Letters to the Editor

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The Index of Refraction for Neutrons*

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IN a recent paper on variational principles in scattering theory, Lippmann and Schwinger[1] have treated the scattering of slow neutrons by bound protons as an application of their time independent formulation, which they obtain from time dependent scattering theory by means of several devices including the adiabatic reduction of the interaction to zero at $t=\pm\infty$. The merits of this formulation are (a) the exact transition rate is obtained in the form $\omega_{ba}=2\pi\delta(E_b-E_a)|T_{ba}|^2$, where T is an operator which in Born approximation is the interaction energy; (b) a variational principle is given for the T-matrix; (c) a sum rule is given which states that the total rate of transition out of the initial state Φ_a is $-2\,\mathrm{Im}T_{aa}$.

By a simple generalization from the neutron scattering by one bound proton we have considered the scattering by a thin slab of material containing many nuclei. The Φ_a component of the final state $S\Phi_a$, where S is the Heisenberg S-matrix, is $[1-i(m\lambda/2\pi)T_{aa}]\Phi_a$. When the sum rule is invoked, the imaginary part of T_{aa} gives the customary amplitude extinction factor $(1-\frac{1}{2}N\sigma x)\approx\exp(-\frac{1}{2}N\sigma x)$, where x is the thickness of the slab, and N is the nuclear density. Similarly, the real part of T_{aa} represents a phase shift, which one might interpret as due to the index of refraction of the slab. When the Fermi approximation to T_{aa} is used, the phase shift is in agreement with the usual formula for the index of refraction, $n=1-N\lambda^2a_c/2\pi$, where a_c is the isotopic average of the *bound* coherent scattering amplitudes of the nuclei.

One cannot expect an actual derivation of the index of refraction to follow from time dependent scattering theory, because from the time dependent viewpoint $S\Phi_a$ is the final state after the interaction has been adiabatically reduced to zero, and it is not clear why the phase shift in $S\Phi_a$ should necessarily be the same as that which the interaction would produce in a stationary wave function. To overcome this objection we have found a formal solution to the stationary eigenvalue problem identical with the $\Psi_a^{(+)}$ of Lippmann and Schwinger, which we show represents an incoming plane wave and an outgoing scattered wave. This stationary formulation is similar to that of Moller,[2] who introduced the S-matrix into stationary scattering theory. The connection between the S-matrix and the wave function is that the outgoing part of $(S-1)\Phi_a$ is identical with the scattered wave. On the basis of this theory we arrive at a general derivation of the index of refraction for gases, liquids, and solids.

This derivation of the index clearly shows that there is no doppler effect due to the motion of the individual nuclei, because the λ in the formula is the neutron wavelength relative to the boundary of the slab. The variational principle gives, besides the Fermi approximation, a temperature dependent correction to the simple formula, thereby taking into account the effect of virtual inelastic interactions of the neutron with the material. We have not as yet evaluated the correction in a specific case, but Lippmann[3] finds a correction of 0.2–0.3 percent to the scattering cross section of parahydrogen in its lowest rotational state for neutrons of zero energy. In liquids or solids at fairly high temperature and for neutrons of nonzero energy it would seem possible for the correction to be of the order of 1 percent. A change in the critical angle of reflection of a mirror of this amount would be significant in the experiment of Hughes, Burgy, and Ringo,[4] and might cause a detectable change in the intensity of a reflected beam of pile neutrons.

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[2] C. Moller, D. Kgl. Danske Vidensk. Selskab, Mat-fys. Medd. XXIII, Nr. 1 (1945).
[3] B. A. Lippmann, Phys. Rev. 79, 481 (1950).
[4] Hughes, Burgy, and Ringo, Phys. Rev. 77, 291 (1950).

On the Ferroelectric Curie Point of Tungsten Oxide

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SINCE Nagasawa[1] pointed out the possible existence of ferroelectric phenomena in tungsten oxide, the physical properties of this substance have been studied extensively at several laboratories in Japan. Matthias[2] also reported independently the experimental investigations of the ferroelectric properties of tungsten oxide. According to the preceding investigators, this substance was found to have many of the properties peculiar to ferroelectrics, i.e., the very high value of permittivity, the characteristic variation with a biasing field, the hysteresis loop, the domain structure, and the sensitive variation with external stress. On the other hand, the domain structure has not been influenced by an electric field, and its transition temperature has not been discovered up to about 550°C, which state of affairs makes us hesitate to conclude that tungsten oxide has ferroelectric properties. In order to clarify this situation, we have recently extended our experimental observations to higher temperatures and have found that tungsten oxide really has a spontaneous polarization with the Curie point at about 710°C. In the following the results of our observations are reported briefly.

The domain structure of a single crystal of area 0.01 mm^2 and thickness 0.01 mm was observed, up to 950°C, by a polarized-light microscope, using a very small electric furnace. On heating the crystal, the domain structure did not vary at all up to about 710°C, at which temperature the domain patterns vanished with the field of view becoming dark almost suddenly, and at still higher temperatures the birefringence was no longer discernible. On cooling, the domain patterns appeared almost suddenly at about 685°C, and the original features were approximately realized again at room temperature.

Next we prepared a rectangular crystal (0.4×0.2×0.02 mm) with a few bands inclined by 45° to the edges, which had two fired-on silver paste electrodes separated by 0.1 mm along the longest direction of the crystal. It was then connected to the 100-v terminals of a battery through a 20-kΩ rheostat, the resistance of the crystal being 2.5 kΩ. On account of the rather high conductivity, the temperature of the crystal is raised by the Joule's heat by decreasing the resistance of the rheostat, and at about 550°C the new 45° bands begin to appear. As the temperature is raised more new bands appear, and finally at about 700°C the whole crystal becomes dark suddenly. The successive disappearance of the 45° bands is observed as the temperature is lowered, i.e., as the resistance of the rheostat is increased, although nearly half of the newly-appeared 45° bands still remain at room temperature. It is shown by x-ray analysis that the a- and b-axes of a plate crystal lie parallel or perpendicular to the edges in the plane of the plate, and hence the abovementioned results can be clearly understood by assuming that tungsten oxide has a spontaneous polarization with Curie point at 710°C in the direction of the b-axis, and further by taking into account the generally accepted fact that the coersive field decreases with increasing temperature. The results of the x-ray analysis of the crystal structure suggest to us that the spontaneous polarization does not lie along the direction