Angular Yield of Deuterons and Alphas from the Proton Bombardment of Beryllium

J. A. NEUENDORFFER,* D. R. INGLIS,† AND S. S. HANNA
Department of Physics, Johns Hopkins University, Baltimore, Maryland‡

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A photographic target chamber incorporating limited magnetic deflection of product particles was used to measure the angular yield of deuterons and alphas from the proton bombardment of beryllium at energies up to 1 Mev. The angular variation observed is of the form $1+A_1\cos\theta+A_2\cos^2\theta$, with relatively large values for A_1 indicating strong interference between states of the compound nucleus of opposite parity. The variation of the total cross sections with energy suggests the presence of a state of the compound nucleus at $E_p=0.68$ Mev, in addition to the states previously known at $E_p=0.33$ and 0.94 Mev. Range in emulsion *vs* energy measurements for the light nuclei observed in the reaction are also given.

I. INTRODUCTION

ENERGY levels in B^{10} have been studied by means of the $Be^9(d,n)B^{10*}$ and $Li^7(\alpha,n)B^{10*}$ reactions by several investigators.[1] In addition, levels in B^{10} above 6.5 Mev can be examined by means of the reactions produced by the proton bombardment of Be^9, in which B^{10} is formed as a compound nucleus:

$$Be^9(p,\gamma)B^{10} \quad Q=6.49 \text{ Mev},$$
$$Be^9(p,p)Be^9$$
$$Be^9(p,d)Be^8 \quad Q=0.56 \text{ Mev},$$
$$Be^9(p,\alpha)Li^6 \quad Q=2.12 \text{ Mev}.$$

Measurements of the γ-ray yield[2] reveal resonances in the low energy range at 0.31, 0.67, 0.88, 1.00, and 1.09 Mev, indicating levels in B^{10} at 6.8, 7.1, 7.3, 7.4, and 7.5 Mev. Thomas, Rubin, Fowler, and Lauritsen[3] have measured the heavy particle yields at bombarding energies between 0.25 and 1.3 Mev and at a laboratory angle of 138°. The proton scattering cross section shows anomalies corresponding to the γ-ray resonances at 0.31, 1.00, and 1.09 Mev. The alpha- and deuteron yield curves are reproduced in Fig. 5. They both exhibit a strong resonance structure at 0.33 Mev and a considerably less pronounced structure in the region from 0.9 to 1.0 Mev. In addition, the deuteron yield shows a small anomaly at 0.47 Mev and appears to be rising toward a resonance above 1 Mev. It is possible that some of this structure in the differential cross section arises from interference effects and not from the presence of levels in the B^{10} nucleus. To investigate this question and to obtain information concerning the nature of the states involved, the variation of yield with angle has been studied for the alpha- and deuteron reactions.

II. METHOD AND PROCEDURE

In the method employed in observing the angular distributions, photographic plates were used as particle detectors. This choice makes it possible to observe the low energy particle groups in the target chamber vacuum, and the intensities can be recorded at several angles simultaneously.

In addition, it is necessary to identify reliably and rapidly the alphas, deuterons, and protons from the reaction. This cannot be done entirely by range in the emulsion alone, for the protons have approximately the same track length as do the alphas at the lower, and the deuterons at the higher bombarding energies. Appropriate photographic technique and careful analysis of track characteristics would facilitate the identification, but this was thought to be impracticable when thousands of tracks must be counted. Moreover, the protons, which are distributed approximately according to the Rutherford law, predominate and would almost completely mask the other particles at all but the most backward angles. For these reasons, limited magnetic deflections of the product particles was incorporated into the design of the experiment. The technique employed served to separate the scattered protons from the alphas and deuterons at all the energies investigated but contributed, to a lesser extent, to the discrimination between the alphas and the deuterons, which have approximately the same magnetic deflection for many of the angles and energies investigated. Electrostatic deflection, technically more difficult, would not have improved the situation over the range of angles and bombarding energies employed.

The target chamber incorporating these features is shown in Fig. 1. It was machined from a duraluminum casting, having an approximately semicircular cross section with a closed bottom and an open top for which an appropriate cover was provided. The iron pieces indicated in the diagram were shaped and arranged inside the chamber so as to provide seven radial gaps through which particles from the target could pass in

* Now with the Operations Evaluation Group, Office of the Chief of Naval Operations, Department of the Navy.
† Now at Argonne National Laboratory, Chicago, Illinois.
‡ Assisted by a contract with the AEC.
[1] O. Haxel and E. Stuhlinger, Z. Physik **114**, 178 (1939); T. W. Bonner and G. Brubaker, Phys. Rev. **50**, 308 (1936); H. Staub and W. E. Stephens, Phys. Rev. **55**, 131 (1939); Rasmussen, Hornyak, and Lauritsen, Phys. Rev. **76**, 581 (1949); Bonner, Butler, and Risser, Phys. Rev. **79**, 240 (1950).
[2] Herb, Kerst, and McKibben, Phys. Rev. **51**, 691 (1937); Curran, Dee, and Petrzilka, Proc. Roy. Soc. (London) **A169**, 269 (1939); W. J. Hushley, Phys. Rev. **67**, 34 (1945); Fowler, Lauritsen, and Lauritsen, Revs. Modern Phys. **20**, 236 (1948); S. Devons and M. G. N. Hine, Proc. Roy. Soc. (London) **A199**, 56 (1949).
[3] Thomas, Rubin, Fowler, and Lauritsen, Phys. Rev. **75**, 1612 (1949).

reaching the photographic plates. This internal iron constitutes the yoke of an external electromagnet of suitable design. The iron magnetic circuit is broken only by the walls of the chamber, there only ⅛ in. thick, and the seven radial gaps, each approximately 0.15 in. wide. The target, a suspended beryllium foil either 15 or 30 microinches thick,[4] is mounted on a rod extending through a seal in the bottom of the chamber. The photographic plates are mounted on a platform which also can be raised and lowered from the outside, allowing several exposures to be made with one set of plates. The incident beam from the electrostatic accelerator arrives at the target through a tube passing through a hole in the iron yoke, and the beam is collimated by suitable apertures, the last of which is approximately 0.03 in.×0.06 in. Despite the smallness of this opening, which improves the resolution of the apparatus, adequate beam currents reached the target. Within each gap in the iron yoke several vertical baffles are placed to reduce scattering, and a single horizontal slit, about in the middle of the gap, completes the collimation of the product particles. The path followed by a typical particle through the magnetic field in a gap is illustrated in the cross section drawing in Fig. 1.

During most of the experimental runs, the target was set at an angle of about 55°, approximately midway between two of the gaps. To reduce the air gaps in the electromagnetic yoke, the width of each gap was adjusted to accommodate the varying projected target width as seen from the gap. This procedure restricts the settings of the target, however, and subsequent experience indicates that the precaution was not necessary. Fields up to at least 3000 gauss in each gap were readily obtainable, and this range proved to be more than adequate for the problem under investigation. Because all sections of the magnetic yoke within the vacuum were of the same thickness (1 in.), with no adequate provision for compensation for leakage flux, the deflecting field in the central gaps was somewhat weaker than in the forward and backward gaps.

Following an experimental run, in which the spectrum of the reaction is recorded photographically at each angle, the developed plates are viewed in a microscope. Particles having a given momentum and charge are observed within a well defined region or "plateau," the extent of which is determined by the size of the target spot and the collimating slits. Any portion of this region can receive particles from any portion of the

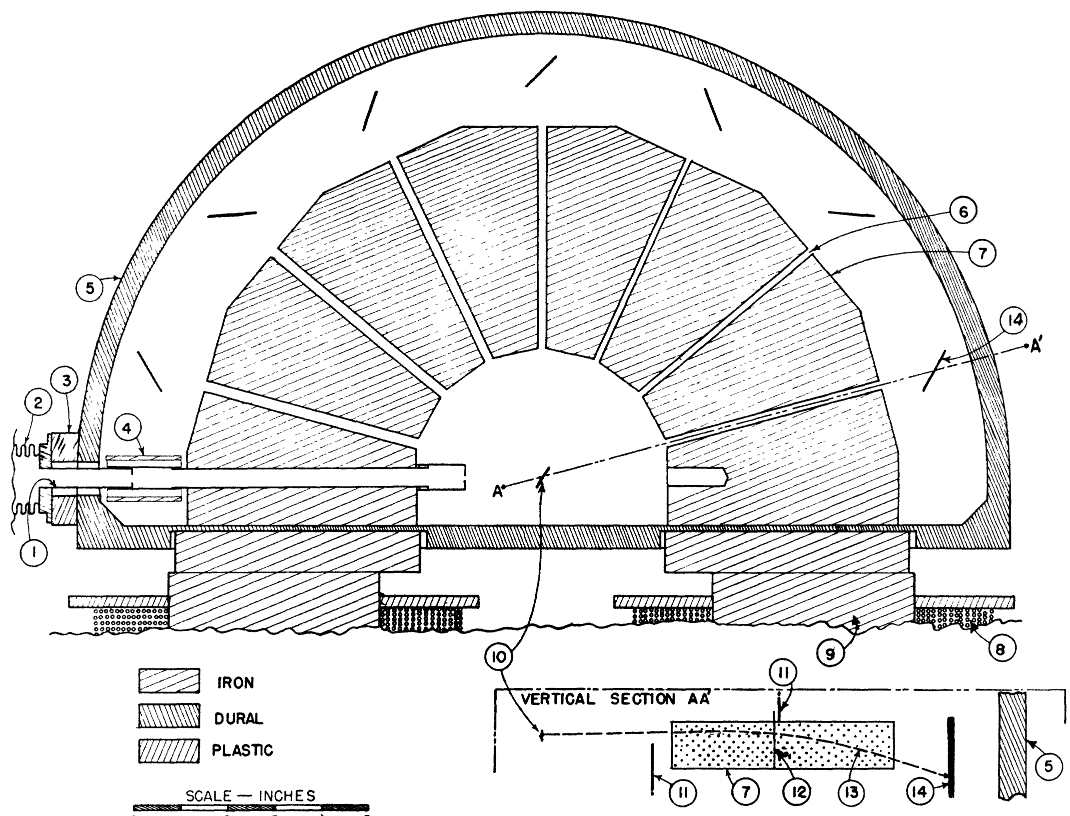

FIG. 1. Photographic chamber incorporating magnetic deflection. (1) Entrance tube; (2) Sylphon connection to accelerator; (3) Insulating washer; (4) Magnetic shield; (5) Chamber wall, 8 in. high; (6) Air gap in magnetic circuit; (7) Magnet pole piece; (8) Coil of electromagnet; (9) Core of electromagnet; (10) Target; (11) Shields; (12) Horizontal slit; (13) Typical particle path; (14) Nuclear plate.

[4] These foils were very generously supplied to us by Dr. Hugh Bradner of the University of California at Berkeley.

target spot. The shape of the plateau is that of a slightly skewed rectangle, since the collimated beam of rectangular cross section, after deflection in the magnetic field, is no longer horizontal when it strikes the plate set at 45°. Surrounding the plateau is a region of partial exposure, the extent of which depends on the particular geometry of the slit system.

The plates are analyzed by determining the extent of the plateau and measuring the particle density within it. Only two small geometric corrections need to be made to this density: an inverse square correction and an area correction, which are necessitated by slight irregularities in plate location and inclination in the target chamber. In principle, the particle density depends also on the deflection in the magnetic field. It was demonstrated that this effect is quite negligible when yields of a given particle are compared at different angles for which the deflections are approximately the same. The corrected densities are converted to the center-of-mass coordinate system,[5] and for convenience the yields are normalized to unity at the 90° angle in the center-of-mass system.

In addition to these routine corrections, several other questions required attention. It is well known that in traversing matter charged particles capture and lose electrons. In the range of energies encountered in this investigation the effect was appreciable only for the alpha-particles. Rutherford,[6] Henderson,[7] and Briggs[8] found that when alpha-particles traverse various kinds of matter, an equilibrium ratio is quickly established between the numbers of He^+ and He^{++} ions (the ratio depending on the alpha-energy and not significantly on the material traversed). In the present experiment, the alphas establish this equilibrium before leaving the target, and the subsequent passage through the magnetic field divides the beam into He^+ and He^{++} components. The He^+ ions are easily identified since they receive one-half of the deflection of the He^{++} ions, and they were counted on four different plates. The He^+ to He^{++} ratios thus determined were in agreement with the results of Henderson and of Briggs. The He^{++} yields on all of the plates were corrected accordingly, to give total alpha-yields, using the equilibrium curve of these authors.

Since the alpha-particles and deuterons receive approximately the same magnetic deflection at several angles and energies, identification was made by track length and appearance alone. It was felt that this was a reliable procedure, but in order to test its validity a series of plates was taken in which a 0.05-mil nickel foil was placed around the target. This foil served either to filter out the alphas completely or at least to separate them from the deuterons after magnetic deflection. Deuteron yields obtained in this manner agreed satisfactorily with those measured without a foil.

The most serious problem encountered in this work arose from the asymmetry introduced by the setting of the target. The effective target thickness for emergent

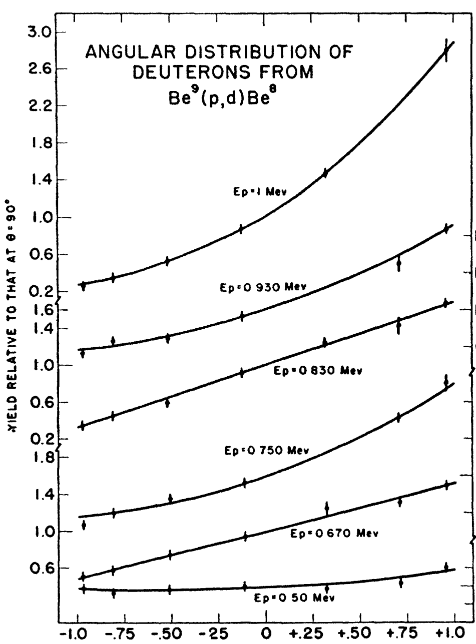

FIG. 2. Angular distributions in the center-of-mass system of the reaction $Be^9(p,d)Be^8$. Yields have been normalized to unity at 90°.

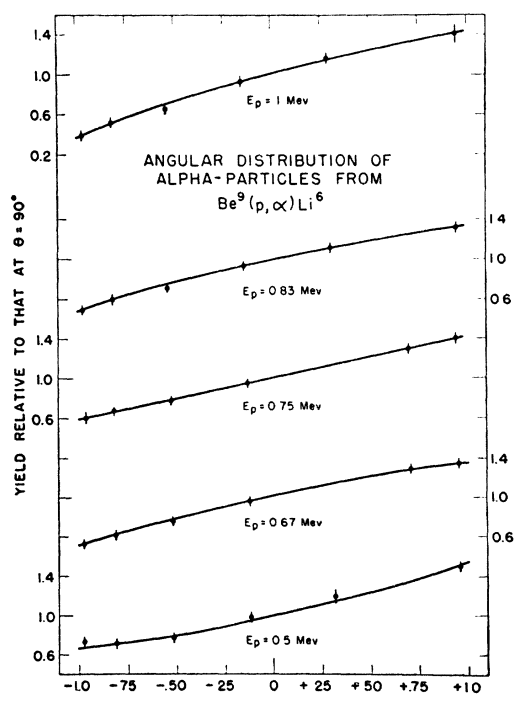

FIG. 3. Angular distributions in the center-of-mass system of the reaction $Be^9(p,\alpha)Li^6$. Yields have been normalized to unity at 90°.

[5] M. Moskow, Master's Thesis (Johns Hopkins University, 1948).
[6] E. Rutherford, Phil. Mag. **47**, 277 (1924).
[7] G. H. Henderson, Proc. Roy. Soc. (London) A **109**, 157 (1925).
[8] G. H. Briggs, Proc. Roy. Soc. (London) A **114**, 341 (1927).

TABLE I. Coefficients in the expression

$$Y(\cos\theta)=Y(0)(1+A_1\cos\theta+A_2\cos^2\theta),$$

obtained from a least-square analysis of the angular yields. The values listed for $Y(0)$ were based on the measurements of Thomas, Rubin, Fowler, and Lauritsen at $\cos\theta=-0.80$. The values are 4π times the cross section in $cm^2\times10^{-27}$ per steradian.

	0.50 Mev	0.67 Mev	0.75 Mev	0.83 Mev	0.93 Mev	1.0 Mev
			Deuterons			
$Y(0)$	222	283	248	300	312	301
A_1	0.11	0.49	0.70	0.70	0.66	1.25
A_2	0.09	−0.06	0.25	0.02	0.21	0.52
			Alphas			
$Y(0)$	225	230	206	273	(354)	260
A_1	0.42	0.41	0.41	0.42		0.53
A_2	0.09	−0.09	0.01	−0.08		−0.11

particles varies with the angle of observation and becomes quite appreciable for the two glancing angles on either side of the target plane. For particles of low energy, originating at various depths within the target, this produces an appreciable spread in energy at these angles and a corresponding spread along the photographic plate. If this spread is smaller than the width of the plateau for monoenergetic particles, the effect is not serious and results chiefly in a narrowing of the plateau within which the particle density is to be computed. If, on the other hand, the spread exceeds the width of the "geometric plateau," a false plateau is created inside of which, at a given point, one observes only particles originating from a restricted layer within the target. A calculation shows that at the two glancing angles on either side of the target set at 55° the latter effect occurs for both the alphas and the deuterons, and, unsatisfactory plateaus and consistently low yields were indeed observed on the corresponding plates. The situation naturally improves with increasing bombarding energy and is less critical for the deuterons than for the alphas; both these effects were observed experimentally. To obviate these difficulties, runs were made with two additional settings of the target, 40° and 60°, each of which sacrifices one of the glancing angles but improves the situation for the other. By intercomparison of the plates taken with these three target settings and rejection of those observations subject to the glancing angle effect, it is believed that reliable yields were obtained at the 40° and 65° angles. After the shape and general trend of the distributions were established, it was not thought worthwhile to make this detailed investigation at every energy, but sufficient data were always taken to obtain a reliable yield at at least one of the glancing angles.

III. RESULTS

The angular yields are presented in Figs. 2 and 3. Except for apparently minor fluctuations, it is possible to fit the distributions by means of polynomials in $\cos\theta$ of the form $1+A_1\cos\theta+A_2\cos^2\theta$. The coefficients, ob-

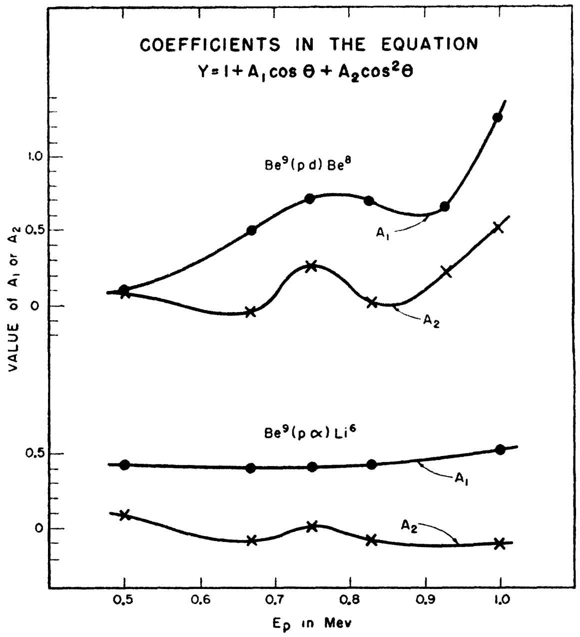

FIG. 4. Energy dependence of the coefficients A_1 and A_2 for the deuterons and alphas from the proton bombardment of beryllium.

tained from a least-square analysis, are listed in Table I and plotted against bombarding energy in Fig. 4. The curves for A_1 are rather reliable inasmuch as the straight line slopes of the distributions are little affected by uncertainties in the individual points. The curves for A_2 are naturally subject to greater uncertainty. For the alpha-particles, A_2 deviates only slightly from zero, and although the corresponding curvatures in the angular distributions seem real, the presence of small systematic errors cannot be ruled out. For the deuterons the existence of a positive squared term is well established, at least for the highest energies, because any reasonable attempt to draw a straight line through the observations at the high energy would result in negative yields at the backward angles.

The presence of a squared term in the angular distributions leads to the following expression for the total cross section in terms of the differential cross section at the center-of-mass angle of 143° (approximately equivalent to the laboratory angle of 138° at which the measurements of Thomas, Rubin, Fowler, and Lauritsen[3] were made):

$$\sigma_T=(4\pi\sigma_{143}/Y_{143})(1+\tfrac{1}{3}A_2).$$

TABLE II. Comparison of ratios of alpha- to deuteron yields.

E_p (Mev)	Y_α/Y_d (Thomas *et al.*, $\theta_L=138°$)	Y_α/Y_d (JHU, $\theta_L=140°$)
0.50	0.50	0.72
0.67	1.0	0.91
0.75	1.2	0.94
0.83	1.4	1.2
1.00	1.6	1.9

$4\pi\sigma_{143}$ are the values given by Thomas *et al.* Y_{143} is the yield at 143° relative to that at 90° and is given by $(1-0.80A_1+0.64A_2)$. The 143° angle is approximately equal to one of the angles of observation of the present experiment. A comparison of the ratios of alpha- to deuteron yields from the two experiments is given in Table II. The discrepancies are probably within the experimental error, except at the lowest energy where our measurement gives a proportionately greater alpha-yield. We have computed the total cross sections in three ways: in the first, the curves of Thomas *et al.* were used as given; in the second, their alpha-yields were adjusted to agree with our alpha- to deuteron ratios; and in the last, both yields were adjusted to give these ratios, but the combined alpha- and deuteron yield was kept the same. The three results differ chiefly in the general trend of the curves and not significantly in the structure displayed, and we present in Fig. 5 the total cross sections obtained in the last manner. The alpha-cross section varies smoothly, but the resonance structure between 0.9 and 1.0 Mev is enhanced. The deuteron curve exhibits more structure. The peak at 0.47 Mev results from the corresponding structure in the 143° curve and occurs in a region where the angular distribution measurements were extrapolated on the basis of rather sketchy observations at 0.44 and 0.40 Mev (these measurements and one at 0.30 Mev are not included in the results because the counting and identification of alphas and deuterons were too unreliable). The structure at 0.68 Mev results chiefly from the variation in the A_1 and A_2 coefficients, but there is a trace of this structure in the 143° curve and in the alpha-curves also, and a gamma-ray resonance is observed in this vicinity.[2] To investigate the possibility that the variation of A_2 in this region is not real, we have made the calculations assuming $A_2 \simeq 0$ to agree with the measurements at adjacent energies. The result

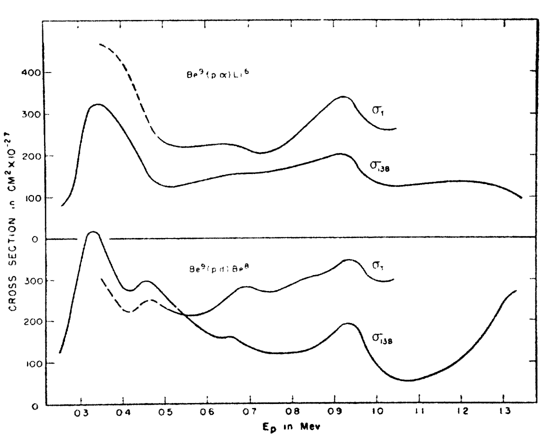

FIG. 5. Energy dependence of the cross sections for the alphas and deuterons from the proton bombardment of beryllium. σ_{138} is 4π times the differential cross sections at a laboratory angle of 138°, as given by Thomas, Rubin, Fowler, and Lauritsen, σ_T is the total cross section calculated from the 138° curves and the measured angular distributions.

indicates a broadening and a shifting to higher energy for the resonance structure, but does not appear to eliminate it.

In the course of this investigation we had opportunity to observe the ranges in photographic emulsion of several different particles, H_1^+, H_2^+, He_4^+, He_4^{++}, Li_6^+, and Li_6^{++}, for a variety of known energies. The results of a series of range measurements are presented in Fig. 6. The proton and alpha-curves are in agreement with the measurements from other laboratories.[9] The deuteron curve agrees substantially with the theoretical prediction based on the proton curve. A comparison of these curves with the corresponding range-in-air curves shows that the stopping power of the emulsion relative to air varies only slightly from 1600. The lithium range-energy curve is not greatly displaced from the alpha-curve, which reflects the fact that at these energies the triply ionized state of lithium is somewhat improbable. This

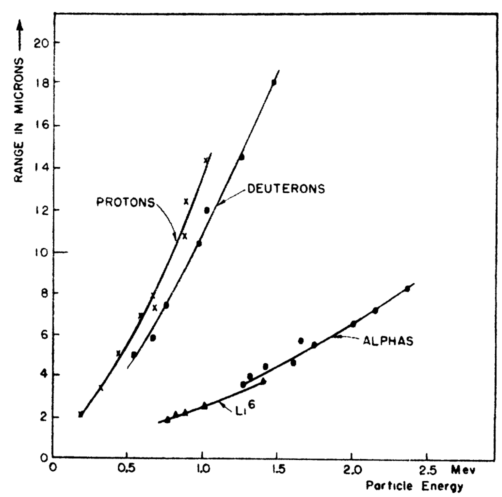

FIG. 6. Range *vs* energy curves for the light nuclei observed in this experiment in Ilford C2 plates.

is confirmed by the failure to locate any appreciable number of Li^{+++} ions on the plates, whereas Li^{++} and Li^+ ions were observed in abundance. Del Rosario,[10] observing the same reaction at lower bombarding energies and with higher resolution, did observe Li^{+++} ions but found that the other states of ionization predominated. As expected, no difference in range between He^+ and He^{++} or between Li^+ and Li^{++} ions were detected. This is because the mean free path for electron capture or loss in the emulsion is very much smaller than the total track length.[6]

The most interesting feature of the angular distributions presented here is the relatively large amount of asymmetry about 90°, indicating interference between two or more states of the compound nucleus, not all of the same parity. Because of the prominence and relatively high yield of the resonance at 0.33 Mev, it seems

[9] Lattes, Fowler, and Cuer, Proc. Phys. Soc. (London) **59**, 883 (1947); N. Nereson and F. Reines, Rev. Sci. Instr. **21**. 534 (1950).
[10] L. Del Rosario, Phys. Rev. **74**. 304 (1948).

reasonable to attribute it to s-wave capture forming a state of the compound nucleus of odd parity. This assignment is confirmed by the observation of Jacobs, Malmberg, and Wahl[11] on the angular yield of the radiation from this state in that they found it to be essentially isotropic. In the interest of simplicity one would like to identify the alpha- and deuteron resonance at 0.94 Mev with the gamma-resonance at 1.00 Mev, although the difference in energy appears to be significant and not easily explained. Devons and Hine[12] attribute this gamma-resonance to s-wave capture also (with a small amount of d-wave penetration), on the basis of their observation that the yield of radiation is nearly isotropic. If this identification of states is correct, the observed interference in the alpha- and deuteron cross sections cannot arise between this state of odd parity and the one at 0.33 Mev; it must come from yet another state of even parity formed presumably by p-wave capture. For the deuterons, such a state is possible at bombarding energies of 0.47, 0.68, and 1.35 Mev; for the alphas, there is evidence for broad resonances, perhaps, at the latter two energies. To test whether the interfering state falls between or above the two odd states, calculations have been carried out to determine the trend of the coefficients. The gross features of the A_1- and A_2-curves seem to be explained best by assuming that the even state falls between the other two, the significant feature being the maxima displayed by the curves in passing through the intermediate resonance (the absence of an observed maximum in the A_1-curve of the alpha-particles is a difficulty which can be attributed partially to the greater width of the alpha-resonance).

[11] Jacobs, Malmberg, and Wahl, Phys. Rev. **73**, 1130 (1948).
[12] S. Devons and M. G. N. Hine, Proc. Roy. Soc. (London) A **199**, 73 (1949).

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Reflections in One-Dimensional Wave Mechanics

ROLF LANDAUER
*Harvard University, Cambridge, Massachusetts**
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A method is presented for solving the one-dimensional Schroedinger equation. The method gives the wave function in the form of an infinite series. The first term of the series is the WBK approximation. Each of the higher terms in turn represents the reflections generated by the preceding term. The expansion is obtained by approximating the potential by a "staircase" potential, which is constant over short regions and changes its value discontinuously between these regions. The method is applied to periodic potentials. By taking the WBK approximation and the first reflection term into account, the zones of the periodic potential can be found in terms of a coefficient which gives the probability that a particle will be reflected in passing through one cell of the periodic potential.

I. INTRODUCTION

SCHELKUNOFF[1] and Brillouin[2] have recently given methods of supplying higher order corrections to the WBK approximation, so that a convergent series for the wave function can be obtained. The method to be presented serves the same purpose, but in addition has a simple physical interpretation. The same method had been arrived at independently by Bremmer.[3,4] The presentation below is along somewhat different lines from that given by Bremmer. Similar concepts have also been suggested by Hill.[5]

In a periodic potential a particle must have energies restricted to certain energy bands. The WBK method neglects the reflections that cause this, and is therefore inadequate for dealing with periodic potentials. The method given here takes distributed reflections into account and is capable of giving zone structure.

II. THE METHOD

Consider first a one-dimensional potential which is constant for $x>0$, also constant for $x<0$, but discontinuous at the origin. Let a particle to the left of the origin, traveling to the right, be represented by the wave function $\exp(ik_1x)$. The wave function for $x>0$ must then be of the form

$$A\exp(ik_2x)+B\exp(-ik_2x), \tag{1}$$

where k_2 is the propagation constant to the right of the origin. Matching considerations give

$$A=1-\Delta k/2k_2,\quad B=\Delta k/2k_2, \tag{2}$$

where $\Delta k=k_2-k_1$. If $k_2\gg\Delta k$, then A will be given by $(k_1/k_2)^{\frac{1}{2}}$, to first order in Δk. Under the same condition $A\gg B$. To a first approximation the term $B\exp(-ik_2x)$ can therefore be neglected. The wave function is then given by a wave which moves to the right everywhere

* Now with the National Advisory Committee for Aeronautics, Cleveland, Ohio.
[1] S. A. Schelkunoff, Q. Appl. Math. **3**, 348 (1946).
[2] L. N. Brillouin, Q. Appl. Math. **7**, 363 (1950).
[3] Presented at the Symposium on the Theory of Electromagnetic Waves, at New York University, June 7, 1950.
[4] H. Bremmer, Physica **15**, 593 (1949).
[5] E. L. Hill, Phys. Rev. **38**, 2115 (1931).