A Symmetry Principle in the Fermi Theory of Beta-Decay*

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A GENERAL interaction operator for beta-decay in the form[1,2]

$$V=G\sum_{k=1}^{5}\left[C_kV_k+C_k^*V_k^*\right] \tag{1}$$

can be constructed from the five invariant operators

$$V_k=\sum_l Q_l\,{}^k\Omega_l(\psi^*(-)\,{}^k\Omega\phi(-))_l \tag{2}$$

in which Q_l is a displacement operator acting on the charge variable of the lth nucleon, $\psi(-)$ is a quantized wave amplitude for negative electrons, and $\phi(-)$ the corresponding quantity for neutrinos. The symbols ${}^k\Omega$ denote the scalar (1), polar vector (2), tensor (3), axial vector (4), and pseudoscalar (5) type operators constructed from products of Dirac matrices.

In a theory containing only one invariant operator V_k the phase of the coupling constant C_k has no physical consequences. Conventionally C_k is taken real.[1,2] In the general theory containing linear combinations the relative phases of the C_k's enter directly into the transition probabilities and influence the half-lives and energy distributions. The conventional choice of real coupling constants then appears to be without physical justification.[3] We report here on some consequences of permitting arbitrary relative phases. The recent paper by De Groot and Tolhoek should be consulted for background and details of calculation.

In the approximation which neglects the distortion of the electron wave functions by the coulomb field of the nucleus the allowed transition probability for $\beta^\pm$ emission per unit energy range and unit time is simply

$$P_\pm(E)=(1/2\pi^3)pE(E-E_0)^2G^2\bigg[(|C_1|^2+|C_2|^2)\left|\int 1\right|^2 +(|C_3|^2+|C_4|^2)\left|\int\sigma\right|^2\mp(1/E)\bigg\{(C_1C_2^*+C_1^*C_2)\left|\int 1\right|^2 +(C_3C_4^*+C_3^*C_4)\left|\int\sigma\right|^2\bigg\}\bigg]. \tag{3}$$

Existing measurements place a fairly small upper limit on the ratio of cross to diagonal terms. The cross terms vanish if

$$\begin{aligned}C_1C_2^*+C_1^*C_2&=0,\\ C_3C_4^*+C_3^*C_4&=0.\end{aligned} \tag{4}$$

In the same way, assuming real coupling constants, De Groot and Tolhoek obtain $C_1C_2=C_3C_4=0$. These drastic conditions appear as highly singular special cases in the context of complex coupling constants.

Equation (4) is also a consequence of the symmetry principle introduced by De Groot and Tolhoek. These authors postulate that transition probabilities for positron and negatron emission have the same form when effects produced by the coulomb field of the nucleus are neglected. From the postulate they obtain either $C_1=C_4=C_5=0$ or $C_2=C_3=0$.

In the usual representation of the Dirac matrices the operator for charge conjugation is $\mathfrak{C}=i\beta\alpha_y$. Wave amplitudes for positively charged particles are provided by the operation of charge conjugation applied to $\psi(-)$ and $\phi(-)$: i.e.,

$$\psi(+)=\mathfrak{C}\bar{\psi}(-),\quad \phi(+)=\mathfrak{C}\bar{\phi}(-), \tag{5}$$

the bar denoting a Dirac function identical with the starred quantity except that it is written as a four element column vector. Similarly a bar on Ω replaces each element by its conjugate complex (transpose of $\Omega^\dagger$). It is then possible to write

$$\begin{aligned}V_k^*&=\sum_l Q_l^\dagger\,{}^k\Omega_l^\dagger(\bar{\psi}^*(-)\,{}^k\bar{\Omega}\bar{\phi}(-))_l\\ &=\sum_l Q_l^\dagger\,{}^k\Omega_l^\dagger(\psi^*(+)\mathfrak{C}\,{}^k\mathfrak{C}\bar{\Omega}\phi(+))_l\\ &=\epsilon_k\sum Q_l^\dagger\,{}^k\Omega_l(\psi^*(+)\,{}^k\Omega\phi(+))_l\end{aligned} \tag{6}$$

with $\epsilon_1=\epsilon_4=\epsilon_5=-1$, $\epsilon_2=\epsilon_3=1$.[2,4] Except for a numerical factor and the appearance of $Q^\dagger$ in place of Q, V_k^* is the same functional of $\psi^*(+)$ and $\phi(+)$ as V_k is of $\psi^*(-)$ and $\phi(-)$. Now, restating the postulate of De Groot and Tolhoek, we ask for conditions on the coupling constants required to insure the relation[5]

$$P_+(E,Z)=P_-(E,-Z) \tag{7}$$

in all orders. These conditions are

$$\epsilon_kC_k^*=e^{i\theta}C_k,\quad k=1,2,3,4,5, \tag{8}$$

with θ an arbitrary phase. Equation (4) is a consequence of Eq. (8). Choosing $\theta=\pi$ for convenience, C_1, C_4, and C_5 are real numbers while C_2 and C_3 are pure imaginary. The antisymmetric interaction of Critchfield and Wigner[6] is a special case, with C_1, C_4, and C_5 assigned particular values and $C_2=C_3=0$.

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1 M. Fierz, Z. Physik **104**, 553 (1937).
2 S. R. De Groot and H. A. Tolhoek, Physica **16**, 456 (1950).
3 We are indebted to Dr. Fierz for a communication on this point.
4 W. H. Furry, Phys. Rev. **51**, 125 (1937).
5 Between a given pair of nuclear states.
6 C. L. Critchfield, Phys. Rev. **63**, 417 (1943).