The Beta-Ray Spectrum of Tl^{206}

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THE beta-decay of 4.23-minute Tl^{206} has been studied because of its relationship to previously reported work[1] on Bi^{206} and Po^{210}. All three of these nuclides decay to Pb^{206}, and it therefore seemed worthwhile to determine whether any of the excited states of Pb^{206} are involved in Tl^{206} decay. Previous work[2] with absorption methods indicated that Tl^{206} emits beta-rays of about 1.7-Mev maximum energy and no gamma-rays.

Samples of pure thallium metal were irradiated in the Brookhaven nuclear reactor and a beta-activity of 4.3±0.1-min half-life was observed. A search for gamma-radiation from strong sources was made with a scintillation spectrometer by photographing the pulse height distribution presented on an oscilloscope screen. These photographs revealed no gamma-ray structure, other than a continuum of energies mostly below 500 kev which can be accounted for only as bremsstrahlung from the beta-rays.

A lens spectrometer set for 3 percent resolution was used to study in detail the electron spectrum from evaporated foils of thallium metal approximately 2 mg/cm² thick. The foils were activated in the reactor for 5 minutes and delivered in a "rabbit" through the pneumatic tube system to an outlet near the spectrometer. A monitor counter, arranged to detect beta-rays directly from the source, allowed the counting interval to be matched to the source strength, thereby removing the effect of source decay.

In a search for gamma-rays no internal conversion lines were found. The Kurie plot of the beta-spectrum is shown in Fig. 1. Relativistic Fermi functions from Feister's table[3] were used in the calculations, and calibration was taken from the conversion line of Cs^{137}. The plot is linear above about 0.6 Mev and has an end point at 1.51±0.01 Mev. The deviation from linearity is about what one would expect from the source thickness and does not contradict the conclusion that the spectrum is simple and of the allowed type.

The result is in general agreement with an analysis of the internally converted gamma-rays of Bi^{206} and Po^{210} by Goldhaber and Sunyar,[4] according to which the first excited state of Pb^{206} at 803 kev is assigned a spin of 3 and odd parity. Since the decay of Tl^{206} to the ground state of Pb^{206} is allowed and therefore involves

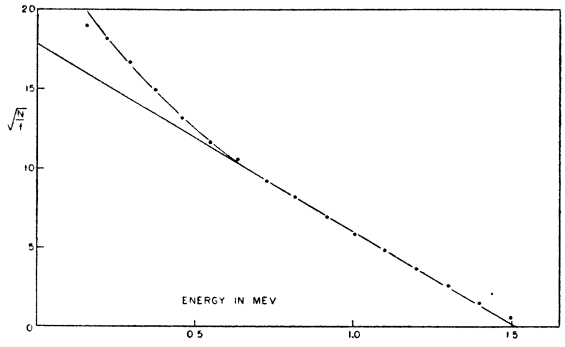

FIG. 1. Kurie plot of the Tl^{206} beta-ray spectrum.

a small nuclear spin change, one would expect that partial beta-decay to the first excited state is forbidden. Further analysis has not been possible, because the complete level scheme of Pb^{206} is not yet established.

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The Combination of Resistivities in Semiconductors*

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THE analysis[1] of resistivity *vs* reciprocal temperature curves for semiconductors, particularly germanium alloys, has shown that the experimental resistivity may be explained as due to a combination of a resistivity ρ_L due to the scattering of carriers by lattice ions and a resistivity ρ_I due to scattering by impurity ions; a complete analysis must also consider scattering processes due to neutral impurity atoms,[2] grain boundaries between crystallites, and the presence of ionized impurity centers of both signs.[3] Shockley and Schottky in discussions have first pointed out that the total resistivity is not given by the arithmetic sum of the partial resistivities;[4] more recently this fact has been discussed by Jones.[5] The following discussion of the proper combination of ρ_L and ρ_I corrects errors in Fig. 1 and Eq. (6) of reference 4 and also points out that our original argument[6] for obtaining higher mobility values from observed data is modified only slightly by this correction.

If one assumes that the two scattering processes are approximately independent of each other, the effective mean free path is given by

$$1/l=1/l_L+1/l_I, \tag{1}$$

where l_L and l_I are the mean free paths associated with lattice and impurity scattering separately. If one assumes the Rutherford scattering model[7] for evaluating ρ_I, the resistivity when only impurity scattering is present, then l_I is proportional to the fourth power of the velocity. Let ρ_L represent resistivity due to lattice scattering alone, with ρ_I negligible. With both lattice and impurity scattering present, the use of Eq. (1) in the usual expression for the resistivity of semiconductors leads to the following relation between the sum $\rho_L+\rho_I$ and ρ, the total resistivity:

$$F=(\rho_L+\rho_I)/\rho=(1+b^2/6)\left\{1-b^2\int_0^\infty xe^{-x}(x^2+b^2)^{-1}dx\right\}, \tag{2}$$

where $b^2=6\rho_I/\rho_L$. Figure 1 shows the dependence of the ratio F upon the fraction $\rho_I/(\rho_L+\rho_I)$. By private communication, we have learned that Shockley had previously calculated this function and obtained results identical with those of Fig. 1.

By using Eq. (1) in the calculation of the Hall coefficient R, one obtains the following relation [equivalent to Jones' Eq. (3)]:

$$r=R/(1/ne)=(\pi^{1/2}/48)(b^2+6)^2F^{-2}\int_0^\infty x^{9/2}e^{-x}(b^2+x^2)^{-2}dx. \tag{3}$$

The dependence of r upon $\rho_I/(\rho_L+\rho_I)$ is shown in Fig. 2. The abscissas in Jones' figure are values of ρ_I/ρ; if allowance for this difference is made, it is found that our curve and that of Jones are in substantial agreement. The figure is also consistent with Shockley's proof that $r\geqslant 1$.

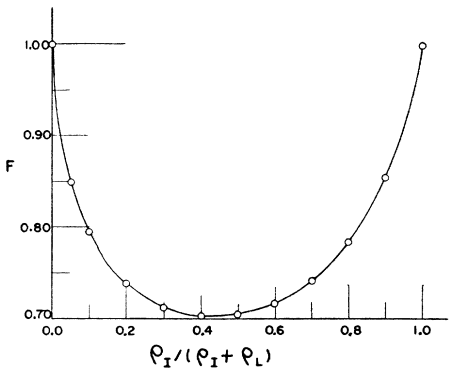

FIG. 1. The dependence of $F=(\rho_I+\rho_L)/\rho$ upon the ratio $\rho_I/(\rho_I+\rho_L)$.

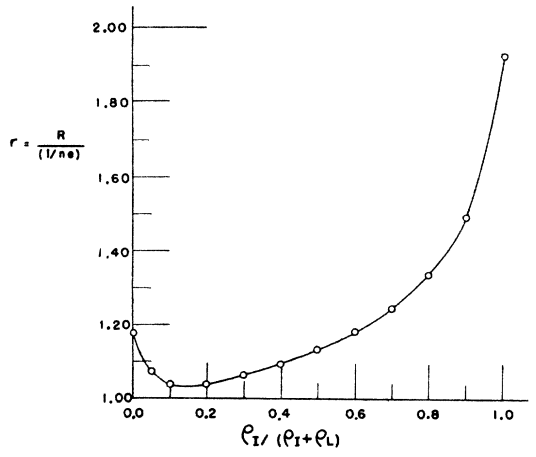

FIG. 2. The dependence of r, the ratio of the Hall coefficient R to $1/ne$, upon the ratio $\rho_I/(\rho_I+\rho_L)$.

We calculate mobility values higher than those suggested by Jones, for the following reason. Since the mobility associated with lattice scattering, b_L, is given by $b_L=(ne\rho_L)^{-1}$ and since $R=r/(ne)$, we find

$$b_L=R/(r\rho_L). \tag{4}$$

However, the experimental resistivity curve is the ρ-curve, and from its analysis one obtains ρ_L'- and ρ_I'-curves, which are determined as the curves of appropriate temperature behavior that add to give the ρ-curve. Since $\rho_L+\rho_I=F\rho$, and since F is constant to within four percent as the ρ_I/ρ_L ratio varies from $\frac{1}{4}$ to 3, it should be a good approximation to take $F\rho_L'$ as ρ_L, $F\rho_I'$ as ρ_I, and ρ_L'/ρ_I' as ρ_L/ρ_I. The r value can be found from Fig. 2 with ρ_L/ρ_I known, and thus b_L has been estimated for a number of N-type germanium samples, both polycrystalline and single crystal. The result is that the mobility values of about one-third of the samples is in agreement with the directly determined value,[8] and the values for the other two-thirds are too low by ten to thirty-five percent. However, the mobilities obtained in this way exceed those calculated from $(8/3\pi)(R/\rho_L)$ by factors up to a maximum of 1.65. There is a tendency for the mobility b_L to decrease with increasing impurity content, whereas one expects b_L to be determined by the properties of the germanium lattice and thus independent of impurities. Since the calculated mobilities for the impure samples tend to be too low, it is likely that the discrepancy is due to failure to consider scattering mechanisms other than lattice and impurity scattering.

* Work assisted by Signal Corps Contract.
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Criterion for Superconductivity

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WE have taken interactions between electrons and lattice vibrations into account by using a wave function for the complete system of the form:[1]

$$\Psi=\varphi_e(x_i, q_r)Q(q_r). \tag{1}$$

The wave function for the electrons, φ_e, which involves the vibrational coordinates, q_r, parametrically, is approximated by Slater-Fock determinental wave functions in which the wave functions for the individual electrons are of the form:

$$\Psi_k=a_k(q_r)\psi_k+\Sigma_{k'}a_{kk'}(q_r)\psi_{k'}. \tag{2}$$

The interactions important for superconductivity are those for which the Bloch states have energies close to the Fermi surface:

$$\epsilon_k\simeq\epsilon_{k'}\simeq E_F. \tag{3}$$

In the paper cited, k and k' were chosen in such a way that

$$E_F<\epsilon_k<E_F+\Delta E;\quad E_F-\Delta E<\epsilon_{k'}<E_F, \tag{4}$$

where ΔE is of the order of the energy change resulting from the interaction terms. The criterion for superconductivity was obtained from the condition that the states Ψ_k have a lower energy than the Bloch states they replace. This requirement is

$$\Delta E\sim N(E)\,|\mathfrak{M}_{kk'}q_{kk'}|^2{}_{Av}>(\hbar\omega_{kk'})_{Av}. \tag{5}$$

The distribution of states and criterion are then similar to those of Fröhlich's theory.[2]

It has been pointed out to the author by Van Vleck[3] that one will get a still lower energy by taking the k in the lower energy range and the k' in the higher range. When Eq. (5) is satisfied, the difference in energy is not large. However, there will always be some decrease in energy by forming the linear combinations no matter how weak the interaction. How should one then distinguish between interactions which occur in the normal phase and those which give rise to superconductivity? While a satisfactory answer has not been obtained, the following indicates the considerations which are involved.

Superconductivity is believed to be associated with a high curvature of the energy surface in k-space and consequent small effective mass of electrons with energies near E_F, as illustrated in Fig. 1.[4] The interactions in the normal state must be such that the effective mass is not altered very much.

We have calculated the first-order change in wave functions for a Fermi distribution of electrons at temperature T and find[5]

$$\Psi_k=\psi_k+\Sigma_{k'}\mathfrak{M}_{kk'}q_{kk'}\psi_{k'}/\{\epsilon_k-\epsilon_{k'}+\hbar\omega_{kk'}\coth[(\epsilon_k-\epsilon_{k'})/2\kappa T]\}. \tag{6}$$

The sum over k' is over all states, occupied as well as unoccupied. The term in $\hbar\omega_{kk'}$ comes from derivatives of φ_e with respect to the q's. The energy is changed in the second order:

$$E_k=\epsilon_k+\Sigma_{k'}|\mathfrak{M}_{kk'}q_{kk'}|^2/\{\epsilon_k-\epsilon_{k'}+\hbar\omega_{kk'}\coth[(\epsilon_k-\epsilon_{k'})/2\kappa T]\}. \tag{7}$$

The interaction energy does not depend very strongly on the wave vector k nor on the distribution of electrons in k-space. The interaction terms are of the sort expected for the normal phase; they give a small decrease in energy of the electrons but

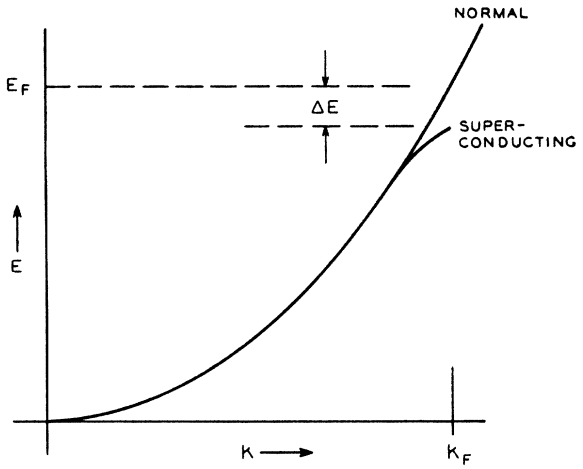

FIG. 1. Schematic diagram of energy, E, *versus* wave vector, k, for electrons in the normal and superconducting phases. The energy decrease $\Delta E\sim\kappa T_c$ is greatly exaggerated.