Current Fluctuations in the Direct-Current Gas Discharge Plasma*

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The noise power from a gas discharge plasma may be ascribed to the electron current fluctuations in the plasma due to collisions of electrons with atoms or ions. The noise power, in general, is derived from both the thermal velocities, which are characterized by the electron temperature, and from the dc power, which is characterized by the average current.

I. INTRODUCTION

RADIOFREQUENCY energy produced in a dc discharge plasma in different gases has been measured by Goldstein.[1] Mumford[2] has recently indicated the use of such tubes as noise sources in the microwave region. This noise power can be accounted for by a study of the electron-current fluctuations in a gas discharge plasma, and, in general, in any electron gas. It will be seen that the electron-current fluctuations can be separated into two parts. One can be ascribed to the electron temperature and the other to the direct current in the gas discharge tube.

II. CALCULATION OF CURRENT FLUCTUATIONS IN A GAS DISCHARGE TUBE

Let us consider the electrons in the gas discharge (Fig. 1) whose thermal speeds lie between v and $v+dv$ with a mean collision frequency Z_v. This means that on the average an electron in this range will suffer Z_vT collisions in a time T. Actually, there will be fluctuations in this number, and the probability $p(K)$ that an electron will experience K collisions in a time T is[3]

$$p(K)=(Z_vT)^K[\exp(-Z_vT)]/K! \tag{1}$$

The probability that the time between consecutive collisions of an electron lies between θ and $\theta+d\theta$ is

$$q(\theta)=Z_v\exp(-Z_v\theta)d\theta. \tag{2}$$

The convection current measured between the electrodes due to an electron that has collided at time t_k with a subsequent free time θ_k is

$$\left.\begin{aligned} i_x(t-t_k;\theta_k)&=(e/d)[v_x+a(t-t_k)]\\ i_y(t-t_k;\theta_k)&=(e/d_y)v_y\\ i_z(t-t_k;\theta_k)&=(e/d_z)v_z \end{aligned}\right\} t_k\leq t\leq t_k+\theta_k \tag{3}$$
$$=0$$

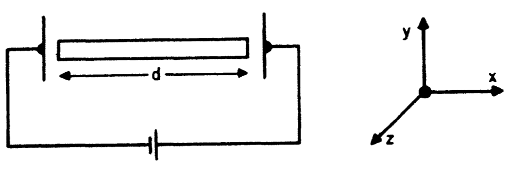

FIG. 1. Direct-current-maintained gas discharge plasma.

everywhere outside this time interval, where d is the length of the tube parallel to the direction of applied electric field E; $a=eE/m$; d_y and d_z are transverse dimensions; v_x, v_y, v_z are components of the thermal velocity; and, similarly, i_x, i_y, i_z are components of the convection current.

We shall now restrict ourselves to the calculation of i_x as the calculations for the others are quite similar. Now let

$$\phi_k=t-t_k. \tag{4}$$

Hence, we have

$$\begin{aligned} i_x(\phi_k;\theta_k)&=(e/d)(v_x+a\phi_k),\quad 0\leq\phi_k\leq\theta_k\\ &=0\ \text{elsewhere.}\end{aligned} \tag{3a}$$

The current $I(t)$ will be a random function of time, depending on the values of ϕ_k and θ_k, which may be looked on as random values. Thus, the methods described by Rice[3] for the study of shot-effect processes may be used to compute the average value and the spectrum of $I(t)$.

Computation of $\langle I(t)\rangle_{\text{Av}}$

Following Rice,[3] the time average of the current $I(t)$ due to a single electron is found by averaging $I(t')$ for a given t' over M intervals of duration T; M being very large. Thus, we have

$$\langle I(t)\rangle_{\text{Av}}=\lim 1/M\sum_{i=1}^{M}I_i(t'). \tag{5}$$

$I_i(t')$ is the current at time t' in the i'th interval. In $Mp(K)$ of these intervals, an electron will experience K collisions that occur at times $t_1, t_2\cdots t_K$. The t_i will vary in a random manner over these $Mp(K)$ intervals. For these intervals, the contribution to the time average is

$$p(K)\langle I_K(t-t_i;\theta_i)\rangle_{\text{Av}}. \tag{6}$$

The latter average is now over all the random-varying t_i in these $Mp(K)$ intervals. Now let us consider those intervals for which $t_i<t'<t_{i+1}$. The average contribution for such intervals would be

$$\int_0^T\int_0^{\theta_i} i_x(\phi_i)T^{-1}d\phi_iZ_v\exp(-Z_v\theta_i)d\theta_i, \tag{7}$$

$$\theta_i=t_{i+1}-t_i,$$

* This work was sponsored by the Signal Corps Engineering Laboratories of the United States Army.
[1] L. Goldstein and N. Cohen, Phys. Rev. 73, 83 (1948).
[2] W. W. Mumford, Bell System Tech. J. 28, 608 (1949).
[3] S. O. Rice, Bell Sys. Tech. J. 23, 282 (1949).

which equals

$$(Z_v e/Td)(v_x/Z_v^2+a/Z_v^3) \quad \text{as} \quad T\to\infty. \tag{8}$$

Hence, summing over all t_i, we obtain

$$\langle I_x(t)\rangle_{\text{Av}}=\sum_{K=0}^{\infty}\frac{Kp(K)}{T}\frac{e}{d}\left(\frac{v_x}{Z_v}+\frac{a}{Z_v^2}\right), \tag{9}$$

$$\langle I_x(t)\rangle_{\text{Av}}=\frac{ev_x}{d}+\frac{ea}{d}\frac{1}{Z_v}, \tag{10}$$

since

$$\sum_{K=0}^{\infty}Kp(K)=Z_vT.$$

Similarly, we have

$$\left.\begin{aligned}\langle I_y(t)\rangle_{\text{Av}}&=ev_y/d_y\\ \langle I_z(t)\rangle_{\text{Av}}&=ev_z/d_z.\end{aligned}\right\} \tag{11}$$

III. CALCULATION OF CORRELATION FUNCTION

Following Rice, the correlation function $\psi_{\text{Av}}(\tau)$ is given by

$$\langle\psi(\tau)\rangle_{\text{Av}}=\lim_{T\to\infty}1/T\int_0^T\langle I(t)I(t+\tau)\rangle_{\text{Av}}dt, \tag{12}$$

where the average is, as previously, over many intervals of duration T. Thus, we have

$$\langle I(t)I(t+\tau)\rangle_{\text{Av}}$$

$$=\sum_{K=0}^{\infty}p(K)\langle I_K(t-t_i;\,\theta_i)I_K(t+\tau-t_j;\,\theta_j)\rangle_{\text{Av}}, \tag{13}$$

where the average is now over the random variables t_i and θ_i. Now for those intervals (t_i, t_{i+1}) for which t and $t+\tau$, $\tau>0$, both lie in this interval or $0<\phi_i<\theta_i-\tau$, the contribution is

$$\langle\psi_1(\tau)\rangle_{\text{Av}}=\sum_{K=0}^{\infty}Kp(K)\frac{Z_v}{T}\int_\tau^T\int_0^{\theta_i-\tau}i_x(\phi_i)i_x(\phi_i+\tau)\times\exp(-Z_v\theta_i)d\phi_i d\theta_i. \tag{14}$$

For those intervals (t_i, t_{i+1}) and (t_j, t_{j+1}) for which t and $t+\tau$ lie in different intervals, the contribution is

$$\langle\psi_0(\tau)\rangle_{\text{Av}}=\sum_{K=0}^{\infty}K(K-1)p(K)T^{-2}Z_v^2\times\left[\int_0^T\int_0^{\theta_i}i_x(\phi_i)\exp(-Z_v\theta_i)d\phi_i d\theta_i\right]^2. \tag{15}$$

Substituting for i_x from (3a) and neglecting those terms that are proportional to v_x inasmuch as they will not contribute to the average over-all velocity classes, we obtain

$$\psi_{\text{Av}}(\tau)=\langle\psi_1(\tau)\rangle_{\text{Av}}+\langle\psi_0(\tau)\rangle_{\text{Av}},$$

$$\langle\psi_1(\tau)\rangle_{\text{Av}}=\frac{e^2}{d^2}\left[v_x^2+\frac{2a^2}{Z_v^2}+\frac{a^2\tau}{Z_v}\right]\exp(-Z_v\tau_i), \tag{16}$$

$$\langle\psi_0(\tau)\rangle_{\text{Av}}=\frac{e^2}{d^2}\left[v_x^2+\frac{a^2}{Z_v^2}\right]. \tag{17}$$

IV. CALCULATION OF THE SPECTRUM $\langle w(f)\rangle_{\text{Av}}$ OF CURRENT FLUCTUATIONS

$$\langle w(f)\rangle_{\text{Av}}=4\int_0^\infty\psi(\tau)\cos2\pi f\tau d\tau \tag{18}$$

The spectrum resulting from $\psi_0(\tau)$ does not represent a fluctuation as it is due to $\langle I(t)\rangle_{\text{Av}}$. Thus, we have

$$\langle[I(t)-\langle I(t)\rangle_{\text{Av}}]^2\rangle_{\text{Av}}=\int\langle w_1(f)\rangle_{\text{Av}}df=\langle\psi_1(0)\rangle_{\text{Av}}=e^2d^{-2}(v_x^2+2a^2/Z_v^2) \tag{19}$$

and

$$w_1(f)=4\int_0^\infty\langle\psi_1(\tau)\rangle_{\text{Av}}\cos2\pi f\tau d\tau=\frac{4e^2}{d^2}\left[\left(v_x^2+\frac{2a^2}{Z_v^2}\right)\frac{Z_v}{Z_v^2+4\pi^2f^2}+\frac{a^2(Z_v^2-4\pi^2f^2)}{Z_v(Z_v^2+4\pi^2f^2)^2}\right]. \tag{20}$$

A similar derivation for metals with no direct current has been carried through by Bakker and Heller.[4] In their case, the current fluctuations, assuming classical statistics, is given by (20) with $a=0$.

V. SUMMATION OVER ALL ELECTRONS

It now remains to sum over all electrons. It is assumed that all electrons act independently of each other. Naturally, some assumptions will have to be made about the number of electrons per unit velocity range F and the variation of Z_v with v.

We shall assume that F in the gas discharge plasma is maxwellian,[5]

$$\begin{aligned}F(v_x,v_y,v_z)&=A\exp[-\beta^2(v_x^2+v_y^2+v_z^2)],\\ A&=N(m/2\pi kT_e)^{\frac{3}{2}},\\ \beta^2&=m/2kT_e,\end{aligned} \tag{21}$$

where N is the total number of electrons $=n_0Sd$, n_0 is the number of electrons per cubic centimeter, S is the cross-sectional area of the tube, and T_e is the electron

[4] C. J. Bakker and G. Heller, Physica 6, 262 (1939).

[5] I. Langmuir and G. Mott-Smith, Phys. Rev. 28, 727 (1926).

temperature. It is also assumed that Z_v is independent of v and equal to Z. This is true for some gases; e.g., A, Kr, Xe for velocities less than 1 volt and Hg for still higher velocities.[6]

With these assumptions, the total quantities being denoted (sum over all electrons) by a subscript N, the average total current is given by

$$\langle I_N(t)\rangle_{\mathrm{Av}}=eaN/dZ. \tag{22}$$

Hence, the dc resistance R_0 is given by

$$R_0=md^2Z/Ne^2. \tag{23}$$

Furthermore, in accordance with the theory of conductivity of electrons in gases,[7] the ac admittance is given by

$$Y_\omega=\frac{Ne^2}{md^2}\left[\frac{Z+j2\pi f}{Z^2+4\pi^2f^2}\right]=G(\omega)+jB(\omega), \tag{24}$$

$$\langle[I_N(t)-\langle I_N(t)\rangle_{\mathrm{Av}}]^2\rangle_{\mathrm{Av}}=\frac{Ne^2}{md^2}kT_e+2\frac{[\langle I_N(t)\rangle_{\mathrm{Av}}]^2}{N}. \tag{25}$$

Except for a numerical factor of 2, this checks with a result obtained by other considerations by Brillouin[8] for electrons in metals

$$\langle w_1(f)_N\rangle_{\mathrm{Av}}=4kT_eG(\omega)+4\frac{[\langle I_N(t)\rangle_{\mathrm{Av}}]^2}{N}\left[\frac{2Z}{Z^2+4\pi^2f^2}+\frac{Z(Z^2-4\pi^2f^2)}{(Z^2+4\pi^2f^2)^2}\right]. \tag{26}$$

For I_y and I_z, the spectrum will be given by just the first term of (26).

VI. CALCULATION OF AVAILABLE NOISE POWER IN WAVE GUIDES

We shall now apply these results to calculate the available noise power from a gas discharge plasma placed in the transverse plane of a rectangular wave guide propagating only in its lowest mode.

The available noise power

$$P_\omega=\tfrac{1}{4}|I_E|^2/G_\omega, \tag{27}$$

where I_E is the current in the direction of the E vector. Hence, in this case we have

$$P_\omega=\left\{kT_e+\frac{P_0}{NZ}\cos^2\theta\left[2+\frac{Z^2-4\pi^2f^2}{Z^2+4\pi^2f^2}\right]\right\}df, \tag{28}$$

θ is the angle between the E vector and axis of tube, P_0 is the dc power dissipated in the tube; and at $\theta=\pi/2$ this reduces to the case of Mumford,[2] for which

$$P_\omega=kT_edf. \tag{29}$$

VII. CONCLUSIONS

The noise power from gas discharge plasma may be ascribed to the electron-current fluctuations in the plasma due to collisions of electrons with atoms or ions. The noise power, in general, is derived both from the thermal velocities, which are characterized by the electron temperature, and from the dc power, which is characterized by the average current. The fluctuations due to the positive-ion current may be neglected, since the ion gas temperature is usually much lower than that of the electron gas, and the ionic current is much less than the electronic current. It has been assumed that the electron velocity-distribution function is maxwellian and that the collision frequency of an electron is independent of speed. This has been verified by experiment in certain cases. However, for those discharges where these assumptions are not experimentally verified, the current fluctuations and noise power may still be computed by a similar method from the experimentally obtained distribution function and variation of collision frequency with speed. The current fluctuations due to the fluctuations in the various electron removal and production processes in the plasma have not been accounted for by this method.

In general, the frequency-sensitive portion of the available noise power is only a few percent of the kT_e term. However, it may become important in low electron temperature gas discharge plasma, such as occur in caesium vapor.

[6] R. B. Brode, Revs. Modern Phys. 5, 257 (1933).
[7] H. Margenau, Phys. Rev. 69, 508 (1946).
[8] L. Brillouin, Helv. Phys. Acta 7, 47 (1934). This discrepancy is due to the fact that in Brillouin's model the average current is not produced by an accelerating dc electric field but is assumed to be given. Brillouin's answer may be obtained by putting $a=0$ and increasing the thermal velocity by a constant drift velocity.