increasing Z, the intensity of Compton electrons emitted in the forward direction decreases with $\log(Z+1)$. For high atomic number materials the additional emission of photoelectrons again becomes apparent.

The results obtained with the radium γ-rays are not plotted in Figs. 1 and 2, since radium emits a large number of γ-rays of various energies. The radium curves appear close to those of Zn^{65}, and are in good agreement with those obtained by other investigators.[2]

Finally, the ratio of forward to backward emission of secondary electrons from various absorbers is given in Fig. 3 as a function

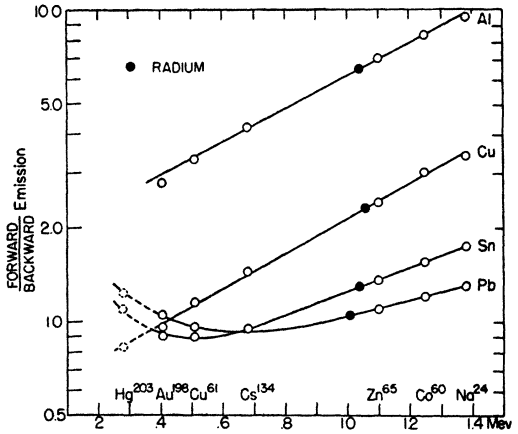

FIG. 3. Ratio of forward to backward emission of secondary electrons from various absorbers as a function of the γ-ray energies.

of the γ-ray energies. Since the Na^{24} values are not much different from those obtained with Co^{60}, they have been plotted arbitrarily at 1.38 Mev neglecting the 2.76-Mev Na^{24} γ-rays. The radium values fit on these curves which have been determined with monochromatic γ-rays, at an energy of about 1 Mev. For aluminum and copper absorbers, the ratio of forward to backward emission increases rapidly with increasing γ-ray energy. Lead, however, emits about equal amounts of secondary electrons in both directions rather independent of the γ-ray energy.

The results indicated here have to be taken into account for the construction of efficient γ-ray detectors. For high energy γ-rays, the front wall should be made of low Z, the back wall of high Z material. Furthermore, two methods become apparent which allow a quick and reasonably accurate determination of the "effective" γ-ray energy of any source. Either the ratio of emergent to backscattered secondary electrons from an about 0.4 g/cm² aluminum or copper absorber or the ratio of the backscattered intensity from aluminum and from a high Z material can be used.

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† Special Research Fellow of the National Cancer Institute, Bethesda, Maryland.
[1] G. J. Hine, Nucleonics 7, No. 4, 18 (1950).
[2] W. H. Bragg and J. P. V. Madsen, Phil. Mag. 16, 918 (1908).
[3] B. P. Burtt, Nucleonics 5, No. 2, 28 (1949).
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The Atomic Masses of H^1, D^2, and C^{12}

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RECENTLY Nier and Roberts[1-3] published new spectrographical data and computed from them the masses of H^1, D^2, and C^{12}. It seems of interest to compare their values with nuclear reaction data, especially as their values for C^{12} do not agree very well (see Table III, columns N^1 and R^2). Therefore, we

TABLE I. Reaction energies (Mev).

1	$D(\gamma, n)H$	2.226 ± 0.003	a
2	$Be^8 \rightarrow 2He^4$	0.091 ± 0.005	b,c
3	$Be^9(p, d)Be^8$	0.560 ± 0.003	c
4	$B^{11}(p, \alpha)Be^8$	8.567 ± 0.011	c
5	$B^{11}(d, \alpha)Be^9$	8.018 ± 0.007	c
6	$C^{13}(d, \alpha)B^{11}$	5.160 ± 0.010	d
7	$C^{12}(d, n)N^{13}$	-0.281 ± 0.003	c
8	$C^{13}(p, n)N^{13}$	-3.003 ± 0.003	c
9	$C^{13}(d, p)C^{14}$	5.948 ± 0.008	c
10	$C^{14}(\beta^-)N^{14}$	0.156 ± 0.001	c
11	$O^{16}(d, \alpha)N^{14}$	3.112 ± 0.006	d

[a] R. C. Mobley and R. A. Laubenstein, Phys. Rev. 80, 309 (1950).
[b] J. Crussard, Nature 166, 825 (1950).
[c] Reference 11.
[d] Reference 12.
[e] Sperduto, Holland, Van Patter, and Buechner, Phys. Rev. 80, 769 (1950).

computed from the reactions mentioned in Table I the mass doublets of Table II (column Q); column NR in the same table gives the values of Nier and Roberts and column M those of Mattauch,[4] except for the doublet D_2-He^4 which has been measured by Ewald.[5] For the difference in mass of neutron and H atom we assume the value[6] 782 ± 2 kev and for the energy-mass conversion factor 1 MU = 931.15 ± 0.05 Mev (from a value $299{,}790 \pm 2$ km/sec for the velocity of light[7,8] and $96{,}520 \pm 3$ coulomb[9,10] for the faraday). The reaction energy value for the D_2-He^4 doublet was computed from reaction cycles as mentioned in a former paper,[6] using however new values for various reaction energies.[11,12] The results from different cycles agree very well: the values from 16 cycles scattered only between 25.608 and 25.572 mMU. In the computation of the last two doublets in Table II the mean value

TABLE II. Mass doublets (mMU).

	Q	M	NR
H_2-D	1.551 ± 0.004	1.539 ± 0.002	1.5519 ± 0.0017
D_2-He^4	25.596 ± 0.009	25.604 ± 0.008	25.612 ± 0.009
$C^{12}H_2-N^{14}$	12.579 ± 0.012	12.578 ± 0.021	12.586 ± 0.013
$C^{12}H_4-O^{16}$	36.388 ± 0.016	36.381 ± 0.028	36.478 ± 0.022
$D_3-\frac{1}{2}C^{12}$	42.316 ± 0.015	42.239 ± 0.021	

of the three results for the D_2-He^4 doublet was used, yielding a difference in binding energies $E(He^4)-2E(D) = 23.838 \pm 0.006$ Mev. Using the result of reaction 1 of Table I for the binding energy of the deuteron, which agrees with the results of various reaction cycles,[6] the binding energy of the α-particle becomes 28.290 ± 0.009 Mev.

The results for the mass doublets D_2-He^4 and CH_2-N^{14} agree very well. The disagreement for the H_2-D doublet is not serious, as the reaction energy value seems to be entirely reliable;[6] therefore, it looks as though there must be an error in Mattauch's value. However, the doublets CH_4-O^{16} and $D_3-\frac{1}{2}C^{12}$ cause difficulties. The reactions (Table I) used in their computation can all be checked by cycles of other precisely measured reactions, except reactions 6 and 11. It is however not likely that there is a serious error in these two (d, α) reactions, as various other (d, α) reactions measured by the same group[12] agree very well with other nuclear reaction data. Moreover, the Q-value of the $N^{14}(d, \alpha)C^{12}$ reaction agrees with the two measurements of the $C^{12}H_2-N^{14}$ doublet in Table II.

In view of these facts we believe the nuclear reaction values for the $C^{12}H_2-O^{16}$ and $D_3-\frac{1}{2}C^{12}$ doublets to be the most reliable.

TABLE III. Nuclear masses: $M-A$ in 10^{-6} MU.

	N	R	Nr	Rr	Q
H^1	8154 ± 6	8168 ± 5	8135 ± 5	8152 ± 5	8146 ± 3
D^2		14785 ± 10		14751 ± 9	14740 ± 4
C^{12}	3850 ± 6	3803 ± 13	3850 ± 6	3781 ± 13	3807 ± 19

Therefore, we computed the masses of H, D, and C^{12} from these doublets combined with a value 1.552±0.002 mMU for H_2-D; the results are collected in column Q, Table III. For comparison we also included values computed from Nier's and Roberts' energy cycles, using however the reaction energy value for $C^{12}H_2-O^{16}$ (columns Nr and Rr). It is seen, that the values of Roberts' cycle are in agreement with the nuclear reaction data, but in Nier's cycles there remains an unsolved discrepancy.

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Note on the General Theory of Scattering

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THE usual presentation of scattering theory proceeds from a discussion of the integral equations[1]

$$\Psi_a^{(\pm)}=\Phi_a+\frac{1}{E_a\pm i\epsilon-H_0}H_1\Psi_a^{(\pm)}. \tag{1}$$

The transition probability per unit time from a state characterized by Φ_a to an initially unoccupied state Φ_b may be expressed as

$$\begin{aligned} w_{ba}&=(2\pi/\hbar)\,|\,\mathbf{R}_{ba}\,|^2\delta(E_b-E_a)\\ \mathbf{R}_{ba}&=-(\Phi_b,\,H_1\Psi_a^{(+)})=-(\Psi_b^{(-)},\,H_1\Phi_a). \end{aligned} \tag{2}$$

The matrix $\mathbf{R}_{ba}$ is defined only on the energy shell $E_a=E_b=E$. (Bold-face quantities hereafter always refer to the energy shell, E.) The integral equation (1) may be replaced by a pair of integral equations: Introduce a "standing wave" state vector $\Psi_a^{(1)}$ which, except for a normalizing factor, is $(\Psi_a^{(+)}+\Psi_a^{(-)})/2$ by the equation

$$\Psi_a^{(1)}=\Phi_a+P\left(\frac{1}{E_a-H_0}\right)H_1\Psi_a^{(1)}. \tag{3}$$

It is then easy to show that (1) is satisfied if

$$\Psi_a^{(+)}=\Psi_a^{(1)}+i\pi\,\Sigma_c\,\Psi_c^{(1)}\delta(E_a-E_c)\mathbf{R}_{ca} \tag{4}$$

from which, using (2), we find the Heitler integral equation[2]

$$\mathbf{R}_{ba}=\mathbf{G}_{ba}+i\pi\,\Sigma_c\,\mathbf{G}_{bc}\delta(E-E_c)\mathbf{R}_{ca}, \tag{5}$$

with

$$\mathbf{G}_{ba}=-(\Phi_b,\,H_1\Psi_a^{(1)})=-(\Psi_b^{(1)},\,H_1\Phi_a)=\mathbf{G}_{ab}{}^*. \tag{6}$$

From (3) we obtain an integral equation for G (*not* on the energy shell):[3]

$$\begin{aligned} G_{ba}&=B_{ba}-P\,\Sigma_c\frac{B_{bc}G_{ca}}{E_a-E_c},\\ B_{ba}&=-(\Phi_b,\,H_1\Phi_a). \end{aligned} \tag{7}$$

B_{ba} is the Born approximation matrix element of the elementary scattering act. It should be carefully noted that G in contrast to $\mathbf{G}$ is *not* hermitian. This may in fact be utilized to derive an expression for the energy dependence of G near the energy shell.

A formulation of scattering theory in which (5) and (7) are regarded as fundamental rather than (1) is attractive for several reasons: (1) Only the existence of the wave functions Φ_a of the separated system, needed for finding B_{ba}, is presumed. (2) Approximation procedures based on approximate solution of (7) and exact solution of (5) have proved very useful in specific applications mentioned below and in all cases have the property of preserving the unitarity of the collision matrix. (3) The whole formalism may be derived from variational principles which are very useful in applications.

A variational basis for (5) and (7) will now be presented. Recalling that for a large class of problems the collision matrix

$$S_{ba}=\delta_{ba}+2\pi i\delta(E_b-E_a)\mathbf{R}_{ba} \tag{8}$$

is both symmetric and unitary, that consequently $\mathbf{R}$ is symmetric, and finally that $\mathbf{G}$ is real and symmetric,[4] it is easy to show[5] that the matrix I,

$$I=\mathbf{RG}+\mathbf{GR}+i\pi\mathbf{RGR}-\mathbf{RR}, \tag{9}$$

with matrix multiplication defined by

$$(\mathbf{RG})_{ba}=\Sigma_c\,\mathbf{R}_{bc}\delta(E-E_c)\mathbf{G}_{ca}, \text{ etc.},$$

is stationary under arbitrary variations about the correct $\mathbf{R}$. Conversely, the requirement of vanishing variation leads to (5). An alternate normalization independent expression of the same stationary principle is

$$I_{ba}'=(\mathbf{RG})_{ba}(\mathbf{GR})_{ba}[(\mathbf{RR})_{ba}-i\pi(\mathbf{RGR})_{ba}]^{-1}. \tag{10}$$

The stationary value obtained (the same for (9) and (10)) is $(\mathbf{R}-\mathbf{G})/i\pi$. These expressions have proved very useful for approximate determinations of $\mathbf{R}$. They have also been used to investigate the nature of the errors involved using the Heitler prescription of replacing $\mathbf{G}$ by $\mathbf{B}$.[6] An analogous pair of relations may be given for $\mathbf{G}$. Consider the matrix J_{ba} with $E_a=E_b=E$ defined by

$$\begin{aligned} J_{ba}=P\,\Sigma_c\frac{G_{bc}{}^\dagger G_{ca}}{E-E_c}+P\,\Sigma_{c,d}\frac{G_{bc}{}^\dagger B_{cd}G_{da}}{(E-E_c)(E-E_d)}\\ -P\,\Sigma_c\frac{G_{bc}{}^\dagger B_{ca}+B_{bc}G_{ca}}{E-E_c} \end{aligned} \tag{11}$$

J_{ba} is stationary under arbitrary variations about the correct G and $G^\dagger$, the demand of vanishing variation yields (7) and its adjoint. A normalization independent form of (11) is

$$J_{ba}'=-(G^\dagger B)_{ba}(BG)_{ba}[(G^\dagger G)_{ba}+(G^\dagger BG)_{ba}]^{-1}, \tag{12}$$

where matrix multiplication is defined by

$$(G^\dagger G)_{ba}=P\,\Sigma_c\,G_{bc}{}^\dagger G_{ca}/(E-E_c), \text{ etc.}$$

The stationary value of J and J' is $\mathbf{G}-\mathbf{B}$.

A detailed discussion of the above material together with the results of applications to nucleon-nucleon and meson-nucleon scattering problems, and to the nonrelativistic Compton effect will be presented shortly.

[1] B. Lippmann and J. Schwinger, Phys. Rev. **79**, 469 (1950). We follow the notation of these authors except $\mathbf{T}\rightarrow-\mathbf{R}$, $\mathbf{K}\rightarrow-\mathbf{G}$.
[2] See, for example, W. Pauli, *Meson Theory of Nuclear Forces* (Interscience Publishers, Inc., New York).
[3] Equation (7) is usually given only in iterated form, reference 2; this is neither desirable nor necessary.
[4] J. A. Wheeler, Phys. Rev. **52**, 1107 (1937); W. Heisenberg, Z. Physik **120**, 513, 673 (1943); E. P. Wigner and L. Eisenbud, Phys. Rev. **72**, 29 (1947). In cases for which S is not symmetric in the elementary sense, a somewhat more complicated formulation can be made. This will be discussed in a later publication.
[5] See also, S. T. Ma and C. F. Hsueh, Phys. Rev. **67**, 303 (1945). I am indebted to Dr. Ma for a discussion of this work and the work of the present paper.
[6] W. Heitler, *Quantum Theory of Radiation* (Oxford University Press, London, 1944), second edition.

A Comparison of Theories of Secondary Emission

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THE theory of secondary emission using a quantum-mechanical attack has achieved reasonable success in predicting the variation of the secondary emission ratio with primary energy for several materials.[1] Recently an admittedly approximate theory using the free-electron approximation has appeared.[2] It is