Polarization Effects in $(n-\gamma)$ Reactions

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THE use of polarized neutron beams permits one to induce reactions in which the reaction products may show certain characteristic polarization properties. Perhaps the simplest case of interest can be described as follows:

A polarized neutron $\sigma_z=\frac{1}{2}\hbar$ of long wavelength is captured by a nucleus A of spin 0 leading to a nucleus $A+1$ of spin $i=\frac{1}{2}\hbar$ and a single γ-ray. We denote the total angular momentum of radiation and nucleus by $f\hbar$; then for reasons of conservation $|f|=f_z=\frac{1}{2}\hbar$. It is clear that the γ-ray must be due to dipole emission; all higher poles are forbidden.

Now the only spin eigenfunction for $f=f_z=\frac{1}{2}\hbar$ is given by

$$(1/\sqrt{3})[\sqrt{2}\varphi_i(-\tfrac{1}{2})\chi_i(1)-\varphi_i(\tfrac{1}{2})\chi_i(0)].$$

Here φ_i's denote the two spin eigenfunctions of the resultant nucleus $A+1$, and χ_j's the three symbolic angular momentum eigenfunctions of the right quantum created by dipole radiation. The resulting spin eigenfunction contains the following results: The γ-ray intensity is radiated isotropically. The resultant nucleus is partially polarized with $i_z=-\frac{1}{2}\cdot\frac{1}{3}\hbar$. The γ-rays radiated perpendicularly to the direction of the neutron spin are unpolarized, while those emitted along the z-direction are circularly polarized.

The discussion of a proper analyzer for a circularly polarized γ-ray and its connection with electron polarization will be left to a later paper, in which also other cases of interest will be treated.

A Variational Method for Inelastic Collision Problems

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A GENERALIZATION of Hulthèn's variational method[1] to the inelastic scattering of electrons by atoms has been derived using the integral

$$L\equiv\int\Psi^*(H-E)\Psi d\tau, \tag{1}$$

where H is the hamiltonian of the system.

In the case of the scattering of electrons by hydrogen atoms

$$H=-\nabla_1^2-\nabla_2^2-2/r_1-2/r_2+2/r_{12}, \tag{2}$$

where r_1, r_2, r_{12} are the distances of the electrons from the nucleus and from each other, respectively, in atomic units.

For slow incident electrons exciting the ground $1s$ state of the hydrogen atom to the $2s$ state, the total wave function of the system may be written in the form

$$\Psi(r_1,r_2)=r_2^{-1}\{f_0(r_2)\psi_0(r_1)+f_1(r_2)\psi_1(r_1)\}\pm r_2^{-1}\{f_0(r_1)\psi_0(r_2)+f_1(r_1)\psi_1(r_2)\}, \tag{3}$$

where

$$f_0(r)\sim\sin kr+a\cos kr,$$
$$f_1(r)\sim d\exp(ik_1r),$$

as $r\to\infty$, corresponding to the zero order partial wave only; and ψ_0 and ψ_1 are the $1s$ and $2s$ hydrogen atom wave functions, respectively. The initial energy of the incident electron is k^2, while the energy of the scattered electron after the $2s$ state has been excited is k_1^2. It is important to notice that a and d are complex quantities. With the above asymptotic forms it follows by Green's theorem that

$$\delta L=4\pi k\delta a-8\pi ik_1d^*\delta d, \tag{4}$$

if $(H-E)\Psi=0$. Hence

$$\vartheta(\delta L)=4\pi k\delta(y-|d|^2k_1/k), \tag{5}$$

where $\vartheta(\delta L)$ is the imaginary part of δL, and $a=x+iy$. It can be shown that application of the principle of conservation of charge[2] results in the relation

$$y=|d|^2k_1/k. \tag{6}$$

Therefore, if the condition $L=0$ is imposed for all small variations of the wave function Ψ so that $\delta L=0$, then $\delta a=2i(k_1/k)d^*\delta d$ and the conservation of charge equation (6) remains satisfied. This suggests the following procedure for the approximate determination of a and d.

Form a trial wave function Ψ_t containing n parameters c_i in addition to the parameters a and d, and satisfying the required boundary conditions at the origin and at infinity, then

$$\delta L_t=\left(\frac{\partial L_t}{\partial d}+2i\frac{k_1}{k}d^*\frac{\partial L_t}{\partial a}\right)\delta d+\frac{\partial L_t}{\partial c_i}\delta c_i+\left(\frac{\partial L_t}{\partial d^*}-2i\frac{k_1}{k}d\frac{\partial L_t}{\partial a^*}\right)\delta d^*+\frac{\partial L_t}{\partial c_i^*}\delta c_i^*, \tag{7}$$

if $\delta a=2i(k_1/k)d^*\delta d$. Therefore $\delta L_t=0$ if

$$\frac{\partial L_t}{\partial d}+2i\frac{k_1}{k}d^*\frac{\partial L_t}{\partial a}=0 \quad\text{and}\quad \frac{\partial L_t}{\partial c_i}=0,$$

since

$$\frac{\partial L_t}{\partial d^*}-2i\frac{k_1}{k}d\frac{\partial L_t}{\partial a^*}=0 \quad\text{and}\quad \frac{\partial L_t}{\partial c_i^*}=0$$

are their respective complex conjugate equations. Hence the $2n+4$ equations

$$L_t=0,\quad \frac{\partial L_t}{\partial d}+2i\frac{k_1}{k}d^*\frac{\partial L_t}{\partial a}=0,\quad \frac{\partial L_t}{\partial c_i^*}=0 \tag{8}$$

may be used to determine the complex parameters $a, d, c_1, \cdots, c_n$.

But, in contrast to Hulthèn's variational method applied to the *elastic* scattering of electrons, the condition $L=0$ does not imply that $\delta a=0$. A correction to the parameter may be obtained by considering the integral

$$L'=\int\Psi(H-E)\Psi d\tau. \tag{9}$$

For it can be shown that

$$\delta L'=4\pi k\delta a. \tag{10}$$

Therefore the corrected value λ of the parameter a is given by

$$\lambda=a-L'/4\pi k. \tag{11}$$

The variational method outlined above can be extended to include any order partial wave, and excitation to any state of the atom.

Calculations are now in progress on the $1s-2s$ excitation of hydrogen. The preliminary results obtained seem promising.

In conclusion I wish to thank Professor H. S. W. Massey for his assistance and interest in this work, and I am also indebted to Dr. R. B. Makinson and Mr. H. H. Robertson for a helpful discussion of the problem.

[1] L. Hulthèn, K. Fys. Sallsk, Lund Forhandl 14, No. 21 (1944); H. S. W. Massey and B. L. Moiseiwitsch, Proc. Roy. Soc. (London) 205, 483 (1951).
[2] N. F. Mott and H. S. W. Massey, *Theory of Atomic Collisions* (Clarendon Press, Oxford, 1949), second edition, Chapter VIII.

Erratum: Q-Values for (α,p) Reactions on Aluminum and Boron by Means of Photographic Emulsions

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THE following correction should be made: line 10 of column 1, p. 642, should read ". . . There is, however, a difference of about 0.21 Mev . . ." .