where P indicates that the principal value has to be taken with respect to the point $x=(\eta+1)^{-1}$. It is readily seen that the principal value integral can be written as

$$-(i/4\pi)\mathcal{P}\frac{dx}{\{1-(\eta+1)x\}\{2-x\}}\ln\frac{2-x+i(x-x^2)^{\frac{1}{2}}}{2-x-i(x-x^2)^{\frac{1}{2}}} \quad \text{(IV.1)}$$

taken along the contour I of Fig. 2 which encircles the branch points $x=0, 1$ of $(x-x^2)$ in the negative sense (the plane is cut along the segment $0\leq x\leq 1$). The contributions of the two infinitesimal semi-circles near $x=(\eta+1)^{-1}$ cancel each other; indeed, the value of the logarithm in any given point of the path above the cut is equal and opposite in sign in the corresponding point below it.

We next deform I to the contour II, coming from $x=\infty$, encircling the pole of the integrand at $x=2$ and the branch point of the logarithm at $x=4/3$ in the negative sense, and then returning to ∞. The plane is cut along that part of the real axis for which $x\geq 4/3$. Care has to be exercised in continuing $(x-x^2)^{\frac{1}{2}}$ to points on path II as well as in choosing the branch of the logarithm on the upper and lower half of this contour. The contributions near the pole $x=2$ cancel again and the final result is

$$D_2=-(1/2\eta+1)\{\tfrac{1}{2}\ln[2(\eta+1)/(4\eta+1)]+i\tan^{-1}(\eta^{\frac{1}{2}}/(2\eta+1))\},$$

which was used in Eq. (61).

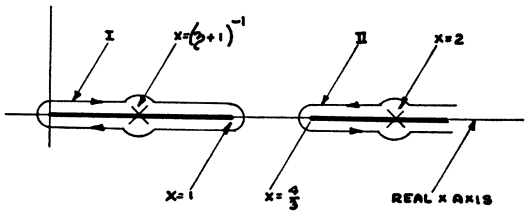

FIG. 2. Deformation of the integration path of the integral (IV.1).

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Transport Rates of the Helium II Film Over Various Surfaces*

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Liquid helium II film transport over various surfaces has been studied by a new method in which a cylindrical capacitor using liquid helium as the dielectric is employed as a depth gauge. Changes in liquid level resulting from film transport produce changes in capacitance which in turn cause frequency changes in a high frequency circuit. The details of this method are described. The film transport rates, measured to 1.25°K, were found to depend on the substrate; at 1.25°K, the highest rate observed was 51×10^{-5} cm³/cm sec for etched copper and the lowest, 7.5×10^{-5} cm³/cm sec for glass. The rates were also measured over iron in the magnetized and unmagnetized state and over a superconductor in the superconducting and in the normal state. No differences were noted. In the latter case the thermal conductivity of the container is abruptly changed and the absence of an effect supports the view that heat transfer plays no significant role in determining the transport rate.

INTRODUCTION

SINCE the discovery[1] of the film transport phenomenon of liquid helium II, various investigations have been made to determine the characteristics of the transport. Among the phenomena studied was the effect of the underlying surface material.[2] These studies indicated that the transport rate is independent of surface material. However, a recent examination of this property by Mendelssohn and White[3,4] and the present authors[5,6] has shown that this is not the case.

Theories of the HeII film have been advanced by Frenkel,[7] Schiff,[8] Temperley,[9] and Bijl, de Boer, and Michels.[10] The first two authors have considered helium atoms to be under the influence of gravity and of the van der waals attractive forces of the walls. This treatment indicates that film thicknesses on conducting surfaces are greater, by about a factor of two, than films on dielectrics. Film flow, according to Frenkel, should be limited by viscosity. For the case of HeII, the theoretical result becomes ambiguous owing to the presence of a zero viscosity, or superfluid component. Temperley has treated the film as an adsorbed phase in which He atoms occupy bound sites on the surface of the solid walls. The adsorbed layers farthest from the wall are assumed to occupy only a fraction of the available sites, and film flow is considered to arise from a transition of atoms from their existing sites to empty neighbors. The influence of the wall is considerably

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[1] B. V. Rollin and F. Simon, Physica **6**, 219 (1939).
[2] J. G. Daunt and K. Mendelssohn, Proc. Roy. Soc. (London) **A170**, 423, 439 (1939).
[3] K. Mendelssohn and G. K. White, Nature **166**, 27 (1950).
[4] K. Mendelssohn and G. K. White, Proc. Phys. Soc. (London) **A63**, 1328 (1950).
[5] H. A. Boorse and J. G. Dash, Phys. Rev. **79**, 734 (1950).
[6] H. A. Boorse and J. G. Dash, Phys. Rev. **79**, 1008 (1950).
[7] J. Frenkel, J. Phys. U. S. S. R. **II**, No. 5, 365 (1940).
[8] L. I. Schiff, Phys. Rev. **59**, 839 (1941).
[9] H. N. V. Temperley, Proc. Roy. Soc. (London) **A198**, 438 (1948).
[10] Bijl, de Boer, and Michels, Physica **8**, 655 (1941).

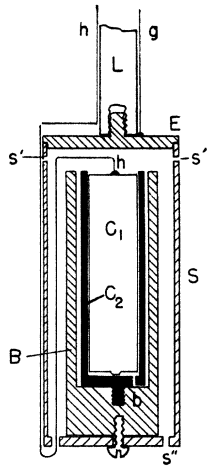

FIG. 1. Sectional view of container, depth gauge, and radiation shield assembly.

smaller for these layers than the influence of the underlying He layers, and film flow rates should be essentially the same for all wall materials. Bijl, de Boer, and Michels limit the film thickness by means of the de Broglie wavelength of the superfluid particles, thus implying that film thickness and transport rate are independent of surface material. Finally, it may be mentioned that no attempt has been made to account for the fact that the film flow is directed toward a region of lower gravitational potential.

The unsatisfactory predictions of, and the general disagreements between the proposed explanations of the HeII film, together with the paucity of observations of rates on surfaces other than glass, have led the authors to undertake a new study of the transport phenomenon.

EXPERIMENTAL METHOD

Measurement of the film transport through direct visual observation of the rise or fall of the liquid helium surface in the transport vessel was considered to hold two disadvantages for this research. First, while the optical method is straightforward for transparent containers, it is not as convenient for measuring rates over opaque materials. Second, possible disturbing effects of thermal radiation incident upon the film could not be entirely eliminated in this method. For these reasons, it was decided to abandon the direct viewing procedure and to develop a more favorable technique.

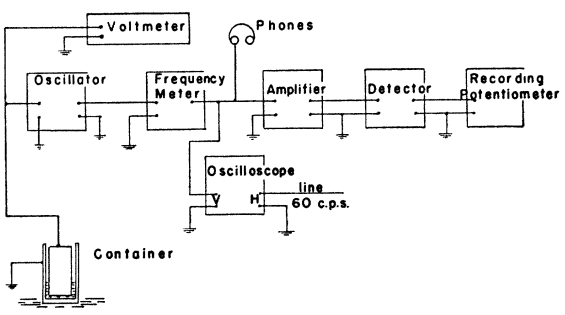

FIG. 2. Block diagram of the measuring circuits.

The use of liquid HeII as the dielectric in a suitably constructed capacitor appeared to offer a simple and convenient method for measuring film transport. Experiment showed that as the liquid flowed from the capacitor, the resulting change in capacitance could easily be determined with the aid of a high frequency circuit. The fact that the electric polarization of HeII is almost independent of the temperature[11,12] and that its dielectric loss is very small[12] gave added assurance of the suitability of this method.

The capacitor design is given in Fig. 1. The container B, which provides the surface for film creep to or from the surrounding bath, was constructed of the material over whose surface the transport rate was desired. The varying level of liquid HeII within B was indicated by the varying capacitance of the cylindrical capacitor C_1-C_2, the hole b permitting equilibrium between the two levels. C_1 was centered within C_2 by four longitudinal strips of Teflon sheet. The mean annular separation between the walls of the capacitor was 0.0050 ± 0.0001 inch, the capacitor being centered within the container by means of the threaded post. C_1-C_2 therefore served as a depth gauge for the liquid in B and operated for transport rate measurements over conducting and nonconducting surfaces. Typical dimensions for the container B were $\frac{5}{8}$ in. inner diameter, and 1.7 in. over-all length.

The container and depth gauge assembly were mounted within a copper radiation shield S. The copper cap E soldered to S provided additional radiation shielding and was connected to a raising and lowering device by means of the Lucite rod L. During transport rate measurements, the lower part of the shield was immersed in the bath, hence any possible temperature gradients between the container and the bath were minimized. As a further precaution in this direction, holes s' and s'' were pierced in the shield to allow He liquid and vapor to pass between the surrounding bath and the container assembly.

The inner cylinder C_1 of the capacitor was electrically connected to the external measuring apparatus by the fine copper wire h. C_2 was grounded outside the cryostat by means of the direct contact between C_2 and S, and the copper wire g.

The circuit for the measurement of the capacitance changes is given by the block diagram in Fig. 2. The cylindrical capacitor is connected in parallel with the tuned circuit of a radiofrequency oscillator of high stability.[13] A voltmeter provides a continuous reading of the tuned circuit excitation which is set by a bias control. Bias may be set in excess of cut-off value to stop oscillation by opening the cathode-to-ground connection. An amplifying and buffering stage feeds the oscillator signal to a General Radio Company hetero-

[11] M. Wolfke and W. H. Keesom, Physica **3**, 823 (1936).
[12] C. J. Grebenkemper and J. P. Hagen, Phys. Rev. **80**, 89 (1950).
[13] J. K. Clapp, Proc. Inst. Radio Engrs. **36**, 356 (1948).

dyne frequency meter, where the note is beat against one of nearly the same frequency. The beat note is observed on an oscilloscope and fed to an audio amplifier. This amplified signal is connected to a simple frequency discriminator consisting of a large resistance in series with an air core inductance. The voltage across the inductance, proportional to frequency, is rectified, filtered, and supplied to a Brown recording potentiometer. It is thus possible to obtain an instantaneous and continuous record of the liquid level in the capacitor. The frequency dependence of the potentiometer circuits is obtained by calibrating the recording potentiometer response by means of the harmonics of line frequency as seen on the oscilloscope. Alternatively, depth measurements can be made by reading the vernier scale of the frequency meter, with the beat frequency set to zero on the oscilloscope.

The fractional frequency change of the oscillator from the completely full to the completely empty condition of the capacitor was approximately 1 percent of the mean frequency. The frequency instability of the oscillator and heterodyne frequency meter was approximately 60 cps per hour at 100 kilocycles. Under the various experimental conditions, emptying times ranged between extremes of about two and thirty minutes. The over-all reproducibility of the observations is estimated at about 5 percent.

CRYOSTAT DESIGN

The mechanical mounting of the depth gauge and container assembly within the cryostat is shown in Fig. 3. The inner He Dewar is of Pyrex glass completely silvered except for a half-inch clear vertical stripe. Copper-glass seals J_1 and J_2 provide vacuum tight joints between the Dewar and its top cap, and a pump line, respectively.

Liquid He was introduced through the filling tube F, and was subsequently cooled by pumping through the tube P. He bath temperatures were determined by a vapor pressure thermometer, consisting of mercury manometers attached to the low conductivity tube V, which ended below the surface of the liquid.

A vacuum tight winch W was used to raise and lower the container assembly. Wires h and g were brought out of the cryostat through the seals K. Several Pyrex glass containers M, attached to the supporting rod L, were used to check transport rates over glass by the visual method.

EXPERIMENTAL DETAILS

Bowers and Mendelssohn[14] have demonstrated that films of solid air deposited on container surfaces lead to He transport rates considerably higher than those obtained with clean surfaces. Accordingly, precautions were taken to avoid this contamination. Before filling the inner Dewar with liquid helium, the liquid space and pumping lines were successively evacuated and filled with pure He gas.

Precooling to liquid nitrogen temperatures was achieved by filling the outer Dewar with liquid nitrogen; He transfer gas was then introduced to the annulus A. After the establishment of temperature equilibrium within the He space, A was evacuated and a thermal isolation vacuum was obtained. Liquid He was then transferred into the cryostat.

Vapor pressures of the liquid He were determined by a 12-mm bore mercury manometer and a calibrated Dubrovin scale-of-nine vacuum gauge. Vapor pressures were maintained constant to 0.1 mm by a manostat-controlled[15] solenoid valve located in the pumping line. Heat influx to the He bath, measured by the rate of descent of the liquid level, was approximately 50 cal/hr, and a single filling of liquid sufficed for seven hours' experimentation.

Calibration of the depth gauge with changes in depth of the liquid He in the container was carried out in the following manner. The container was lowered into the bath until fully immersed, and the bath maintained at a constant temperature below the lambda-point. Oscillator frequency readings and cathetometer observations of the bath level were taken simultaneously, as normal evaporation caused the height of liquid in the He Dewar to fall. The rate of decline of the bath level at the calibrating temperatures was considerably smaller than the rate of fall of liquid level in the container arising from film transport. This condition assured that the levels of the liquid in the bath and in the container dropped at the same rate. The calibration procedure therefore gave an over-all measure of depth gauge "sensitivity" in frequency meter scale divisions per centimeter change in liquid level. The most frequently used depth gauge had a sensitivity of 4.10 $\pm$0.06 divs./cm. A sample calibration is given in Fig. 4.

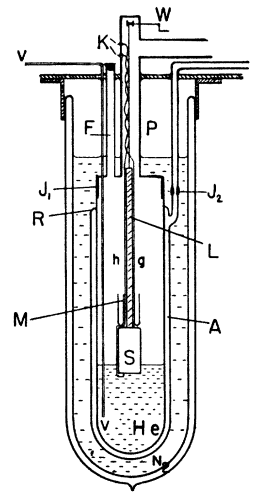

FIG. 3. Sectional view of the helium cryostat.

[14] R. Bowers and K. Mendelssohn, Proc. Phys. Soc. (London) **A63**, 1318 (1950).

[15] J. J. Beaver and D. J. Beaver, J. Ind. Eng. Chem. **15**, 359 (1923).

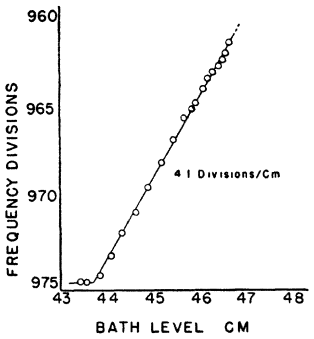

FIG. 4. Typical depth gauge calibration curve.

To verify that the rf field within the capacitor had no effect on the transport rate, the oscillator signal was interrupted for a short time during the course of the transport. The result of such a check is shown in Fig. 5. No indication of rf disturbance was found in any of the many checks which were made.

Optical determinations of transport rate over the auxiliary containers of Pyrex glass were made at the beginning of each experiment to determine whether the surfaces had been contaminated with solid air. If contamination had occurred, measurements were abandoned for the day and the apparatus warmed to room temperature and flushed thoroughly with pure He gas. In order to detect the possible progressive contamination of surfaces during the course of a run, measurements were taken at successively lower and then at successively higher temperatures. Transport rates taken at the beginning and at the end of the run were then compared.

Flow rates were obtained both for filling and emptying the containers and were compared in order to correct for evaporation losses. No significant differences in rates were observed for these two conditions. This result indicated the effectiveness of the radiation shield.

RESULTS

The dependence of transport rate on the position of the liquid level below the container rim was found to correspond qualitatively with the results of other investigators.[2,4,16] The emptying curve in Fig. 6 shows this initial rate as varying while the liquid level is near the lip of the container, and a constant rate thereafter. Transport rates plotted in the figures correspond to the constant rates determined from the straight portions of the curves.

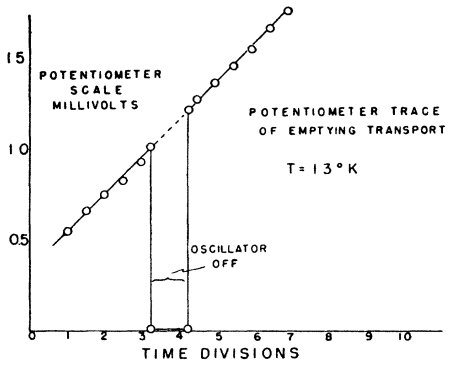

FIG. 5. Effect of capacitor excitation on transport rate.

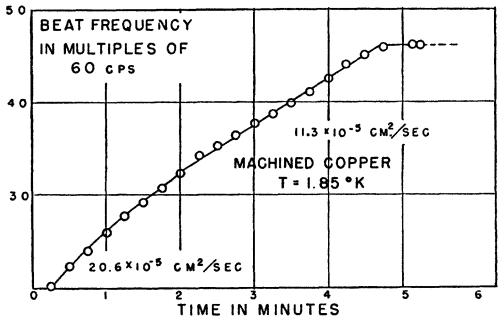

FIG. 6. Typical emptying curve for a machined copper container.

COPPER

Initial measurements with a carefully machined copper vessel yielded a maximum film transport rate approximately twice that previously published for glass. To verify this result, a number of measurements were made with containers of varying dimensions, the annular liquid spaces ranging from 0.005 in. to 0.090 in. Within the experimental error, the containers all gave the same value; results of a typical run are shown in Fig. 7. As a check on these measurements, the transport at 1.3° from a Pyrex glass container was studied by the depth gauge technique. The result was identical with that of Daunt and Mendelssohn.[2]

A machined copper container that had yielded transport rates as shown in Fig. 7 was etched for two minutes in 0.1 N nitric acid and then reinstalled in the cryostat. This container gave the high rates shown in Fig. 8. Owing to an insufficient amount of liquid He in the bath during this experiment, only three rate determinations could be made at temperatures below 1.89°.

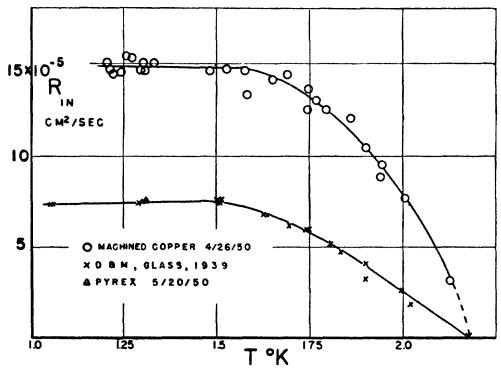

FIG. 7. HeII film transport rates over machined copper (typical curve).

[16] Webber, Fairbank, and Lane, Phys. Rev. **76**, 609 (1949).

The shape of the curve cannot therefore be determined with the same certainty as the other two in this figure; nevertheless, it appears that all three are similar in showing an increasing rate at the lowest temperatures reached.

Microscopic examination of this etched surface indicated considerable surface roughness, and this may contribute to the high transport rate. Attempts were made to determine the increase in perimeter by photographing the container edges under high magnification, but the results were not considered reliable for comparison purposes.

The etched container was then allowed to oxidize in air for two days at room temperature. The result of this treatment, also given in Fig. 8, shows a large decrease in rate, together with some change in temperature dependence.

In order to observe additional effects of copper surface preparation, measurements were made on a burnished copper container. The effect of burnishing, i.e., the production of an amorphous surface layer by high

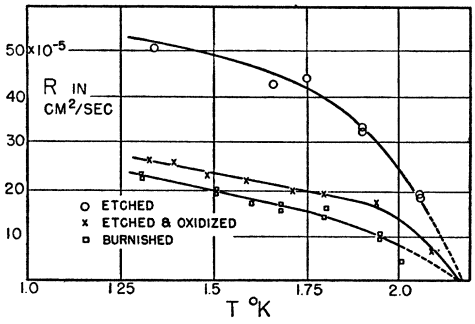

FIG. 8. HeII film transport rates over variously treated copper surfaces.

local temperature and pressure during polishing, is seen from Fig. 8 to reduce the rate still further but to produce a temperature dependence similar to the etched and oxidized material.

STAINLESS STEEL

The results of a single run on machined 18–8 stainless steel are presented in Fig. 9. The structure of this curve resembles the results for burnished copper, there being a continued increase in rate with decline in temperature.

LUCITE

It was considered desirable to determine the transport rate over the surface of a dielectric other than glass; owing to its machinability, Lucite was chosen. The container surfaces were polished with rouge after machining. This vessel showed the unusual temperature dependence exhibited in Fig. 9. Numbers alongside the experimental points refer to the chronological order in which the data were taken; it thus appears that the peak at 1.5° was reproduced. The singular nature of

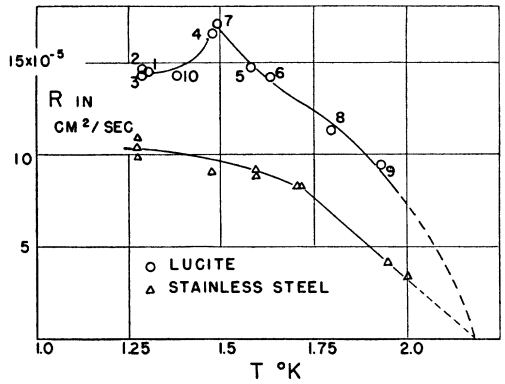

FIG. 9. HeII film transport rates over Lucite and stainless steel.

the curve is noteworthy. The results, however, are submitted as tentative, pending further investigation.

LEAD AND IRON

An exploratory investigation was made of the possible effects of ferromagnetic and superconducting substrates on the transport rate using, respectively, cold-rolled steel and lead vessels. The data for these two materials are given in Fig. 10.

If a transfer of heat plays a part in determining the transport rate, it might be expected that an abrupt change in the thermal conductivity of the container material would disclose this effect. It is rather striking to realize that such an abrupt change can be brought about by applying a magnetic field of sufficient strength to a superconductor, the thermal conductivity in the superconducting state being usually much smaller than in the normal state at the same temperature. Lead, for instance, at 2°K shows conductivities[17] differing by 900 percent.

A lead container was therefore constructed and measurements of the transport rate were made with the metal in the superconducting and in the normal state; as Fig. 10 shows, no appreciable difference in the rate was observed. It should be remarked, however,

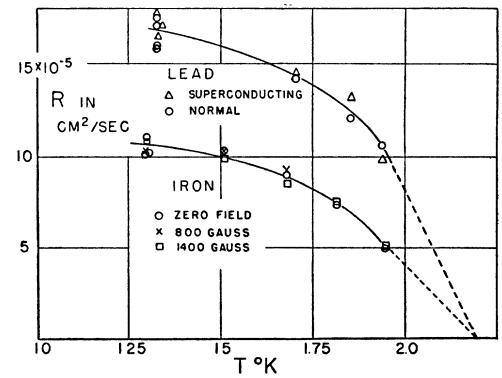

FIG. 10. HeII film transport rates over lead and iron.

[17] W. J. de Haas and A. Rademakers, Physica 7, 992 (1940).

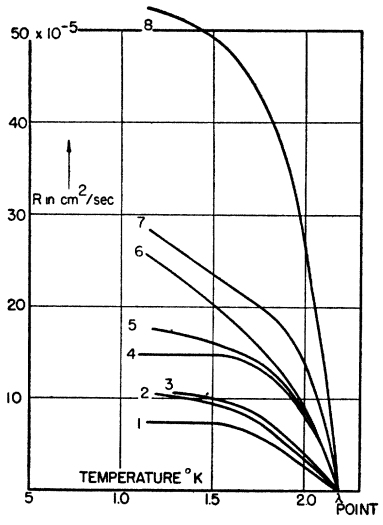

FIG. 11. HeII film transport rates for: (1) glass (D and M), (2) stainless steel, (3) iron, (4) machined copper, (5) lead, (6) burnished copper,(7) etched and oxidized copper, (8) etched copper (all drawn to the same scale).

that the experiment was carried out after a dark oxide layer had formed on the surface of the material. Although it seems unlikely that this circumstance should have completely masked an effect if it were present, further investigation is planned.

Measurements of the transport rate over a magnetized and unmagnetized ferromagnetic surface were made using cold-rolled steel. Although the fields applied in this case (up to 1400 oersteds) were not sufficient to produce saturation, here again the absence of any perceptible effect makes it seem unlikely that any effect exists.

DISCUSSION OF RESULTS

All experiments on the transport of the He film have led to the conclusion that other conditions being the same, the transfer of liquid from a container to the bath is limited by the narrowest part of the containing surface located above the upper liquid level. Under the conditions prevailing in these experiments, this narrowest part is the inner perimeter of the transfer vessel *B* of Fig. 1. Since the film is of the order of 10^{-6} cm thick, it might be expected that the geometric perimeter would have little meaning as far as the transport is concerned and that, instead, the film would "see" the microscopic perimeter and its flow would be adjusted accordingly.

If this were true, it would appear reasonable to expect that a given glass vessel would not maintain its rate with age or with extreme temperature cycling over a period of months; microscopic surface cracks could increase in depth or in number or both; alternatively, it might be argued that a microscopic deposit from whatever source would in time tend to reduce the surface roughness. In any event such changes would be apparent as an altered transport rate. It appears, however, that no effect of this kind has ever been found. This suggests that the microscopic perimeter is not the effective perimeter for transport, a view that is supported by the agreement of the transport rates obtained from the various machined copper containers (Fig. 7).

An extension of this argument against the effectiveness of the microscopic perimeter may be made by noting that if the transport were a function of the temperature and the microscopic perimeter and nothing else, then *all* substances should show rates which, while differing from each other, should exhibit the common characteristic of rising like glass to a maximum value at approximately 1.5°K and remaining sensibly constant thereafter. A glance at Fig. 11, in which the rates for the various substances are plotted on the same scale, shows that this is not the fact. Although glass and *machined* copper show similar curves, the others display the distinctly different characteristic of a rising rate down to the lowest temperatures reached. It thus appears to be a legitimate conclusion that the substrate plays a significant role in the transport phenomenon.

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