tion inside will be $4\pi I/c$ and its total content

$$(4\pi\bar{R}^3/3)\cdot(4\pi I/c). \tag{17}$$

Equating (16) and (17), we get

$$\bar{R}^3=\tfrac{3}{4}(I_\odot/I)R_\odot^2\Lambda \tag{18}$$

or

$$\bar{R}=5.4\times10^{-3}\ \text{pc}=1.1\times10^3\ \text{astr. units.} \tag{19}$$

Assuming a larger than average density of interstellar matter in our neighborhood, Λ and $\bar{R}$ would become smaller.

It seems quite impossible that the sun can maintain a systematic magnetic field of the strength required to deflect hard cosmic rays appreciably within dimensions of the order $\bar{R}$. If we return, however, to the previous assumption that the magnetic field originates in the interstellar matter, there will always be "channels" allowing cosmic-ray particles produced by the sun to escape into galactic space. Then, however, the observed intensity of the general cosmic radiation can again only be explained by combining the theory of the "storage factor" with our idea of the existence of superactive stars all over the galaxy.

I am indebted to Professor Biermann and Dr. Schlüter for many interesting and stimulating discussions on problems of plasma physics and cosmic magnetic fields.

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Cosmic Radiation and Cosmic Magnetic Fields. II. Origin of Cosmic Magnetic Fields

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The equations governing the behavior of a wholly or partly ionized gas moving in the presence of a magnetic field are given. It is emphasized that the electric conductivity is independent of the magnetic field strength in many cases of actual interest. The stationary case of a magnetic field arising from the nonrigid rotation of a gaseous body is considered. Such a field is of the toroidal type. Fields of poloidal type may arise by the contraction of magnetized interstellar matter towards a star. The increase of strength of a magnetic field in turbulent conducting matter (stellar or interstellar) is considered. The conclusion is reached that the turbulence of every order of magnitude leads finally to a magnetic field, the energy density of which corresponds roughly to the energy density of the turbulence considered. If the magnetic field was weak in the beginning, this state was reached at first by the turbulence of smallest scale and smallest velocity, and then by the turbulence of higher orders; hence, the magnetic field strength must change secularly, as long as this process goes on. If our galaxy had only weak fields in its earlier stages, the present magnetic field should correspond to the turbulent velocity associated with distances of the order of 10^3 light years (between 10^{-6} and 10^{-5} gauss), and these fields should be more or less homogeneous over distances of this order. If there were already fields in the beginning, the present fields might be somewhat stronger.

INTRODUCTION

THE problem of the origin of stellar magnetic fields has caused considerable interest since Hale[1] communicated his measurements of the sun's general field. Although these results have not been confirmed lately, the discovery by Babcock[2] of highly magnetized stars has renewed this interest. No definite theory of the origin of these fields has as yet been proposed. We will show that one has to expect that differential rotation and other internal motions inside the stars will quite naturally cause magnetic fields, since electrons and ions will not move in exactly the same way just because of the different masses.

The existence of interstellar magnetic fields has been derived[3] from the properties of cosmic rays. If the cosmic rays occupied the whole universe with the energy density observed in our immediate neighborhood, their total energy would exceed that of light, and it would not be easy to escape the conclusion that more energy is continuously being transformed into that of cosmic rays than into that of heat radiation. Barring this possibility, it has to be assumed that the cosmic rays observed by us are continuously being produced within the galactic system or within a part of it. The approximate isotropy of their intensity is then explained by assuming interstellar magnetic fields of sufficient energy density to overcome the pressure of the cosmic-ray particles (10^{-5}–10^{-6} gauss). In this case the required production of cosmic rays would become much smaller than in the case of the first assumption; as discussed in detail by Unsöld[3a] in the preceding paper, it would be sufficient to assume that only a small fraction of the energy output of the stars is converted into cosmic-ray energy.

Here, of course, the question imposes itself of whether there are independent reasons for assuming galactic

[1] Hale, Seares, v. Maanen, and Ellermann, Astrophys. J. **47**, 206 (1918).
[2] H. W. Babcock, Astrophys. J. **105**, 105 (1947).
[3] H. Alfvén, Arkiv. Mat. Astron. Fysik **25B**, No. 29 (1937); Z. Physik **107**, 579 (1937); E. Fermi, Phys. Rev. **75**, 1169 (1949). See also H. Alfvén, *Cosmical Electrodynamics* (Oxford, 1950).
[3a] A. Unsöld, Phys. Rev. **82**, 857 (1951).

magnetic fields of the required order of magnitude. Alfvén showed that the interstellar gas, owing to its partial or complete ionization and especially because of the large dimensions involved, behaves in some respects like a conductor without resistivity: The Joule losses of electromagnetic fields are very small, and kinetic energy may be transformed by induction effects into magnetic energy until the densities of both have become comparable. The kinetic energy available is that of the interstellar gas, which, according to observation, is concentrated for the most part in clouds and shares the nonrigid rotation around the center of the galaxy of the stars, with irregular components of about 5 km/sec for normal compact clouds (but of up to 100 km/sec for the less compact ones). The growth of magnetic fields in turbulent conductive fluids has been studied further by Batchelor,[4] and by the present authors.[5] Reference should be made also, to the work of Elsasser and to that of Bullard on similar problems arising in the theory of the earth's magnetic field and its secular variations.[6]

The present authors have been especially concerned with the origin of the primary magnetic fields which are afterwards increased by induction effects. One mechanism, which operates as well in interstellar matter as inside stars, arises from the accelerations and pressure gradients because of the different masses of ions and electrons. The equations of the problem have been used in a form given by one of us (A.S.),[7] which takes explicit account of the "plasma" properties of the interstellar gas. This is rather essential in the presence of a magnetic field, especially with regard to the electric conductivity. By defining the original state of the magnetic fields, it becomes possible to draw definite conclusions regarding their rate of growth. The present state is probably not a stationary one, but the magnetic field still increases secularly.

THE PROPERTIES OF A QUASINEUTRAL IONIZED GAS ("PLASMA")

We restrict ourselves to the case in which the relative separation of electric charges is so small that the convection currents arising from mass motions can always be neglected.[8] Then, the following equation holds for the electric current density $\mathbf{j}$ in the presence of a magnetic field of strength $\mathbf{H}$, if the medium is moving with the mass velocity $\mathbf{v}$.

$$\mathbf{j}/\sigma=\mathbf{E}+(\mathbf{v}/c)\times\mathbf{H}-(\mathbf{j}\times\mathbf{H})(m_i-m_e)/e\rho c + (m_i\,\mathrm{grad}p_e-m_e\,\mathrm{grad}p_i)/e\rho. \quad (1)$$

Here, σ denotes the ordinary conductivity (esu), m_i and m_e the masses of the ions and electrons, respectively, p_i and p_e their partial pressures, ρ the mass density of the plasma, and $+e$ the charge of an ion. Here, $\mathbf{E}+(\mathbf{v}/c)\times\mathbf{H}=\mathbf{E}^c$ is the electric field as measured by an observer moving with the velocity of the plasma. It is evident that the field in this co-moving coordinate system is responsible for the relative diffusion of electrons and ions. The next term, containing the vectorial product of $\mathbf{j}$ and $\mathbf{H}$, is the one responsible for the diminution of conductivity in the presence of a magnetic field. The last terms indicate that, in addition to the electromagnetic causes, gradients of the partial pressures of the constituents tend to generate diffusion, i.e., electric current.

Equation (1) has been derived in a former paper[7] by a simple momentum balance for the single constituents. It is also contained essentially in the book by Chapman and Cowling.[8a] It has been used, without the pressure terms, by a great many authors.

Equation (1) is not sufficient for determining the current in a given electromagnetic field, because the current acts by the Lorentz force on the motion of the plasma. The equation of motion of the plasma

$$\rho d\mathbf{v}/dt=\mathrm{grad}p-(\mathbf{j}/c)\times\mathbf{H}$$
$$(d\mathbf{v}/dt\equiv\partial\mathbf{v}/\partial t+(\mathbf{v}\cdot\mathrm{grad})\mathbf{v}\,;\,p=p_i+p_e) \quad (2)$$

must be solved simultaneously with Eq. (1). It is advisable to eliminate by virtue of Eq. (2) the term $\mathbf{j}\times\mathbf{H}$ in Eq. (1). So one arrives at

$$\mathbf{j}=\sigma(\mathbf{E}^c+\mathbf{E}^i), \quad (3)$$

where

$$e\mathbf{E}^i=(m_e\,\mathrm{grad}p_e-m_i\,\mathrm{grad}p_i)/\rho+(m_i-m_e)d\mathbf{v}/dt. \quad (4)$$

$\mathbf{E}^i$ may be called an "impressed" electromotive field; it summarizes the effects of the nonelectrical forces (we have taken into account only pressure and inertia) on diffusion in terms of an equivalent electric field.

While of course Eqs. (3) and (2) are still interdependent, Eq. (3) no longer contains $\mathbf{j}$ explicitly on the right side. Thus, there is no explicit influence (except by the term $\mathbf{v}\times\mathbf{H}$ in $\mathbf{E}^c$) of the magnetic field on the current density. The formulation in Eqs. (3) and (4) is obviously superior to that of Eq. (1) in stationary cases (more precisely, if $d\mathbf{v}/dt=0$), when the elimination of the $\mathbf{j}\times\mathbf{H}$ term is achieved by a modification of the pressure terms (which often lead only to space charges compensating the resulting $\mathbf{E}^i$); but it is also useful in nonstatic cases. It shows that in many cases of actual interest it is the ordinary electric conductivity which determines the dynamical state of the plasma.

Equations (1) and (3) show that the electric phenomena are especially simple when viewed by a comoving observer. We, therefore, also transform Faraday's law. First, we define[9] an operator D/Dt which, when operated on a vector field $\mathbf{A}$, measures the flux of $\mathbf{A}$ through a surface element, the boundaries of

[4] G. K. Batchelor, Proc. Roy. Soc. (London) (A) **201**, 405 (1950).
[5] A. Schlüter and L. Biermann, Z. Naturforsch. **5a**, 237 (1950).
[6] An account of this work has just been given by W. M. Elsasser Revs. Modern Phys. **22**, 1 (1950).
[7] A. Schlüter, Z. Naturforsch. **5a**, 72 (1950).
[8] This will be allowed in practically all cosmic applications.
[8a] S. Chapman and T. G. Cowling, *The Mathematical Theory of Non-Uniform Gases* (Cambridge, 1939).
[9] Abraham-Becker, *Theorie der Elektrizität* (B. G. Teubner, Leipzig, Germany), Vol. I, Sec. 19; Vol. II, Sec. 45.

which are moved and distorted by the velocity field $\mathbf{v}$. According to the rules of vector analysis, this operator can be expressed by

$$D\mathbf{A}/Dt = \partial\mathbf{A}/\partial t + \mathbf{v}\,\mathrm{div}\mathbf{A} - \mathrm{curl}[\mathbf{v}\times\mathbf{A}].$$

When operating on the magnetic field it yields, because of $\partial\mathbf{H}/\partial t = -c\,\mathrm{curl}\mathbf{E}$ and the definition of $\mathbf{E}^c$,

$$D\mathbf{H}/Dt = -c\,\mathrm{curl}\mathbf{E}^c \qquad (5)$$

or, from Eq. (3)

$$D\mathbf{H}/Dt = c\,\mathrm{curl}\mathbf{E}^i - c\,\mathrm{curl}(\mathbf{j}/\sigma).$$

The last term shows that the free decay of a magnetic field is determined by the unchanged value of the conductivity and is therefore extremely slow for fields of cosmic dimensions. The first term describes the production (and annihilation) of magnetic lines of force by the impressed fields.

The preceding results can be generalized on several ways. It is easily possible to take into account other forces beside pressure and inertia. The general form of Eq. (4) is

$$e\mathbf{E}^i = (m_i\mathbf{f}_i - m_e\mathbf{f}_e)/(m_i + m_e), \qquad (6)$$

where $\mathbf{f}_i$ and $\mathbf{f}_e$ represent all nonelectromagnetic forces on a particle (ion or electron, respectively), including pressure gradients, inertial forces, radiative pressure, thermodiffusion, and so on.

If neutral atoms are present in considerable number, the exact equations become more complicated.[10] The result is, however, that to a sufficient approximation, if the Lorentz-force is small compared with grad$p$ in the equation of motion, Eq. (2), the impressed force is given by

$$N_e(\alpha_{in} + \alpha_{en})e\mathbf{E}^i = \alpha_{in}\,\mathrm{grad}p_e - \alpha_{en}\,\mathrm{grad}p_i, \qquad (7)$$

where N_e is the numerical density of electrons; α_{en} and α_{in}(g cm^{-3} sec^{-1}) measure the transfer of momentum between the neutral atoms and the electrons and ions, respectively. Since α_{in} happens to be very large compared with α_{en} because of the mass difference, we can write

$$e\mathbf{E}^i \approx (1/N_e)\mathrm{grad}p_e. \qquad (8)$$

In all cases we can assume the order of magnitude of $\mathbf{E}^i$ to be given by

$$e\mathbf{E}^i = \bar{m}d\mathbf{v}/dt, \qquad (9)$$

where $\bar{m}$ is a certain mean value of the masses of the constituents. In the case of Eq. (4) this is clearly a reasonable approximation. For an *HI* region, Eq. (9) gives a lower limit, since all local differences of the degree of ionization and of electron temperature give higher impressed forces than that given by Eq. (9).

The value of the conductivity may be taken from the work of Chapman and Cowling,[8a] under astrophysical conditions it is roughly proportional to $T^{\frac{3}{2}}$ and depends only to a small degree upon the density. In interstellar space its value may be assumed to be 10^{11} sec^{-1} in *HI* regions, and between 10^{12} and 10^{13} in *HII* regions, but up to 10^{18} in the interior of the sun.

The time scale of the free decay of a magnetic field or of its growth under the influence of a steady impressed electric field is of the order of magnitude $Q\sigma/c^2$ (where Q is a representative cross section), as follows from Eq. (5) and from $\mathrm{curl}\mathbf{H} = 4\pi\mathbf{j}$, if one puts approximately curl curl$\approx 4\pi/Q$. If the time available is small compared with $Q\sigma/c^2$, $\mathbf{E}^i$ is always very nearly equal to $-\mathbf{E}^c$. In interstellar space the smallest distances of interest are of order 10^{16} cm (1/100 light year), and the corresponding time scale is of order $\sigma\times 10^{11}$ sec or $\geqslant 10^{14.5}$ years.

STELLAR MAGNETIC FIELDS

We consider first the stationary magnetic field produced by nonrigid rotation of a gaseous body.[11] A magnetic field is maintained if the impressed force $\mathbf{E}^i$ arising from the inertia of the rotating masses cannot be compensated for by a suitable distribution of space charges, that is if $\mathrm{curl}\mathbf{E}^i \neq 0$. If $\mathbf{E}^i$ is due to rotation, this means $\partial\boldsymbol{\omega}/\partial z \neq 0$ ($\boldsymbol{\omega}$ is the angular velocity of rotation z the axis of rotation).

As an example, we assume the vector $\boldsymbol{\omega}$ proportional to $\mathbf{a}(R/|\mathbf{r}|)$ ($\mathbf{a}$ is a constant vector parallel to z, $\mathbf{r}$ the radius vector, R the radius of body). Furthermore, the conductivity inside the body is taken equal to a constant σ_0; outside the body it is equal to zero. The solution is

$$\mathbf{H} = 6\pi\sigma_0(\bar{m}/ec)[(R^2 - r^2)/r^2][\mathbf{a}\times\mathbf{r}](\mathbf{a}\cdot\mathbf{r}) \qquad (10)$$

inside the body, and $\mathbf{H} = 0$ outside of it. If we put $\sigma_0 = 10^{17}$ esu, $\bar{m} = 1.66\times 10^{-24}$ g, and $\boldsymbol{\omega}$ equal to the equatorial angular velocity of the sun, we get (φ latitude)

$$H = 0.94\cdot 10^3[(R^2 - r^2)/R^2]\sin 2\varphi \quad \text{(gauss)}. \qquad (11)$$

Since there is no reason to expect $\partial\boldsymbol{\omega}/\partial z = 0$ in actual stars, and the time scale considered above is comparable with the age of most of the normal stars, it appears that the magnetic fields directly created by diffusion effects in the interior of stars may be quite considerable.

The geometrical properties of the simple model discussed above—two toroidal fields of opposite direction symmetrical to the axis of rotation, one in each hemisphere—do not lead to fields observable at the surface; but it has been long recognized[12] that, for instance, the sunspot fields must be carried to the surface by induction effects.

In this special model the magnetic field is not disturbed by convection, since $\mathbf{v}\times\mathbf{H}$ vanishes everywhere.

[10] A. Schlüter, Z. Naturforsch. **6a**, 73 (1951).

[11] L. Biermann, with an appendix by A. Schlüter, Z. Naturforsch. **5a**, 65 (1950).

[12] T. G. Cowling, Monthly Notices Roy. Astron. Soc. **94**, 39 (1934).

This does not hold, of course, in more complicated types of motion.

The problem of how magnetic fields of poloidal types can arise by convection has been treated by Elsasser and by Bullard[6] for the case of the earth's magnetic field. It appears difficult to understand fields of this type and of their observed strength on stars.

If the existence of *interstellar* magnetic fields (which will be discussed in the following section) is assumed, another explanation suggests itself. These fields are supposed to be rather homogeneous over distances comparable with those between neighboring stars.

It seems very plausible to assume that stars of early spectral type augmented their mass to some extent by accretion of interstellar matter within the last 10^8 or 10^9 years. We consider the simple case of a homologous contraction towards the star. Then, it follows from $D\mathbf{H}/dt=0$, that

$$d(\mathbf{H}\rho^{-\frac{2}{3}})/dt=0;\quad d/dt=\partial/\partial t+\mathbf{v}\cdot\mathrm{grad};\qquad (12)$$

this may also be seen intuitively. If the original field has a strength of 10^{-6} gauss, a contraction by 10^{24} in the density would give a field of 10^{10} gauss. In this case the magnetic forces would overcome gravitation by several powers of 10. This means that the contraction must be nonisotropic; the matter will be attracted by the star mainly along the magnetic lines of force. The result will be a magnetized star with a field of poloidal type. It is difficult to estimate its probable strength, and only an upper limit can be given. Since the mean value of $\mathbf{H}^2/4\pi$ must be small compared with GM^2/R^4 (M is the mass, R the radius of the star), it follows that

$$H\ll 10^8 \text{ gauss.}$$

Furthermore, the relatively small conductivity in the outer layers has as a consequence that their currents maintaining the field decay much faster than in the interior, where the time scale $Q\sigma/c^2$ is of the order 10^9–10^{10} years. Hence, at the surface one would expect a field smaller by perhaps two powers of ten than in the far interior. The smaller density of these layers works in the same direction. Therefore, fields above 10^5 gauss at the surface would not be expected from this argument.

MAGNETIC FIELDS IN A TURBULENT FLUID

At the end of the last section, we considered the increase of a given magnetic field by a regular motion. Now the influence of irregular motions will be treated.

At first we study the consequence of the assumption $D\mathbf{H}/Dt=0$. It may be seen intuitively that any line of force is on the average continuously lengthened as long as the magnetic forces do not react on the motion. This is the case when the energy density of the magnetic field becomes comparable with that of the motion. The increase is essentially exponential with time until this limit has been reached.[13] This can be shown in the following way:

From $D\mathbf{H}/Dt=0$ and the equation of continuity, it follows that

$$\rho d(\mathbf{H}/\rho)/dt=(\mathbf{H}\cdot\mathrm{grad})\mathbf{v}\qquad (13)$$

(ρ is the mass density, and d/dt is the time derivative). If there are no substantial changes in the density and if the velocity field is essentially constant during a certain time τ, the magnetic field strength will therefore change exponentially (increase or decrease) during this time. Denoting a characteristic length of the irregular motion by L, a relation of the kind

$$\ln\{H(t+\tau)/H(t)\}=\alpha(v/L)\qquad (14)$$

will be valid, where α represents a positive or negative number of order of magnitude 1. According to the usual assumptions in the theory of turbulent motion, the time τ during which the state of motion remains essentially unchanged is just of the order v/L. If we, furthermore, assume that at any instant $t+T$ the velocity field is no longer correlated with that at the time t, where $T>\tau$, the value of $\ln\{H(t+T)/H(t)\}$ (which we shall denote by ξ) will be a statistical sum of T/τ numbers α of order of magnitude 1. For $T\gg\tau$, the probability of arriving at a certain value of ξ will be given by a gaussian distribution

$$\omega(\xi)=\omega_0\exp\{-2\xi^2/(T/\tau)\},\qquad (15)$$

where we have taken $\alpha=\pm 1$. The expectation value of H^2 will then be given by

$$\langle H^2(t+T)\rangle_{\mathrm{Av}}=H^2(t)\frac{\int\omega(\xi)e^{2\xi}d\xi}{\int\omega(\xi)d\xi}\qquad (16)$$

$$=H^2(t)\exp\{T/2\tau\}.$$

The exponential growth of the mean magnetic field strength in a turbulent fluid on which the magnetic field does not react, is characterized by a doubling time of the order of the lifetime of the turbulence elements. This increase, of course, takes place only if the time constant of the exponential decay by Joule losses is greater than this doubling time. In this way we arrive at the criterion

$$L^2\sigma/c^2>\tau,\qquad (17)$$

or, using again the relation between the characteristic

[13] This was first pointed out by G. K. Batchelor (reference 4), to whom we are indebted for valuable remarks. However, the conclusions arrived at by us regarding interstellar fields differ from those of Dr. Batchelor. *Note added in proof:* The result, that ultimately equilibrium between turbulent and magnetic energy density is reached in *every* order of magnitude, is supported by Elsasser's result (Phys. Rev. **79**, 183 (1950)), that (for an incompressible fluid) the hydromagnetic equations are symmetrical with respect to $\mathbf{H}(4\pi\rho)^{-\frac{1}{2}}$ and $\mathbf{v}$.

quantities of turbulence $\tau = L/v$,

$$Lv\sigma/c^2 > 1. \quad (18)$$

If this condition is satisfied, any magnetic field will grow to the limit given by the reaction of the field on the motion. The properties of the original field, e.g., its local derivations, do not enter into the discussion and are therefore quite inessential.

The field reacts on the motion when the term $(\mathbf{j}/c) \times \mathbf{H} \equiv (1/4\pi)[\text{curl}\mathbf{H} \times \mathbf{H}]$ in the equation of motion, Eq. (2), becomes comparable with the other terms. This amounts to saying that the magnetic energy density will finally reach the energy density of the turbulence considered (if the time scale permits this to happen).[14]

In the interior of all stars, electromotive forces are to be expected if there are any internal motions. These will generate magnetic fields, as discussed in the preceding section, and these magnetic fields will be amplified wherever turbulent motions exist. Let us consider for instance the sun. In that region below the surface where the hydrogen is just partly ionized, convection will take place up and down, and it will cause a kind of turbulent motion. The granulas are the outermost part of this hydrogen-convection zone. If we follow Schwarzschild's recent analysis,[15] the typical velocity in the most energetic elements is of the order 2 km/sec; they are too small to be directly visible, but their diameter has been estimated to some 100 km. The mean lifetime of these elements should be of the order 1 min, as compared with the time constant for spontaneous decay of 10 days. ($\sigma \approx 10^{13}$ esu). Therefore, our criterion for turbulent amplification is fulfilled. The doubling time being extremely short as compared with the age of the sun, any original magnetic field will have grown to the upper limit given by the energy density of the motion. This, in the granulas, is of the order 10^3 erg/cm,^{3} so that a magnetic field of order 10^2 gauss would result. This discussion is rather superficial, because we see only the outermost part of the hydrogen convection zone, which is perhaps not typical. On the other hand, the density gradient will cause a diminution of the magnetic field somewhat similar to the $\rho^{\frac{2}{3}}$-law of the preceding section. It is not yet known how rapidly the density increases inward within the zone; a value of 10^{-7} gr/cm^3 has been adopted before, which is probably a lower limit for the interior of the zone.

In the same way in all other stellar convection zones, magnetic fields should be present (e.g., in the central convective core of the sun, if this core exists) but not enough is known about their extension and state of motion to draw any definite conclusion.

[14] The variations of **H** over smaller distances tend to average out in Eq. (2). For the detailed argument see Sec. VIII of our paper (reference 5).

[15] R. S. Richardson and M. Schwarzschild, Astrophys. J. **111**, 351 (1950).

INTERSTELLAR MAGNETIC FIELDS

The same principles apply also to the matter between the stars. But the situation is rather different in several respects. Let us take for instance $L \approx 300$ to 400 parsec $\approx 10^{21}$ cm as a characteristic length, and the corresponding irregular velocity as $v = 5$ km/sec. That value of the velocity is an approximate mean of the peculiar velocities of the heavy interstellar gas clouds, the absorption lines of which have been observed with great accuracy by Adams.[16] It follows that the doubling time is of the order L/v $10^{15.3}$ or $10^{15.4}$ sec or 1/50 to 1/40 of the age of the galaxy, if the latter is assumed to be 10^{17} sec$=3\times10^9$ years. During the existence of the galaxy, any weak original magnetic field will have grown by a factor 10^{12}–10^{15}. The most conservative estimate of the present field strength is obtained in the following way. We assume that in the earliest stages of the galaxy, during which its state of motion was already similar to the present one, no magnetic fields were present except those which were produced by the electromotive forces of the galaxy itself. The order of magnitude of these is given, according to Eq. (4), by

$$eE^i \approx m_H dv/dt \approx m_H v^2/L \quad (19)$$

(m_H = mass of a proton) or by

$$|D\mathbf{H}/Dt| \approx (cm_H/e)(v^2/L^2). \quad (20)$$

Then, after the first characteristic time of the turbulence, we arrive at a magnetic field of the order

$$H_0 \approx (cm_H/e)(v/L), \quad (21)$$

since during the time v/L the creation of magnetic flux will be in the same sense. Inserting the values for v and L used above, Eq. (21) yields

$$H_0 \approx 10^{-19} \text{ gauss.}$$

This we take as the starting field of the amplification process, and we arrive at a present magnetic field of

$$H \approx 10^{12} \cdots 10^{15} \cdot H_0 \approx 10^{-7} \cdots 10^{-4} \text{ gauss,}$$

the energy density of which is roughly of the order of the kinetic energy of the turbulent motion considered (10^{-13} erg/cm^{-3}). The latter is the upper limit for the magnetic energy density which can be produced by the turbulence of this characteristic velocity.

Had we taken a smaller value for the characteristic length, the starting field as well as the amplification factor would have been greater. Then the limit given by the turbulent energy density would have been reached in the past. When this was the case, the turbulence of this order of magnitude was no longer able to lengthen the magnetic lines of force. We may assume that the kinetic energy of turbulence associated with turbulence of a certain characteristic length L is greater, the

[16] W. S. Adams, Astrophys. J. **97**, 105 (1943); **109**, 354 (1949); see also A. Schlütter and P. Stumpff, Z. Astrophys. (to be published).

greater the value of L.[17] Then the turbulence of greater scale is not much changed by the magnetic field belonging to turbulence of a smaller scale.

The motions which bring about the amplification in the largest dimension considered take place mainly in the plane of the galaxy. Hence, it must be concluded that the magnetic lines of force also run more or less parallel to this plane. Furthermore, for the reasons given above, the magnetic field should be rather homogeneous over distances comparable with this same L. Local differences are to be expected, especially in places where the mass density is above average.

The magnetic term in the equation of motion, Eq. (2), acts in the direction perpendicular to the plane of the galaxy—on the average as a force directed away from the plane. Also, the pressure exerted by the cosmic-ray particles acts in the same direction. Both must be compensated for by the gravitational force. It now seems likely that these are the most essential factors governing the lateral extension of interstellar matter.

The interstellar gas clouds commonly observed with high dispersion[16] should be strongly affected by the magnetic field. Components of motion perpendicular to H are not prohibited, but they must be of such character that no permanent lengthening of the lines of force results (e.g., oscillatory).

[17] This is equivalent to the statement that the mean square of the (irregular) velocity difference v between two points, whose separation is L, increases with increasing L. The Kolmogoroff-Weizsäcker theory of isotropic statistical turbulence predicts $v \sim L^{\frac{1}{3}}$ for an incompressible fluid.

The influence of the non-ionized part of the gas has been considered in detail in our paper.[5] It is shown there that it does not lead to any essential modification of this picture, because the relative motion of the ionized and the non-ionized part of the gas ("ambipolar diffusion") is rather slow (probably $\leqslant 1$ km/sec). If the dissipation of turbulent energy corresponds to the values of v and L used above (from which results 10^{-4} erg/g sec), the ambipolar diffusion would provide a comparatively important mechanism converting turbulent into thermal energy. The dissipation of energy may be higher if the relatively fast moving clouds are frequent enough.

It follows from the analysis outlined before that the age of the galaxy determines essentially the value of L/v of the largest elements whose motion is just affected by the magnetic field. Their kinetic energy determines the magnetic field strength and their extension (namely, L) the approximate radius of curvature. On the other hand, the present field depends very little (by a logarithmic term) on the initial field.

From these considerations it seems likely that in other galaxies conditions may be fairly different, either in the sense that the dynamics of the interstellar gas are more greatly affected by the magnetic field or in the opposite sense.

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Electron Removal in Mercury Afterglows*

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Observations of the electron density, visible and near ultraviolet light intensity, and spectrum associated with a 2800-Mc pulsed electrodeless discharge through mercury vapor have been made. Our results indicate ambipolar diffusion as the principal mechanism of electron removal at low pressure, with attachment becoming increasingly important at higher pressure. The data also yield a considerable amount of information regarding electron temperatures, recombination coefficient, and other parameters necessary to a detailed description of the processes taking place in the discharge.

I. INTRODUCTION

As part of a general program of studying the mechanism of the disappearance of electrons from gas discharge plasmas, an investigation of mercury afterglows has been made. The method used includes simultaneous spectroscopic and electron density measurements together with relative and absolute light intensity measurements.

Previous work on metallic vapors has been confined largely to mercury and cesium. Mohler and Boeckner[1] and Mohler[2] have investigated cesium afterglows after cut-off of an intense direct current discharge at a pressure of 0.1 mm with electron densities of the order of 10^{12} electrons/cm^3. Electron temperatures in the afterglow were of the order of 1200°K. The spectrum was

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[1] F. L. Mohler and C. Boeckner, Natl. Bur. Standards J. Research 2, 489 (1929).
[2] F. L. Mohler, J. Research Natl. Bur. Standards 19, 446 (1937).