interesting to observe that by means of a simple manipulation of Wooldridge's result a remarkable similarity between the two theories is evident.

The result of the free-electron approximation attack can be written

$$\delta/\delta_{\max}=1.85F[0.92E_p/E_{p\,\max}], \quad (1)$$

where

$$F(x)=\exp(-x^2)\int_0^x \exp(t^2)dt,$$

where δ is the secondary ratio, E_p is the primary energy, and $E_{p\,\max}$ is the primary energy for the maximum secondary ratio, $\delta_{\max}$. The numerics in Eq. (1) result from the behavior of $F(x)$.

The result of the quantum-mechanical method given by Wooldridge is

$$\delta/\delta_{\max}=1-\exp[-K(E_p/E_0-1)]+M\int_0^{K(E_p/E_0-1)}[E_p/E_0-v/K]^{-\frac{1}{2}}e^{-v}dv,$$

where

$$K=\frac{1}{2\delta_{\max}}\left[1-\left(\frac{W_a}{E_0+E_F}\right)^{\frac{1}{2}}\right]$$

$$M=\frac{1}{2}\left[1+\left(\frac{W_a}{E_0+E_F}\right)^{\frac{1}{2}}\right],$$

where E_0 is the average energy given a secondary by a primary, W_a is the height of the energy barrier at the surface, and E_F is the mean Fermi energy of the electrons in the lattice. Introducing the substitutions

$$E_p/E_0-v/K=t^2/K$$

and

$$KE_p/E_0=x^2,$$

the relation becomes after some manipulation,

$$\delta/\delta_{\max}=1-A\exp(-x^2)+BF(x), \quad (2)$$

where $F(x)$ is the same function as in Eq. (1) and the constants are

$$A=e^K+2MK^{\frac{1}{2}}\int_0^K \exp(t^2)dt\cong e^K+2MK(1+K/3)$$

$$B=2MK^{\frac{1}{2}}.$$

It is seen that the same function appears in both theories, although the method of attack varies widely. For materials like gold, silver, and platinum, for which the Wooldridge theory is most accurate, the constants A and B are roughly 1.5 and 0.6, respectively, which means that neither term can be neglected. Figure 1

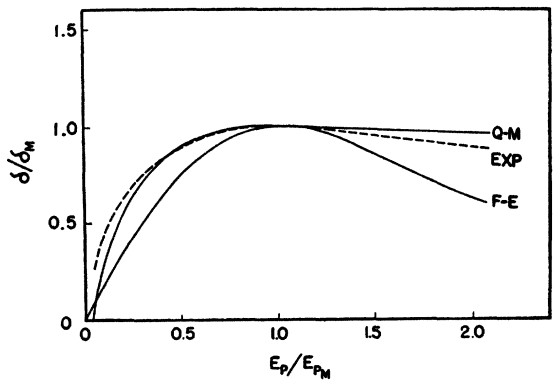

FIG. 1. Comparison of the quantum-mechanical and free-electron results with the universal secondary efficiency curve suggested by Baroody. The experimental curve is taken from reference 2.

compares the results of the two theories with the universal secondary emission curve suggested by Baroody. For this plot the above values of A and B have been used.

In addition to the appearance of the new exponential term in Eq. (2), there is a difference in the dependence on E_p. In Eq. (1) the argument is a linear function of the primary energy while in Eq. (2) the argument is proportional to the square root of this energy. With Wooldridge's relation in the form given by Eq. (2), the application to experimental results is considerably simplified since the graphical integration need be done only once to evaluate $F(x)$.

I am indebted to Dr. P. L. Copeland for his kindness in discussing this work with me.

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Decay of Np^{236}

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RECENT work at this laboratory on the nuclear energy surface[1] in the heavy region has demonstrated the value of precise determinations of the transition energies in radioactive decay in these elements. As part of a program of such measurements, the beta-decay energy of 22-hour Np^{236} came under consideration. The decay cycles involving this isotope of neptunium (Fig. 1)[2] showed that the experiment might be complicated by

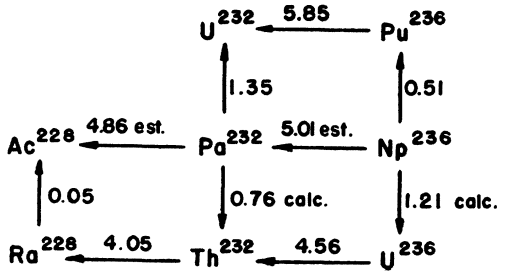

FIG. 1. Closed cycles involving the decay energies of Np^{236}.

orbital electron capture decay to U^{236}, so an attempt has been made to establish the decay scheme of Np^{236}.

Samples of neptunium were prepared by bombardment of U^{235} with the deuteron beam from the 60-inch cyclotron at Crocker Radiation Laboratory. The electron spectrum of purified neptunium fractions was observed on a beta-ray spectrometer employing the double-focusing principle proposed by Svartholm and Siegbahn,[3] and Shull and Dennison.[4] The detector is a Geiger counter filled with 8.8 cm of 90 percent argon, 10 percent ethylene mixture, and has a 30-μg/cm^2 Formvar window supported on a mesh grid. Window absorption corrections are necessary only below about 30 kev and can be made with fair accuracy.

Samples of chemically separated Np^{236} were evaporated from concentrated hydrochloric acid solution on conducting Tygon films about 30 μg/cm^2 thick. Auger electrons, internal conversion electrons, and beta-particles were seen. The beta-spectrum is complex, and resolution of Fermi-Kurie plots of the data indicate beta-energies of 0.51 Mev (59 percent) and 0.36 Mev (41 percent). A pair of internal conversion lines corresponding to the K and L conversion of a 150-kev gamma-ray was also observed. This gamma-ray is approximately 100 percent converted if it is assumed to follow the 0.36-Mev transition. The measured intensities of the electrons from Auger conversion of the K and L x-rays were far in excess of the Auger electrons expected from K and L electron vacancies due to the observed internal conversion of the 150-kev gamma-ray.

The radiations were resolved by beryllium and silver absorption into electrons, L x-rays, and K x-rays; and the relative abundance of each was determined. These were corrected for the counting efficiencies under the experimental conditions, and the L Auger conversion factor was assumed to be[5] 0.5. An estimate of the

amount of K electron capture may be made if the K x-rays observed in the absorption experiments are corrected for the contribution from internal conversion of the gamma-ray. If the L electron vacancies resulting from the K vacancies are subtracted from the total calculated from the absorption experiments, a further estimate of the amount of L electron capture is obtained. When these are related to the number of beta-particles, a decay scheme may be postulated in which the relative abundances of beta-decay and electron captures should be correct to ± 20 percent (Fig. 2).

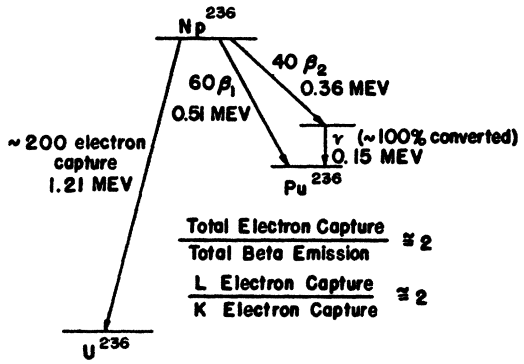

FIG. 2. Decay scheme of Np^{236}.

Apparently there is a considerable amount of L electron capture, even at this large disintegration energy of about 1.2 Mev. However, other heavy nuclides such as Am^{242m}, U^{231}, and Np^{235} have demonstrated predominant capture of L electrons[6-8] at lower transition energies.

We wish to thank Professor G. T. Seaborg for his continued interest and advice.

* This work was performed under the auspices of the AEC.

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The Splitting of l-Shells in Heavy Nuclei by the Tensor Interaction

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TO explain the nuclear "magic numbers,"[1] a strong spin-orbit interaction favoring single-particle states of total angular momentum $j=l+\frac{1}{2}$ is needed. A study has been made of the contribution to this splitting of a tensor interaction

$$V=\Sigma_j J(r_{oj})T_{oj}=\Sigma_j J(r_{oj})[3(\boldsymbol{\sigma}_o\cdot\mathbf{r}_{oj}/r_{oj})(\boldsymbol{\sigma}_j\cdot\mathbf{r}_{oj}/r_{oj})-\boldsymbol{\sigma}_o\cdot\boldsymbol{\sigma}_j]$$

between the outer particle Q_o, and the particles Q_j of a spherically symmetrical spin-saturated core, regarded as moving independently in the states ψ_i of a spherical potential well.

The first-order perturbation energy is zero for all outer particle states and so can contribute nothing to the splitting. The second-order perturbation energy is $\Sigma_i(o|V|i)(i|V|o)/(E_o-E_i)$. A Fermi-Thomas approximation and other crude approximations to the part of this energy dependent on the spin of the outer particle give zero, and a direct evaluation of the above expression incorporating complete antisymmetry is required. The isotopic spin plays no essential role. The only feasible way of treating the denominators in the above summation is to expand them about a mean excitation energy, $\bar{\epsilon}$, and to deal with the first two terms of the expansion,

$$\frac{1}{E_o-E_i}=-\frac{1}{\bar{\epsilon}}+\frac{E_i-\bar{E}_i}{\bar{\epsilon}^2}\cdots.$$

For the spin-dependent part of the perturbation, the first term of the expansion gives

$$(\Delta\epsilon)_1=\frac{-18i}{\bar{\epsilon}}\Sigma_\sigma\int\psi_o^*(\mathbf{r}_1)\chi_o(\sigma)\Big\{\boldsymbol{\sigma}\cdot\Big(\frac{\mathbf{r}_{12}}{r_{12}}\times\frac{\mathbf{r}_{12}'}{r_{12}'}\Big)\Big(\frac{\mathbf{r}_{12}}{r_{12}}\cdot\frac{\mathbf{r}_{12}'}{r_{12}'}\Big)$$
$$\times J(r_{12})J(r_{12}')[\delta(\mathbf{r}_1-\mathbf{r}_1')-\rho_c(\mathbf{r}_1,\mathbf{r}_1')][\delta(\mathbf{r}_2-\mathbf{r}_2')-\rho_c(\mathbf{r}_2,\mathbf{r}_2')]$$
$$\times[\rho_c(\mathbf{r}_2,\mathbf{r}_2')\psi_o(\mathbf{r}_1')\chi_o(\sigma)-\rho_c(\mathbf{r}_2,\mathbf{r}_1')\psi_o(\mathbf{r}_2')\chi_o(\sigma)]\Big\}$$
$$\times d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}_1' d\mathbf{r}_2'.$$

The second term is similar in character.

The integrals are evaluated by expanding the wave functions ψ_o and the core matrix densities ρ_c about $\mathbf{r}_1$, say, chosen as the local coordinate and carrying out the local integrations. We then find that

$$\Delta\epsilon\rightarrow K(\chi_o\psi_o|\boldsymbol{\sigma}\cdot\mathbf{L}V(r)|\chi_o\psi_o),$$

where

$$V(r)=-(1/r)\rho_c(r)d\rho_c(r)/dr.$$

Contributions to the splitting of the correct sign can come only from a region of radially-increasing core density. If the outer particle spends most of its time in such a region and if the above approximations have validity, splitting of the needed sign and the magnitude could be available. The positive quantity K varies as $J_o^2/\bar{\epsilon}$, where J_o is the strength of the tensor interaction and $\bar{\epsilon}$ is the mean excitation energy of the nucleus. K also depends in a complicated way upon the range and form of the tensor interaction and the structure of the core density $\rho_c(\mathbf{r},\mathbf{r}')$. A rough approximation to the energy difference of the $j=l+\frac{1}{2}$ and $j=l-\frac{1}{2}$ levels of a particle moving about a moderately heavy core is given by

$$\epsilon_{l-\frac{1}{2}}-\epsilon_{l+\frac{1}{2}}\simeq(2l+1)J_o^2/(250\bar{\epsilon}).$$

Several particles outside the core will polarize it independently, and show a consequent individual preference for the appropriate j states. The particles will also be acted on by a change in potential due to the core polarization of the other exterior particles. These cross effects may be expected to be small and to tend to cancel as a new shell forms.

The details of this analysis are available in the author's Doctoral thesis, Department of Physics, Harvard University, June 1950.

I would like to thank Professor Julian Schwinger, who sponsored and encouraged this investigation.

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Radiations of Ag^{110}

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FROM studies of the radiations of the 225-day silver activity, Siegbahn[1] has proposed a decay scheme in which the activity originates in a metastable level 116 kev above the ground state. A small complex fraction of the beta-radiation, with end points of 2.86 and 2.12 Mev, is placed with its origin in the ground state and is presumed to be the beta-radiation of the well-known 24-sec activity. Siegbahn's data suggested that still other low intensity beta-rays may connect the Ag^{110} ground state with excited levels