${}^3S_1$, ${}^3D_1$ $\to{}^1S_0$	M_2
	${}^1P_1\to{}^3S_1$	0	${}^1S_0$, ${}^1D_2$ $\to{}^3S_1$	M_3

C. Parity and Angular Momentum Requirements

We can introduce considerable simplification by restricting ourselves to the consideration of pseudoscalar mesons. At the time of writing this seems to be the most reasonable choice since it is known that the π^0 meson cannot have spin one[4] and since the experiments on the absorption of π^- mesons in deuterium indicate that the charged π-meson is not scalar.[1,5,6] If further experimental results should lead to contrary evidence, an analysis such as that given here can be made for any given spin and parity of the meson.

Then, on the assumption that the π-meson is pseudoscalar, we can enumerate all possible transitions to produce π-mesons which are consistent with the Pauli

[3] There is evidence of a nonvanishing nucleon-meson elastic scattering cross section, which implies that the wave function for the outcoming mesons is not rigorously that for a plane wave. This effect would be expected to modify only the low energy end of the spectrum of produced mesons. No evidence for this has been observed experimentally; however, this may well be due to the difficulty of detecting low energy mesons. A further discussion of this point is given by Brueckner, Serber, and Watson (reference 1).

[4] Steinberger, Panofsky, and Steller, Phys. Rev. **78**, 802 (1950).

[5] Panofsky, Aamodt, and Hadley, Phys. Rev. **81**, 565 (1951).

[6] S. Tamor, Phys. Rev. **82**, 38 (1951).

principle, parity and angular momentum conservation, and the assumption that the final nucleons are emitted primarily into s-states—i.e., our deductions will not be valid for the (small) part of the cross section corresponding to nucleons in higher angular momentum final states. The permissible transitions are given in Table III. Of course, under actual experimental conditions, the initial state will be a statistically weighted combination of those listed in the table.

It is clear that if the hypothesis that the meson is pseudoscalar is to be maintained, the experimental results must be consistent with Table III. For example, if the (pp, π^+) process leads to mesons in p-states, the final neutron and proton must be in a triplet state only. Again, if the production of mesons into p-states represents a universal type of coupling for all production processes in nucleon-nucleon collisions, reference to Table III indicates the existence of a general selection rule prohibiting the (pp, π^0) process.

D. Further Symmetry Requirements

If we are to assume that the meson is a pseudoscalar, then the transition matrix R must be invariant under spatial rotations but change sign under a coordinate reflection. The consequences of the hypothesis of charge independence of nuclear forces are also of interest, since we here have the possibility of greater uniqueness to test this hypothesis than is afforded by elastic nuclear scattering.

Because of the considerable interest in this latter hypothesis we shall develop in detail its consequences for meson production. In particular, charge independence of nuclear forces implies invariance with respect to rotations in charge space.‡ We may thus impose this condition of invariance on the transition matrix, R. Presumably meson production is not independent of nuclear forces.

The rotation operator in charge space consists of two parts, one of which is

$$\mathbf{T}=\tfrac{1}{2}(\boldsymbol{\tau}^{(1)}+\boldsymbol{\tau}^{(2)}), \tag{10}$$

where $\boldsymbol{\tau}^{(1)}$ and $\boldsymbol{\tau}^{(2)}$ are the isotopic spin matrices of the two nucleons. This is, of course, the analog of the usual spin angular momentum. We define an orbital angular momentum as the rotation operator, $\mathbf{L}$, for the meson wave functions in charge space. This can be easily done by constructing three-dimensional vector operators $\mathbf{U}$ and $\mathbf{U}^+$ which destroy and create mesons; i.e.,

$$[U_i, U_j^+]=\delta_{ij}, \quad (i, j=1, 2, 3), \tag{11}$$

all other combinations commuting. An invariant is $\boldsymbol{\tau}\cdot\mathbf{U}$. Then

$$\mathbf{L}=i\mathbf{U}\times\mathbf{U}^+. \tag{12}$$

The infinitesimal rotations about an axis $\hat{a}$ are generated by $\hat{a}\cdot\mathbf{J}$, where

$$\mathbf{J}=\mathbf{T}+\mathbf{L}. \tag{13}$$

Invariance of R about the axis $\hat{a}$ implies $[\hat{a}\cdot\mathbf{J}, R]=0$, or, since we assume complete symmetry,

$$[\mathbf{J}, R]=0. \tag{14}$$

The eigenvalues of $Q\equiv 1-J_3$ give the total charge of the system in units of the proton charge.

To develop the consequences of Eq. (14), we choose the following representation. For $\boldsymbol{\tau}$, we decompose the isotopic spin states into a singlet state, s, and three triplet states t^+, t^0, t^-, as usual (t^+ corresponds to a state consisting of two neutrons, etc.). For the representation of $\mathbf{L}$, we note that

$$\mathbf{L}\omega(0)=0, \tag{15}$$

where $\omega(0)$ is a state containing no mesons, or a state of zero "orbital angular momentum." The states $\omega(1^+)$, $\omega(1^0)$, $\omega(1^-)$—that is, states containing one positive, one neutral, and one negative meson, respectively—correspond to an orbital angular momentum of unity ($l=1$). Since

$$\begin{aligned} L_3\omega(1^+)&=-\omega(1^+),\\ L_3\omega(1^0)&=0,\\ L_3\omega(1^-)&=\omega(1^-), \end{aligned} \tag{16}$$

we conclude that these states transform as the three spherical harmonics of order one. We may thus use the usual angular momentum matrix elements for $\mathbf{L}$ in the ω-representation.

It then follows that the eigenstates of $J^2=j(j+1)$ for the final state containing two nucleons and a meson correspond to one set for $j=2$, two sets for $j=1$, and one set for $j=0$. Since the initial state containing two nucleons only corresponds to $j=1$ and $j=0$, we conclude that all transitions to final states with $j=2$ are forbidden. This then leaves us with only *three independent* matrices, R, corresponding to the two representations with $j=1$ and the one with $j=0$.

The explicit relationships can easily be obtained from Eq. (14) by evaluating the matrix of the commutator between a state with no meson and one with one meson of a given charge. The matrix elements of R are written as $(1^-t^0|R|t^+)$, etc., as designating a transition from a nucleon t^+ state to a nucleon t^0 state to produce a negative meson. Similarly $(1^-s|R|t^+)$ designates the same transition, except that the final nucleons are in a singlet isotopic spin state. R also depends on the nucleon spin and momentum coordinates, but this dependence is not explicitly written for the present.

Rotations about the "3" axis give us just charge conservation from Eq. (14). Those about the "1" and

‡ We are indebted to Professor D. L. Falkoff for calling our attention to a paper by W. Heitler, Proc. Roy. Irish Acad. **51A**, 33 (1946), in which similar considerations are applied to the scattering of mesons by nucleons.

"2" axes give us the following relations:

$$\begin{aligned} M_1 &= (1^+t^+|R|t^0)=(1^+t^0|R|t^-) \\ &= (1^0t^+|R|t^+)=-(1^0t^-|R|t^-) \\ &= -(1^-t^0|R|t^+)=-(1^-t^-|R|t^0), \\ M_2 &= (1^+t^+|R|s)=(1^-t^-|R|s)=-(1^0t^0|R|s), \\ M_3 &= (1^0s|R|t^0)=(1^+s|R|t^-)=(1^-s|R|t^+), \\ &(1^0t^0|R|t^0)=(1^0s|R|s)=0. \end{aligned} \quad (17)$$

All other matrices vanish by the requirement of charge conservation. We thus find three sets of matrices, as asserted above.

We can now combine the results of the requirements of the charge symmetry hypothesis with the requirements of conservation of angular momentum and parity. The matrices M_1, M_2, and M_3 corresponding to the allowed transitions given in Table III are listed in columns (a) of that table. We see that if the meson is emitted into an s-state, then the smallness of the (PP, π^0) process (Tables I and II) also requires the smallness of the (NP, π^+) and (NP, π^-) processes in contradiction with experiment. This would also require that the (PP, π^+) cross section be twice the (NP, π^0) cross section (because of the difference of the normalization of the wave functions).

If the meson is emitted into a P-state, then the only restriction is that $\sigma(NP, \pi^0)$ be equal to $\sigma(NP, \pi^+)$ plus $\frac{1}{2}\sigma(PP, \pi^+)$, which is consistent with the present rather inaccurate experimental data. It is interesting to observe that this assumption for the final meson angular momentum state automatically gives a selection rule inhibiting the (PP, π^0) process.

We conclude that to give agreement with the approximate magnitudes of the experimental cross sections it must be assumed that the meson, if pseudoscalar, is emitted predominantly into p-states, while the final nucleons are in s-states. We shall make a more detailed analysis of these conclusions in the following sections.

III. CALCULATED CROSS SECTIONS

If the energy of the final nucleons were sufficiently high, we could take $(2\pi)^3|\psi_F(0)|^2$ as unity. In actual fact, for the energies presently available this quantity is effectively much larger than unity and must be calculated on the basis of some assumed force for the two nucleon system. In particular, we need only the partial wave of zero angular momentum. Also, if the final state is a neutron-proton system with some triplet spin state present, there is a large probability that a deuteron will be formed.

When the nucleons in the final state are not bound, the wave function is a function of their relative momentum p', which is related to the meson kinetic energy, T.

Unfortunately, $\psi_F(0)$ depends more strongly upon the assumed shape of the two nucleon potential than does the low energy scattering data. Calculations have thus been made for both square well and exponential potentials (tensor forces have not been included) whose parameters were chosen to fit the low energy scattering. The n-p (neutron-proton) triplet scattering length and effective range were taken from Christian and Hart.[7] In accordance with an assumed charge symmetry (at low energies) of nuclear forces, the n-n (neutron-neutron), p-p (proton-proton), and n-p singlet potentials were assumed to be the same and were taken from Jackson's and Blatt's work.[8] The effect of the coulomb interaction in the p-p case was included for the square well, but not for the exponential well, and amounted to only a 10 percent reduction in the total cross section for 340-Mev collisions (laboratory frame of reference).

TABLE IV. Values of the arbitrary constants $\Gamma^2 g$, adjusted to give a total cross section (neglecting deuteron formation) at 343 Mev of $2.60(10)^{-28}$ cm². The notation (Triplet, S), etc., refers to the spin state of the final nucleons and the orbital angular momentum state of the meson [Eqs. (5), (6), (7)].

	(Triplet, S)	(Singlet, S)	(Triplet, P-sym)
$\Gamma^2 g$	$1.94(10)^{-44}\frac{\text{cm}^2}{(\text{Mev})^5}$	$2.01(10)^{-44}\frac{\text{cm}^2}{(\text{Mev})^5}$	$4.90(10)^{-48}\frac{\text{cm}^2}{(\text{Mev}^7)}$
	(Singlet, P-sym)	(Triplet, P-cos²θ)	(Singlet, P-cos²θ)
$\Gamma^2 g$	$4.36(10)^{-48}\frac{\text{cm}^2}{(\text{Mev})^7}$	$1.47(10)^{-47}\frac{\text{cm}^2}{(\text{Mev})^7}$	$1.31(10)^{-47}\frac{\text{cm}^2}{(\text{Mev})^7}$

The results for square and exponential wells were in only fair agreement—although the discrepancies were within present experimental errors. For instance, for a final n-p triplet state, the ratio of number of neuterons formed to number of unbound particles agreed to within about 5 percent. Also the cross section for forming a deuteron using an exponential well agreed to within a few percent with that calculated using the wave function of Chew and Goldberger.[9]

The exponential well, as being the more reasonable physically, was chosen for the calculations presented here, and it is felt that the results are sufficiently reliable until much more accurate and complete experimental results are available than at present. Eventually, meson production may provide a means of probing nuclear forces at close distances.

With the values of $|\psi_F(0)|^2$ we are in a position to evaluate the partial cross sections corresponding to the expressions (5) to (9). Giving the constants $\Gamma^2 g$ of these expressions fixed numerical values, we need consider only three types of final states: final unbound triplet and singlet states of the nucleons, and final states with a deuteron formed. The values of $\Gamma^2 g$ were arbitrarily chosen to normalize the total cross sections (disregarding deuteron formation) to $2.60(10)^{-28}$ cm². These values are listed in Table IV.

The cross sections are designated as follows:

$$d\sigma(\text{Triplet, } P\text{-sym}) = \sigma_0(\text{Triplet, } P\text{-sym})(\theta, T)d\Omega_q dT, \quad (18)$$

[7] R. Christian and E. Hart, Phys. Rev. **77**, 441 (1950).
[8] J. D. Jackson and J. M. Blatt, Revs. Modern Phys. **22**, 77 (1950).
[9] G. F. Chew and M. L. Goldberger, Phys. Rev. **75**, 1637 (1949).

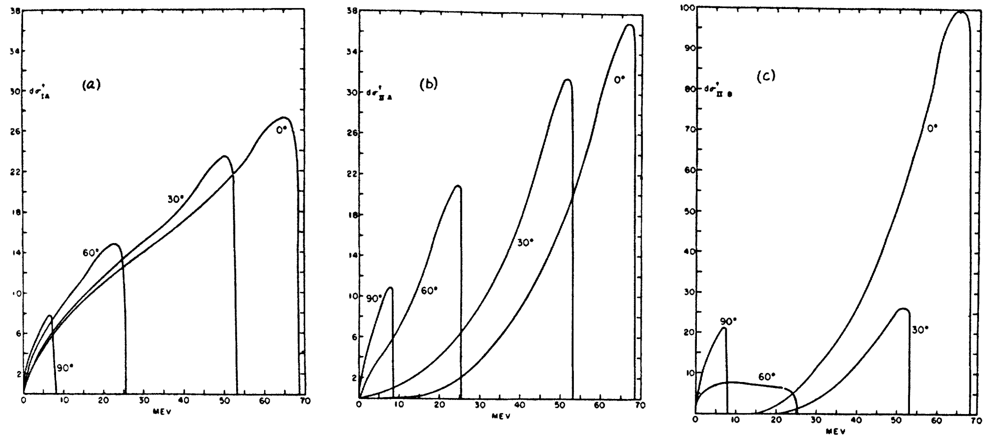

FIG. 1. Differential cross section for meson production in the laboratory system in units of 10^{-31} cm^2 per Mev per unit solid angle [designated as σ_0 in Eq. (18)] at an initial nucleon energy of 343 Mev and for a meson rest-mass energy of 140 Mev. The final nucleons are assumed to be in a triplet spin state. The energy scale in Mev refers to the meson kinetic energy; the angles indicated are the angles between the meson and incident nucleon momenta. (a) is for (S) type coupling; (b) is for P(sym) type coupling; (c) is for $P(\cos^2\theta)$ type coupling [this notation is defined by Eqs. (5), (6), and (7)].

where the spin state of the final nucleons (triplet or singlet) and the type of meson p-state (symmetrical or $\cos^2\theta$) are indicated. Here T is the meson kinetic energy, $d\Omega_q$ is an element of solid angle into which the meson is emitted, and θ is the angle between the meson and beam directions. If a deuteron is formed, the mesons have a fixed energy, T_0, and we have a cross section (symmetrical or $\cos^2\theta$)

$$d\sigma(\text{deuteron}, P\text{-sym}) = \sigma_0(\text{deuteron}, P\text{-sym})(\theta)d\Omega_q dT, \quad (19)$$

where

$$\sigma_0(\text{deuteron}, P\text{-sym})(\theta) = \sigma_0(\text{triplet}, P\text{-sym})(\theta, T_0)\delta(T-T_0). \quad (20)$$

To facilitate comparison with experiment, the differential cross sections of Eq. (18) have been transformed to the laboratory system and are plotted in Figs. 1 and 2 for a beam energy of 343 Mev. Cross sections with the final nucleons in P-states are also given in Fig. 3 to show the marked contrast with the processes in which the final nucleons are in S-states. The quantities are plotted in units of cm^2 (Mev-steradian)$^{-1}$. The corresponding cross sections for deuteron formation, i.e., $\sigma_0(\text{deuteron}, \text{sym})(\theta)$ and $\sigma_0(\text{deuteron}, \cos^2\theta)(\theta)$ of Eq. (19) have also been transformed to the laboratory system and are given in Table V. For the numerical results, the values of $\Gamma^2 g$ given in Table IV have been used.

TABLE V. Differential cross section, $d\sigma/d\Omega$, in units of 10^{-28} cm^2 per unit solid angle for production of a positive π-meson and a deuteron in a 343-Mev p-p collision. θ is the angle between the directions of the initial nucleon beam and meson momenta in the laboratory system. The notation is the same as that used in Table IV.

θ	(Triplet, S)	(Triplet, P-sym)	(Triplet, P-$\cos^2\theta$)	Meson energy (Mev)
0°	1.75	2.87	8.61	73
30°	1.42	2.32	2.38	56
60°	0.82	1.34	0.32	28
90°	0.28	0.46	0.83	11

The rather striking peak in the curves of Figs. 1 and 2 is due to the rapid increase of $|\psi_F(0)|^2$ with increasing meson energy.

Since the quantities $\Gamma^2(p)$ presumably vary much less rapidly with energy than do the other factors in the cross section, it is instructive to observe the variation of total cross section with beam energy on the assumption that these quantities are constant. This variation is given in Figs. 4 and 5. Deuteron formation has been included for final triplet states.

IV. COMPARISON WITH THE EXPERIMENTAL DATA ON (PP, π^+) PRODUCTION

The most detailed experimental data avaiable is for (PP, π^+) production with a beam energy of 341 ± 2 Mev. The experiments of Richman, Cartwright, and Whitehead[10] and of Cartwright, Richman, Whitehead, and Wilcox[11] give the meson energy spectrum at zero degrees ($\pm3°$) with respect to the beam direction. Further results concerning the energy spectrum at 30° have been obtained by Peterson[12] and at 18° by Peterson, Iloff, and Sherman.[13]

The experimental energy spectrum at zero degrees is given by the points in Fig. 6. To fit this data, we are restricted to final nucleon triplet states (see Table III), since we concluded in Sec. II that the mesons were emitted predominantly into p-states.

[10] Richman, Cartwright, and Whitehead, private communication.

[11] Cartwright, Richman, Whitehead, and Wilcox, Phys. Rev. **78**, 823 (1950).

[12] V. Z. Peterson, private communication.

[13] Peterson, Iloff, and Sherman, Phys. Rev. **81**, 647A (1951).

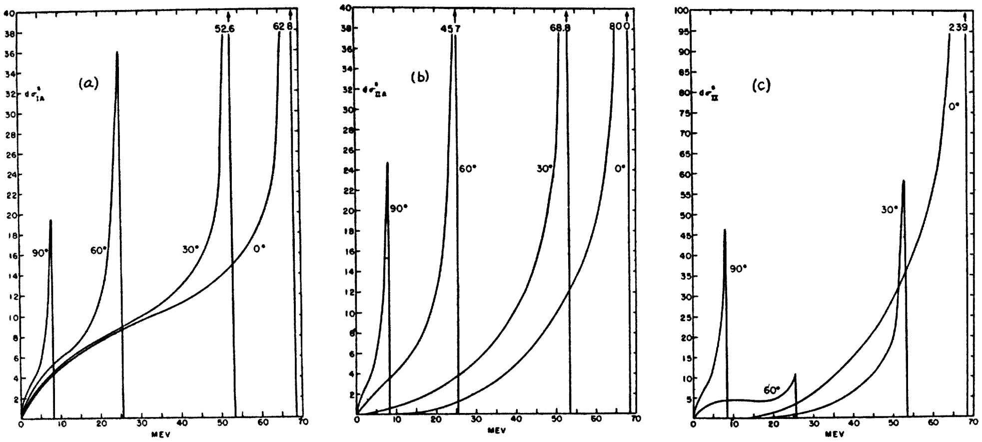

FIG. 2. Differential cross section for meson production with a beam energy of 343 Mev. The final nucleons are assumed to be in a singlet spin state, but otherwise the notation is the same as for Fig. 1.

The corresponding cross section, calculated from the (p-) type coupling of Eqs. (6) and (7) and folded into the experimental energy resolution,[14] is given in Fig. 6. The agreement seems to be quite satisfactory. A small amount of meson s-state could be included to raise the low energy end of the spectrum if this turns out to be necessary.

The angular distribution of the mesons is also of particular interest. Designating the production cross section per steradian at an angle θ' with respect to the beam direction (laboratory frame of reference) by $\sigma(\theta')$, i.e., the area under the meson energy spectrum curve, we have the experimental results given in the first column of Table VI. It is quite apparent that a spherically symmetric angular distribution is incompatible with the experimental results. On the other hand, the $\cos^2\theta$ type cross section lies within the experimental error. We thus conclude that the predominant term in the cross section is of the form (p-$\cos^2\theta$) of Eq. (7), giving additional support to the conclusion of Sec. II that the meson is emitted into p-states.

The experimental error given in Table VI is consistent with a maximum of about 40 percent spherically symmetric contribution to the total cross section at 341 Mev, although, of course, the contribution may be much less. Experiments are now in progress to measure the cross section at 60 degrees. At this angle the $\cos^2\theta$ contribution is quite small ($\cong 1/20$ its value in the forward direction), so deviations from the $\cos^2\theta$ distribution should be readily detectable.

V. SUMMARY OF RESULTS

We have shown that the analysis of the production of mesons in nucleon-nucleon collisions near threshold can be greatly simplified by use of the requirements of angular momentum and parity conservation and of charge symmetry. Further simplification is introduced if the meson is assumed to be pseudoscalar, in conformity with other experimental evidence.

Analysis of the experimental data indicates that the pseudoscalar meson is emitted predominantly into p-states, since this assumption allows prediction of (1) small (PP, π^0) cross section; (2) large ratio of (NP, π^0) and (NP, π^+) to (PP, π^0); (3) approximate $\cos^2\theta$ angular distribution of π^+ mesons produced in PP collisions; (4) predominance of final NP triplet states in (PP, π^+) process and consequent large probability of deuteron formation.

The consequences of the hypothesis of charge symmetry are also of particular interest for the processes of meson production, as is apparent from an examination of Table III. Since experimental evidence indicates that the predominant final state in the production process is a nucleon s-state and a meson p-state, it also

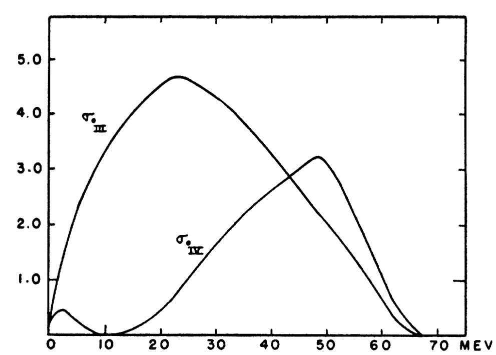

FIG. 3. Differential cross section (in the laboratory system) in the direction of the beam for producing mesons in 343-Mev nucleon collisions when the nucleons in the final state are in p-states. The energy scale refers to the meson kinetic energy; the ordinate is given in arbitrary units. The cross section σ_{0III} and σ_{0IV} refer to the respective couplings of expressions (8) and (9). The lack of high energy peak is readily apparent from a comparison with Figs. 1 and 2. *Note added in proof:* The labels on the cross sections in the figure are incorrect. For "III" read "(p,s)"; for "IV" read "(p,p)."

[14] Richman, Cartwright, and Whitehead (to be published soon). A similar spectrum, with equivalent energy resolution, has been obtained by Peterson, Iloff, and Sherman (private communication) at 18°.

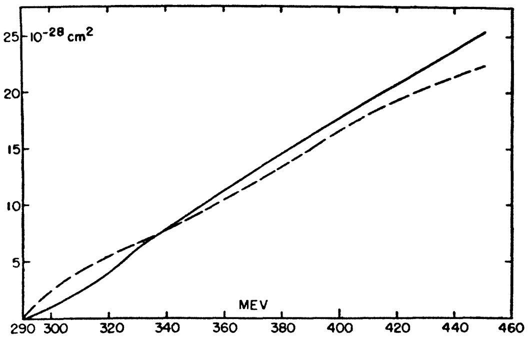

FIG. 4. Variation of the total cross section for meson production with the energy of the incident nucleon. These curves are the consequences of the (*S*) type of coupling of expression (5). The cross sections are arbitrarily normalized to 8×10^{-28} cm^2 at 340 Mev. The solid curve is for a final nucleon singlet state, the dashed curve is for a final nucleon triplet state and includes the possibility of deuteron formation.

follows that

$$\sigma(NP,\pi^0)=\sigma(NP,\pi^+)+\tfrac{1}{2}\sigma(PP,\pi^+). \tag{21}$$

In addition, a comparison of the energy spectrum of π^+ and π^- mesons produced in NP collisions will give a direct comparison of NN and PP singlet potentials through the dependence of the cross sections on $|\psi_F(0)|^2$ [Eq. (2)]. Further information concerning the properties of nuclear forces at close distances may very well be obtained from such experiments.

The relation of these results to meson theory is of some interest. This is so, in particular, since pseudoscalar meson theory has in many cases given reasonable qualitative agreement with experiment. Brueckner[2] found that pseudoscalar theory with pseudovector coupling predicted correctly the $\cos^2\theta$ angular distribution (for proper choice of the coupling constants), the predominance of final nucleon triplet states in (PP,π^+) production, and the smallness of the (PP,π^0) cross section. These results are rather open to doubt, however, since the pseudoscalar theory gives completely incorrect predictions of NP scattering which is, of course, intimately connected with meson production.

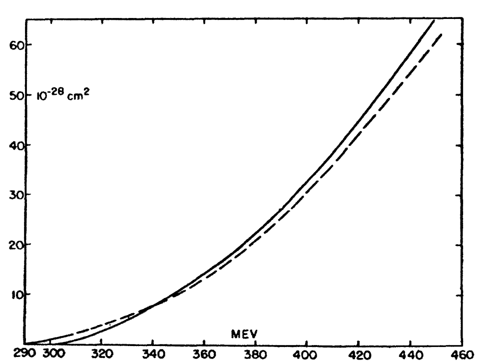

FIG. 5. Variation of the total cross section for meson production. The definitions and symbols are the same as for Fig. 4 except that here the (*p*-) type coupling of expressions (6) and (7) is assumed.

The phenomenological treatment of Marshak and Foldy[15] can give approximate agreement with the angular distribution only if a strong tensor force is included in the N-P interaction. Such methods of analysis, however, are of doubtful validity because of their intimate connection with weak coupling perturbation theory and because of the failure of any one model to explain more than a few of the experimental results.

We wish to express our appreciation to Professor R. Serber, with whom much of the present work has been discussed and to whom we are indebted for many valuable suggestions. We are also indebted to Dr. C. Richman, Dr. H. A. Wilcox, Mr. W. Cartwright, Miss M. Whitehead, Dr. V. Z. Peterson, and their collaborators for aid in interpreting their experiments and for permission to quote their results in advance of publication. Further appreciation is expressed to Mr. D. Clark and Mr. W. Noh for their considerable assistance with the numerical calculations.

APPENDIX

We now consider a more detailed development of the ideas developed in Sec. II.

In Eq. (1) we introduced the transition matrix, $(\mathbf{q},\mathbf{p}'|R|\mathbf{p})$ for the description of meson production. This was defined in the center-of-mass coordinate system, and $\mathbf{p}$ was considered to be the relative momentum of the initial nucleons, $\mathbf{p}'$ that for the final nucleons, and $\mathbf{q}$ the relative momentum of the meson and the center-of-mass of the two nucleons. The partial wave analysis of the final state is most easily effected by such an expansion of R:

$$(\mathbf{q},\mathbf{p}'|R|\mathbf{p})=(q,p'|R_1|\mathbf{p})+q_i(q,p'|R_2|\mathbf{p})_i+(q_iq_j-\tfrac{1}{3}q^2\delta_{ij})(q,p'|R_3|\mathbf{p})_{ij}+p_i'(q,p'|R_4|\mathbf{p})_i+\cdots \tag{A-1}$$

(a summation over repeated indices is implied), where the various R_i $(i=1,2,\cdots)$ are functions of the magnitude only of the vectors $\mathbf{q}$ and $\mathbf{p}'$. The short range hypothesis developed in Sec. II implies that the R_i depend only weakly on the quantities q and p'. The zero range approximation used implies that we neglect any dependence of the R_i on q and p'. Then, neglecting for the present, those terms which depend on factors of p' (i.e., we keep only nucleon s-states), we can put R into the form

$$R=\Sigma_i\,\Gamma_i(p)O_i(\boldsymbol{\sigma},\boldsymbol{\tau},\mathbf{q},\mathbf{p}). \tag{A-2}$$

Here the $\Gamma_i(p)$ are real numerical functions of p only—i.e., the beam energy—and the O_i are homogeneous polynomials in both $\mathbf{q}$ and $\mathbf{p}$, multiplied by various combinations of the vector matrices of spin and isotopic spin, $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$, respectively.

Then the transition matrix $(F|R|I)$ [Eq. (2)] will be given by

$$(F|R|I)=\Sigma_i\,\Gamma_i(p)(F|O_i|I)(2\pi)^{\frac{3}{2}}\psi_F^*(0), \tag{A-3}$$

where $(F|O_i|I)$ is obtained from that of Eq. (A-2) by evaluating the matrix elements of $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$, and $\psi_F(0)$ is the final nucleon wave function, $\psi_F(r)$, evaluated at $r=0$ (because of the zero range hypothesis).

It is the quantities $(F|O_i|I)$ that determine the detailed nature of the cross sections—i.e., lead to cross sections of the type represented by expressions (5), etc. It is therefore of interest to see the symmetry restrictions that can be placed on these quantities.

These requirements can be easily discussed in terms of the operators $O_i(\boldsymbol{\sigma},\boldsymbol{\tau},\mathbf{q},\mathbf{p})$ of Eq. (A-2). On the assumption that the meson is pseudoscalar, these must be invariant under rotations and change signs upon coordinate reflections. Thus, the following

[15] R. Marshak and L. Foldy, Phys. Rev. **75**, 1493 (1949).

TABLE VI. The angular distribution for (pp, π^+) mesons is analyzed on the basis of experimental data (see references 10–14) at 0°, 18°, and 30° with respect to the beam direction. In the second column are given the experimental ratios of cross sections. The calculated ratios for center-of-mass spherical symmetry and $\cos^2\theta$ angular distribution are given [notation is that of Eqs. (6) and (7)] in the remaining two columns.

Ratio	Experimental	(P-sym)	(P-$\cos^2\theta$)
$\sigma(0°)/\sigma(30°)$	3.5±0.7	1.25	4.14
$\sigma_d(0°)/\sigma_d(18°)$[a]	1.6±0.3	1.08	1.54

[a] By σ_d we mean just the cross section for meson production with deuteron formation. Experimentally, this is the area under the high energy peak in the meson spectrum.

combinations of the vectors $\boldsymbol{\sigma}$, $\mathbf{q}$, and $\mathbf{p}$ are possible:

$$\begin{aligned}
A_1(1) &= (p\cdot q)^{n-1}\boldsymbol{\sigma}^{(1)}\cdot\mathbf{p},\\
A_2(1) &= (p\cdot q)^{n}(\boldsymbol{\sigma}^{(1)}\cdot\mathbf{q}),\\
A_3(1) &= (p\cdot q)^{n-2}\boldsymbol{\sigma}^{(1)}\cdot\mathbf{p}\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{p}\times\mathbf{q}),\\
A_4(1) &= (p\cdot q)^{n-1}\boldsymbol{\sigma}^{(1)}\cdot\mathbf{q}\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{p}\times\mathbf{q}),\\
A_5(1) &= \tfrac{1}{2}(p\cdot q)^{n-1}\boldsymbol{\sigma}^{(1)}\times\boldsymbol{\sigma}^{(2)}\cdot\mathbf{p},\\
A_6(1) &= \tfrac{1}{2}(p\cdot q)^{n}\boldsymbol{\sigma}^{(1)}\times\boldsymbol{\sigma}^{(2)}\cdot\mathbf{q},
\end{aligned}\tag{A-4}$$

(where n is a positive integer). The superscript "1" refers to one nucleon, "2" refers to the other. Similar quantities $A(2)$ can be defined by interchanging "1" and "2" and replacing $\mathbf{p}$ by $-\mathbf{p}$ (i.e., interchanging the nucleons). The quantities O_i must be invariant with respect to interchanging the nucleons.

Evaluation of the spin matrices in Eqs. (A-4) for a given process when combined with the results of Eqs. (17) leads immediately to numerical values for the quantities g_{FI} of expressions (5), etc. —that is, to the structure of the $(F|O_i|I)$ of Eq. (A-3). We thus find by a suitable choice of the A's of Eq. (A-4) and by the hypothesis of charge symmetry an almost complete specification of the cross sections—we have then only two arbitrary numerical factors with which to adjust the absolute magnitude of the total cross sections, one for M_2 and one for M_3 of Eqs. (17).

The characteristics of the leading term in the (pp, π^+) cross section determine this choice of the A's uniquely to the approximation that the cross section is represented by the leading $p(\cos^2\theta)$ term—for the matrices M_3, at least. This is a linear combination

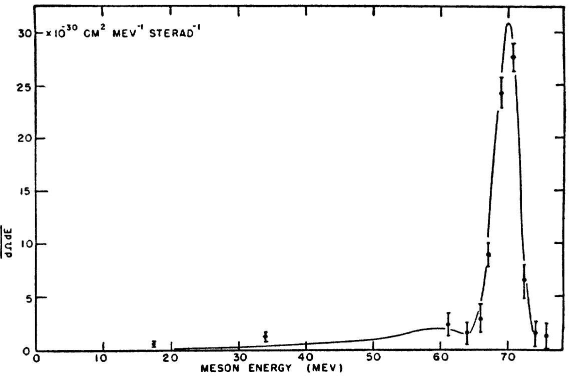

FIG. 6. Differential cross section for meson production in the direction of the beam at 341 Mev. The solid curve is calculated from the (p-) type coupling of expressions (6) and (7), i.e., for mesons emitted into p-states and a final triplet state for the neutron and proton. This curve has been "folded" into the experimental energy resolution. The experimental points are those of Cartwright, Richman, and Whitehead (reference 10).

of A_1 and A_5 with $n=2$. This linear combination does not lead to further ambiguity (i.e., introduce new arbitrary parameters) however, since these terms differ only by factors of the spin singlet and triplet projection operators. This distinction is not significant, as M_2 and M_3 are resolved into spin eigenstates. We can thus consider just the term A_1; although we are strictly justified in assigning it to M_3 only, since the (np, π^+) cross section has not been measured as yet.

If charge symmetry is not assumed, it is easy to enumerate the few possible combinations of τ-operators possible in the $O_i(\boldsymbol{\sigma}, \boldsymbol{\tau}, \mathbf{q}, \mathbf{p})$. This leads to more numerical parameters than we had before, which can be used, if necessary, to adjust to the experimental cross sections.

We have concentrated largely on the leading term in the cross section. When experimental evidence warrants the use of correction terms, it will be necessary to consider others of the A's in Eqs. (A-4).