Scattering of π-Mesons by Deuterium*

S. Fernbach, T. A. Green, and K. M. Watson
Radiation Laboratory, University of California, Berkeley, California
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For sufficiently high meson energies the impulse approximation[1] may be used to express the scattering cross section for deuterium in terms of the free particle cross sections for protons and neutrons. The evaluation of such an expression is of interest, since, under appropriate conditions, it may permit an estimate of the meson-neutron scattering cross section from the directly observable hydrogen and deuterium cross sections. The theoretical background for such a calculation has already been given by Chew,[1] who has treated the very similar but more complicated problem of the scattering of neutrons by deuterium.

The starting point in the calculation is the use of the impulse approximation, in which one assumes that the incident particle is scattered by a free nucleon wave packet whose momentum distribution is the same as that of one or the other of the bound nucleons. Then, following Chew, one argues that the contribution of each nucleon to the scattering amplitude is given, to a good approximation, by the product of the free particle scattering amplitude and an overlap integral which depends on the initial and final wave functions. One may simplify the calculation of the differential cross section by integrating it over the relative coordinates of the outgoing nucleons, thus considering only the scattering angle of the meson as the quantity to be observed. Then, according to the arguments of Gluckstern and Bethe,[2] one can use plane waves for the wave functions in the final state. The differential cross section for the scattering of a meson into the solid angle $d\Omega$, situated about the laboratory scattering angle θ, may then be written:

$$\frac{d\sigma}{d\Omega}(\theta)=\left[\frac{d\sigma_p}{d\Omega}(\theta)+\frac{d\sigma_n}{d\Omega}(\theta)\right]H_1(\theta)+2\cos\omega\left(\frac{d\sigma_p}{d\Omega}\frac{d\sigma_n}{d\Omega}\right)^{\frac{1}{2}}H_2(\theta), \quad (1)$$

where $d\sigma_p(\theta)/d\Omega$ and $d\sigma_n(\theta)/d\Omega$ are the free particle cross sections, $\cos\omega$ is an unknown phase factor, and the weighting factors $H_1(\theta)$ and $H_2(\theta)$ are defined by

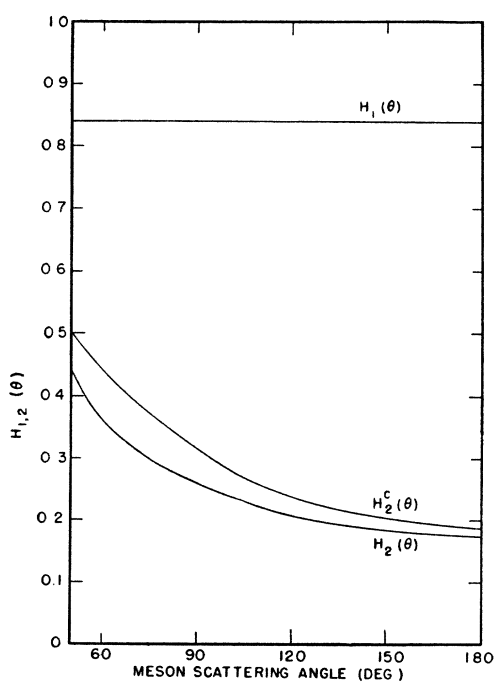

Fig. 1. Free particle and correlation term weighting factors for $\pi-D$ scattering.

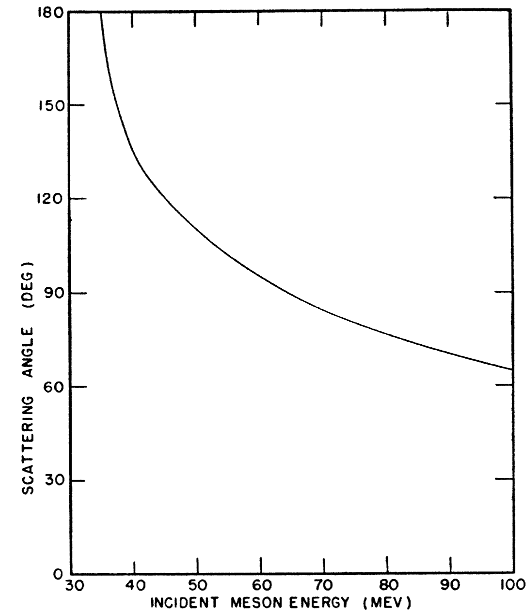

Fig. 2. Meson scattering angle beyond which correlation term is small.

$$H_1(\theta)=(1/J_0)\int d\mathbf{k}J(\mathbf{k})\phi_D^2[\mathbf{k}+\tfrac{1}{2}(\mathbf{q}(\mathbf{k})-\mathbf{q}_0)],$$

$$H_2(\theta)=(1/J_0)\int d\mathbf{k}J(\mathbf{k})\phi_D[\mathbf{k}+\tfrac{1}{2}(\mathbf{q}(\mathbf{k})-\mathbf{q}_0)]\times\phi_D[\mathbf{k}-\tfrac{1}{2}(\mathbf{q}(\mathbf{k})-\mathbf{q}_0)]. \quad (2)$$

In Eq. (2), $\mathbf{q}_0$ and $\mathbf{q}(\mathbf{k})$ are the momenta of the incident and outgoing mesons, respectively, $\mathbf{k}$ is the relative momentum of the outgoing nucleons, ϕ_D is the fourier transform of the deuteron wave function; $J(\mathbf{k})$ is the volume of momentum space available to the scattered meson, and J_0 is the corresponding quantity for scattering by free particles. The integrations are to be extended over all values of $\mathbf{k}$ compatible with energy conservation.

Equation (1) should best approximate the differential cross section for high meson energies and large scattering angles. In Fig. 1 we have plotted $H_1(\theta)$ and $H_2(\theta)$ for 70-Mev incident mesons and for scattering angles greater than 50 degrees.

One may obtain another approximate expression for the differential cross section, this one being best for small scattering angles, by neglecting the restriction imposed on $\mathbf{q}(\mathbf{k})$ by energy conservation. If one replaces $\mathbf{q}(\mathbf{k})$ and $J(\mathbf{k})$ by appropriate average values and integrates over all $\mathbf{k}$-space, there results the closure approximation, for which

$$H_1^c(\theta)=1, \quad H_2^c(\theta)=\langle\sin|\mathbf{q}-\mathbf{q}_0|r/|\mathbf{q}-\mathbf{q}_0|r\rangle_{\text{Av}}. \quad (3)$$

In Eq. (3), $\mathbf{q}$ is defined, for a given angle θ, by the free particle energy-momentum conservation laws, while the average is taken over the square of the deuteron wave function. As θ increases, $H_2^c(\theta)$ decreases from its initial value, unity. For purposes of comparison with the plane wave approximation, we have plotted $H_2^c(\theta)$ in Fig. 1. One sees that the two approximations give values of $H_1(\theta)$ and $H_2(\theta)$ which agree to within 15 percent. This indicates that the readily applied closure approximation may be used with some justification for the discussion of the cross section at other meson energies. On this basis we have calculated, for various incident meson energies, the scattering angle beyond which the correlation effects will be small. As a criterion of smallness we have taken $H_2^c(\theta)<\frac{1}{3}H_1^c(\theta)$. The results are shown in Fig. 2. They are of interest, since they indicate under which experimental conditions one might estimate the meson-neutron cross section without a knowledge of the phase factor $\cos\omega$.

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[1] G. F. Chew, Phys. Rev. **80**, 196 (1950).
[2] R. L. Gluckstern and H. A. Bethe, Phys. Rev. **81**, 761 (1951).