from the dipolar interaction in the second order. The results of Van Vleck[4] for ferromagnetic anisotropy may be carried over approximately if we set the local field $H=4\pi M_s/3$. Then the anisotropy constant at $T=0°\mathrm{K}$ is, apart from a numerical constant of the order of unity,

$$K_1\approx -M_s^2/S\approx -2\times 10^5 \text{ ergs/cc};$$

here $S(\approx 1)$ is the spin quantum number; for classical spins $(S\rightarrow\alpha)$ the effect vanishes. Anisotropy of this magnitude may also occur as the result of an anisotropic g-value, as in some cobalt salts.

Magnetostriction.—On Becker's classical calculation (FD Eqs. (B.5) and (2.3.16)) we estimate the magnetostriction

$$\lambda\approx\kappa M_s^2\approx 10^{-7},$$

where κ denotes the compressibility ($\approx 10^{-12}$ cm²/dyne). This may be modified by quantum effects.

Domain dimensions.—Except in properly bounded cubic crystals the domain width D in a slab of thickness $L(\approx 1$ cm) is given approximately by FD Eq. (4.1.6) or (4.2.5). As now $K\approx M_s^2$, both equations give approximately the same result:

$$D\approx[\sigma L/M_s^2]^{\frac{1}{2}}\approx[aL]^{\frac{1}{2}}\approx 2\times 10^{-4} \text{ cm}.$$

The energy density of the domain structure is $w\approx[M_s^2\sigma L]^{\frac{1}{2}}\approx 10^{-4}M_s^2$. If one has a flux closure arrangement without anisotropy energy FD Eq. (4.2.11) applies, and

$$D\approx[\sigma L\kappa/\lambda^2]^{\frac{1}{2}}\approx 0.2 \text{ cm}$$

and a correspondingly low energy. It is in either case economical to form domains for particles larger than a few unit cells, as is seen from FD Sec. 6.1.

The transition from a nonmagnetic dipole array to a saturated array, predicted by Sauer to occur for prolate spheroid axial ratios in excess of certain values, is not according to domain theory expected to take place. Instead, domains will be expected for almost every conceivable size and shape, if the lattice is appropriate.

Magnetization curves.—The results of Luttinger and Tisza must be modified. In a polycrystalline specimen or in a single crystal in a hard direction of magnetization a field $H_s\approx M_s/S$ will be required for saturation in the direction of the field, provided that the specimen is very long. In other cases H_s is probably determined largely by the demagnetizing factor N of the specimen, so that the magnetization curve is given by $M\approx H/N$, and the saturation field is $H_s=NM_s$.

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[1] For a review see C. Kittel, Revs. Modern Phys. **21**, 541 (1949); this will be referred to as FD.

[2] J. A. Sauer, Phys. Rev. **51**, 142 (1940); J. A. Sauer and A. N. V. Temperley, Proc. Roy. Soc. (London) **176**, 203 (1940).

[3] J. M. Luttinger and L. Tisza, Phys. Rev. **70**, 954 (1946).

[4] J. H. Van Vleck, Phys. Rev. **52**, 1178 (1937).

The Diamagnetism of Free Electrons in Finite Systems

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It has recently been suggested[1] that free electrons contained in a finite box show "almost perfect diamagnetism up to a critical field, and then a triple-valued transition (similar to the condensation of a van der waals gas) to a non-diamagnetic state. This behavior is provisionally identified with the perfectly diamagnetic behavior of a superconductor. . . ."

In the course of a detailed investigation of the magnetic properties of metals I have obtained results[2] which do not support this suggestion. When the magnetic field is sufficiently weak for the bulk of the electron orbits to be of greater radius than the specimen (for most metals this requires that $HR<5$ gauss cm, where R is the radius of the specimen), the volume susceptibility is given by[3]

$$\kappa=-\frac{xe^2R^{1/3}\zeta^{2/3}}{(2m)^{1/3}c^2\hbar^{4/3}}\left\{1-\frac{5}{54}\left(\frac{\pi kT}{\zeta}\right)^2\cdots\right\},$$

where ζ is the Fermi energy at absolute zero, and x a numerical factor equal to 0.0333 for an infinite cylinder (axis parallel to field) and 0.00717 for a sphere. This diamagnetic susceptibility at very low fields is greater than the Landau value (valid for large fields or large specimens) by a factor of the order $(R/d)^{1/3}$, where d is the mean spacing of the electrons. The Landau value is, however, so minute (10^{-6} cgs units for most metals) that the susceptibility at low fields only approaches the value necessary for perfect diamagnetism $(-1/4\pi)$ for specimens of several hundred miles radius—and then only fields less than $\sim 10^{-7}$ gauss would be excluded.

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[1] M. F. M. Osborne, Phys. Rev. **81**, 147 (1951).

[2] To be published in detail elsewhere.

[3] There are also some complicated terms periodic in the magnetic field. These are somewhat analogous to the periodic terms which give rise to the de Haas-van Alphen effect at high fields and, like them, are appreciable only at very low temperatures.

A Modification of the Isotopic Lead Method for Determination of Geological Ages*

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The method of determining geological ages from isotopic analyses of radiogenic lead samples was developed by A. O. Nier.[1,2] He used the formula,

$$\frac{\mathrm{N(Pb^{207})}}{\mathrm{N(Pb^{206})}}=\frac{\mathrm{N(U^{235})}}{\mathrm{N(U^{238})}}\cdot\frac{\{\exp[\lambda(\mathrm{U^{235}})t]-1\}}{\{\exp[\lambda(\mathrm{U^{238}})t]-1\}},$$

where $\mathrm{N(Pb^{207})}$ etc., refer to numbers of atoms of the indicated isotopes in the sample, $\lambda(\mathrm{U^{235}})$ and $\lambda(\mathrm{U^{238}})$ are decay constants, and t is the age of the sample in years.

Nier determined the ratio $\mathrm{N(Pb^{207})/N(Pb^{206})}$, or more simply $\mathrm{Pb^{207}/Pb^{206}}$, by mass spectrometer analysis of lead iodide ($\mathrm{PbI_2}$). Because $\mathrm{PbI_2}$ is a solid, both source and tube must be heated to about 350°C. Free lead is deposited on the inside of the mass spectrometer tube, which causes electrical leakage and necessitates frequent and troublesome cleaning. These experimental difficulties, together with the need for a mass spectrometer of high resolving power, have resulted in few isotopic lead ages being determined.

To assist in interpreting the geology of the Canadian Shield by age determinations, a mass spectrometer of high resolving power (better than one mass unit in 275) has been constructed in this laboratory and a new technique for analysis has been evolved.

Lead tetramethyl has been found satisfactory as a source of ions. It has sufficient vapour pressure at room temperature and does not contaminate the mass spectrometer tube. A method of preparing it from samples of lead chloride as small as 50 mg has been developed by Professor G. F. Wright and Mr. H. L. Cohen of the Department of Chemistry, University of Toronto.

The mass spectrograms obtained from lead tetramethyl are complex, but they can easily be divided into groups associated with $\mathrm{Pb^+}$, $\mathrm{Pb(CH_3)^+}$, $\mathrm{Pb(CH_3)_2{}^+}$, and $\mathrm{Pb(CH_3)_3{}^+}$ ions. The full interpretation cannot be discussed here, but values for the isotopic constitution of lead determined by examining the several ion groups agree with one another and with previous determinations, when corrections are made for the occurrence of such ions as $\mathrm{PbH^+}$, $\mathrm{Pb(CH_2)^+}$, $\mathrm{Pb(CH)^+}$, and $\mathrm{Pb(CH_3)_2(CH_2)^+}$, and for the occurrence of $\mathrm{C^{13}}$. The $\mathrm{Pb(CH_3)_3{}^+}$ ions are the most abundant, and the only related ions found to occur with them are $\mathrm{Pb(CH_3)_2(CH_2)^+}$ ions (1 percent). With a mass spectrometer of high resolving power the small ion currents due to $\mathrm{Pb(CH_3)_2(CH_2)^+}$ ions can be measured accurately, and the corrections for them and for $\mathrm{C^{13}}$ atoms are easily made. For these reasons analyses are being based upon the lead trimethyl ion group.