The Twice-Forbidden Transition of Cs^{137} and the Law of Beta-Decay*

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(Received February 19, 1951)

The shape of the beta-spectrum of the direct 1.17 Mev, 8 percent abundant transition between the $g_{7/2}$ ground state of Cs^{137} and the $d_{3/2}$ ground level of Ba^{137} has been studied in detail in a magnetic spectrometer. The shape of the spectrum can be adequately described by a twice-forbidden factor, C_{2T}, resulting from the tensor form of interaction. Neither the axial vector form nor the combination S–A–P are capable of accounting for the data.

I. INTRODUCTION

THE shape of the beta-spectrum of the weak, twice-forbidden transition of Cs^{137} has been studied in detail in order to determine which of the possible forms of the Fermi theory[1] is the correct one. The general validity of the Fermi theory, as applied to allowed transitions, now appears to be well established.[2–6] The details of the theory[7] can, however, be formulated in different ways, depending essentially on the transformation properties of the product of the wave functions associated with the emitted electron and neutrino. Five invariant forms are possible: scalar, vector, tensor, axial vector, or pseudoscalar.

It happens that the choice of a single one of the interaction forms has no influence on the shape of the spectrum of an allowed transition. Therefore, measurements of such "allowed spectra" offer no means of determining which of the particular forms of the theory is correct.

Certain linear combinations of the interaction forms are also possible. In particular, the combination S–A–P proposed by Wigner and Critchfield[8] is of interest, since it implies a symmetry in the wave functions of the initial and final nuclei as well as those of the electron and neutrino. It has been shown[9] that the combinations S–A–P and V–T are the only ones which need be considered, largely because of an interference effect, an additional energy-dependent term to the distribution formula for the allowed beta-transition. This term is of the form $(1\pm a/W)$, where W is the electron energy and a is a constant. Since the allowed spectrum is already adequately described without this additional term, it is inferred that all other possible combinations may be disregarded.

For the forbidden transitions, the shape of the beta-spectrum may depend upon the form of the interaction. In general, a beta-spectrum is given by the distribution formula

$$Nd\eta \sim C(W_0, W, Z)F(Z, W)\eta^2(W_0-W)^2 d\eta.$$

$Nd\eta$ is the number of electrons in the momentum interval $d\eta$. W_0 is the maximum energy (including rest energy) of the electron. W_0-W is c times the momentum of the neutrino, if that particle has zero rest mass.[10] $F(Z, W)$ is the "coulomb factor" and represents the effect of the electrostatic field on the emitted electron. The factor $C(W_0, W, Z)$ arises from the matrix elements measuring the overlapping of the initial and final nuclear states. For allowed transitions this factor is independent of the energy for all of the interaction forms. In the case of such allowed spectra, the distribution is determined mainly by the "statistical factor" $\eta^2(W_0-W)^2 d\eta$, which would be characteristic of any momentum distribution resulting from the sharing of a given energy release, W_0, between a pair of particles.

It is also to be expected that for some forbidden transitions the spectra will have the allowed shape. This arises from the dominance of certain energy independent terms in the expression for C.[11,12] This is particularly so for once-forbidden transitions involving a change of parity and a spin change of 0 or 1 unit. However, for once-forbidden transitions involving a change of 2 units of angular momentum, all matrix elements but one vanish, and the expression for C yields a unique energy dependence. A spectrum of this type was first reported[13] in the decay of Y^{91}. The shape of the spectrum of this transition can be satisfactorily explained by setting C proportional to $a\sim[W^2-1+(W_0-W)^2]$. This energy dependence is provided by either the axial vector or the tensor forms of interaction. It could, obviously, also result from S–A–P with negligible contributions from S and P. This same characteristic spectrum shape has since been identified in at least a dozen other beta-disintegrations.[14] Whereas these

* This work was assisted by a grant from the Frederick Gardner Cottrell Fund for the Research Corporation and by the joint program of the ONR and AEC.

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once-forbidden shapes have been useful in interpreting the predictions of the nuclear shell model[15] and in confirming the validity of Gamow-Teller selection rules[16] by reducing the form of the interaction to either tensor or axial vector, they alone are not capable of yielding a decision as to which of these two interaction forms is correct.

In addition to the once-forbidden shape, a unique energy dependence is also predicted for the twice-forbidden transition involving no change of parity and a change of 3 units of angular momentum. In this case, the forbidden factor, C, is proportional to $c\sim 3(W^2-1)^2+3(W_0-W)^4+10(W-1)(W_0-W)^2$ and again results from either the tensor or the axial vector interaction. Spectra of this shape have been observed[17,18] for Cl^{36}, and Be^{10}.

The spectrum to be discussed in the present paper is of particular interest because it offers a means of choosing between the tensor and the axial vector interactions. The decay in question is the direct transition from the ground state of Cs^{137} to the ground level of Ba^{137}. An attractive feature of this transition is that the spin of the radioactive parent as well as that of the stable daughter have been measured experimentally.[19,20] The $g_{7/2}$ ground level of Cs^{137} decays in either of two ways with a total half-life of 37 years. In over 90 percent of the time, the decay is to a metastable state of Ba^{137} by a negatron emission with a maximum energy of 0.51 Mev. The metastable level of 2.6 minutes half-life then proceeds to the $d_{3/2}$ ground level of Ba^{137} by the emission of a gamma-ray of 0.661 Mev.[21] The alternative mode of decay is the direct beta-transition to the ground level of Ba^{137}

The low energy beta-transition has been studied by various investigators.[11,22-24] Its shape has been generally interpreted as that of the once-forbidden, $\Delta I=2$, class discussed above, although the exact determination of the shape has always been complicated by the necessity of correcting for the small but finite contribution from the high energy group.

The weak beta-group, with a maximum energy of 1.17 Mev, results then from a twice-forbidden transition involving a spin change of 2 and no change of parity. Because of the nonvanishing of matrix elements, whose evaluation is somewhat arbitrary, the energy dependence of the factor $C(W_0, W, Z)$ is, in this case, not unique. However, it will be shown below that it is not possible to fit the experimental data with the axial vector interaction for any values of the adjustable parameters. Furthermore, the combination S–A–P is also ruled out. On the other hand, the tensor form of interaction can be made to give a very good fit.

Another advantage to be found in working with this 1.17-Mev group of Cs^{137} arises from the fact that, since the transition is only slightly twice-forbidden ($ft\approx 10^{12}$), a source can be obtained of sufficiently high specific activity so as to permit a reliable measurement of the spectrum shape.

II. EXPERIMENTAL METHOD

The measurements of the spectra of the high energy Cs^{137} group were made in the 40-cm radius of curvature, 180 degree focusing, shaped magnetic field spectrometer.[25] This instrument is particularly well suited for measuring the shape of the weak 1.17-Mev spectrum in the presence of the much more intense 0.51-Mev group. The large clearances provide adequate space for effective antiscattering baffles. The large distance and thick absorbing core between the source and the detector results in no background interference from the gamma-radiation even with the relatively intense sources employed in this investigation.

Preliminary measurements indicated that there was some slight amount of scattering of low energy electrons off the edge of the main defining baffle (located midway between source and detector). This baffle, which was 0.625 inch thick, had originally been designed for the universal use of the spectrometer with electrons of much higher energy. Some electrons from the intense low energy group were bouncing off the edge of this baffle and getting to the detector. When the thickness of this baffle was reduced to 0.125-inch Al (sufficient for stopping all electrons from Cs^{137}), the number of scattered electrons became completely negligible. As a check, the spectrum was measured for several different openings of the defining baffle. No further change in the spectrum was introduced by the change in geometry. Also, the addition of an extra antiscattering baffle, installed in a plane 18 degrees up from that of the detecting slit, did not change the measured distribution.

Measurements were made with both single- and double-bead 3-mg/cm^2 mica end-window counters. The use of the double-bead counter and a pulse height selector has the advantage that, in addition to lowering the background, it also discriminates against any scattered electrons which might pass through the detecting slit but whose trajectories would not have the proper curvature for traversal of both sections of the counter.

Some caution must be applied in using a double-bead counter for the measurement of a momentum distribution. In general, such a counter will have a detec-

as being of the once-forbidden $\Delta I=2$ type. This apparently is not correct; see R. B. Duffield and L. M. Langer, Phys. Rev. **81**, 203 (1951). In addition, the decay of Rb^{88} has also been found to exhibit this characteristic "a" shape; see Bunker, Langer, and Moffat, Phys. Rev. **81**, 30 (1951).

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tion efficiency which is a function of the energy of the incident particles. This arises from the fact that, at low energies, there is more large angle scattering of the electrons passing through the mica window. Some of these will not traverse both sections of the counter. Also, at very high energies, the probability of an incident electron producing an ion pair in both sections of the counter will become less with increasing energy if the length of path in each counter section is small. A correction for these effects can be made by determining the ratio of double-size to single-size pulses as a function of energy, with a strong beta-source. It turns out, conveniently, that in our case, the double-bead counter sensitivity is esentially constant over the relatively narrow energy band explored in this investigation.

For the present experiment, both the source and the detector slit had widths of 0.6 cm, so that the resolution of the instrument was about 0.75 percent.

Two different sources were prepared from two different fission product samples obtained from Oak Ridge. The first source contained about 3 mC and had an average surface density of 0.4 mg/cm^2. The second source was about 3 times as intense and had an average density of 1.7 mg/cm^2. Both sources were spread quite uniformly with the aid of insulin[26] over a 0.6-cm by 2.5-cm rectangle. The backing for the sources was 0.0002-inch aluminum covered with a 3-$\mu g/cm^2$ layer of zapon. The source was covered by a 1.5-$\mu g/cm^2$ layer of zapon. Although the thickness of the sources and of the backings is more than might be esthetically desirable, it is felt that no measurable distortion resulted over the energy range of particular interest, *viz.*, from 0.7 Mev to 1.17 Mev. The agreement of the data obtained from both sources is indicative of this, as is the very small amount of energy degeneration evident on the low energy side of the internal conversion line. On the other hand, the aluminum backing provides a mechanically strong, electrically grounded support for the extremely intense, long-lived sources. Repeated runs, taken over a period of weeks, showed that there was no apparent decay of the source because of sublimation or any other causes.

The strength of the sources were such that counting rates in the region of the twice-forbidden spectrum, at energies just above the conversion lines of the 0.661-Mev gamma-ray, were from several hundred per minute to about 2500 per minute, depending on the size of the opening in the defining slit. The background with the double-bead counter was 11 counts per minute. All the data were taken with statistical accuracies of from 1 to 3 percent.

The magnetic field measurements were made by means of a continuously rotating coil arrangement.[27] The instrument is calibrated in terms of the internal conversion line from the 0.4112-Mev gamma-ray of Au^{198}. The K line from the 0.661-Mev[20] gamma-ray of Cs^{137} served as a useful monitor in the present investigation.

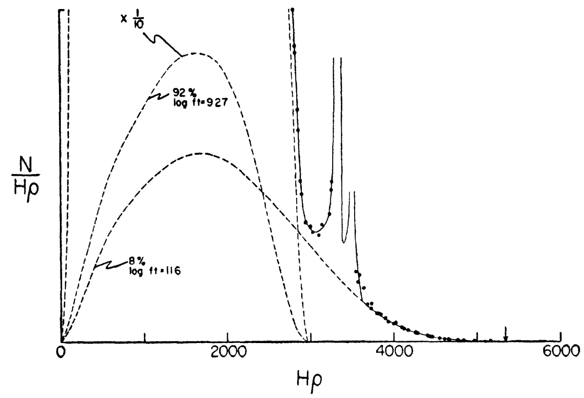

FIG. 1. Beta-spectra of Cs^{137}.

III. RESULTS

Figure 1 is a momentum distribution plot of some typical data obtained in this investigation. The resolution of the spectrum into two groups is done on the basis of the forbidden spectra analysis described below. The extrapolation of the high energy group results from the fit of the tensor "forbidden factor," C_{2T} appropriate for a twice-forbidden transition involving no change of parity and a spin change of 2. After the 1.17-Mev group was properly subtracted, the 0.51-Mev group was extrapolated to lower energies as a once-forbidden spectrum resulting from a transition involving a spin change of 2 and a change of parity. The spectrum shapes of the two groups are obviously quite different from each other and also from an "allowed shape." The relative intensities of the two groups are 92 percent for the 0.51-Mev group and 8 percent for the 1.17-Mev group. This abundance of the high energy group is somewhat greater than earlier estimates of less than 5 percent, based upon less exact analyses.[22,23] The "comparative half-lives," on the basis of the present data are given by $\log ft = 9.2$ for the 0.51-Mev group and $\log ft = 11.6$ for the 1.17-Mev group.

Figure 2 shows the ordinary allowed ($C=1$) and also the appropriate twice-forbidden ($C=C_{2T}$) Fermi plot of the data. The straight line plot obtained with the tensor "forbidden factor," C_{2T}, extrapolates to an end point of 1.17 Mev. This is consistent with the value of $W_0 = 3.29$ mc^2 assumed for the calculation of C_{2T}, and is also in agreement with the sum of the energy of 0.51 Mev obtained for the inner group plus the gamma-ray energy of 0.661 Mev.

The value of C_{2T} was calculated from the very good approximations appearing in Konopinski's review article.[7] To obtain a good fit with C_{2T}, it was only necessary to use the dominant terms containing the matrix elements A_{ij} and T_{ij}. The ratio of A_{ij}/T_{ij}, chosen to obtain the best fit to the data, is 7.43. The fit is not very sensitive to the choice of a value for the

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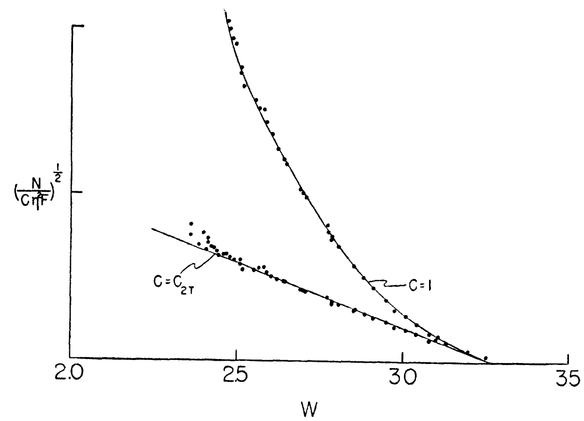

FIG. 2. Ordinary ($C=1$) Fermi plot and tensor forbidden ($C=C_{2T}$) Fermi plot of the 1.17-Mev beta-group of Cs^{137}.

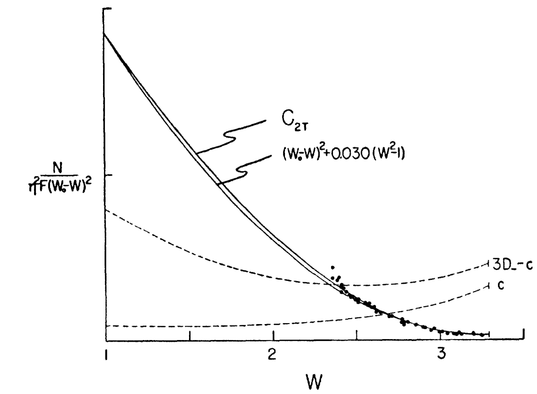

FIG. 3. Energy dependence of "forbidden factors" for the twice-forbidden Cs^{137} beta-group.

nuclear radius (*viz.*, the distance at which the wave functions are evaluated). A very good approximation for the energy dependence of C_{2T} is given by $(W_0-W)^2+k(W^2-1)$. For the present case, $k=0.030$ gives a good fit. Figure 3 shows how the experimental and theoretical "forbidden factors" depend on the energy W. Such a plot is a more sensitive test of the fit than the Fermi plot. The good agreement provided by the tensor interaction is apparent.

The broken line curves in Fig. 3 illustrate the energy dependence of the two terms which, when multiplied by positive coefficients, can combine to make up the axial vector "forbidden factor," C_{2A}.[7] It is quite obvious that there is no way of adding these curves so as to obtain the energy dependence demanded by the data.

Furthermore, since the linear combination S–A–P can only add more D_- energy dependence, it appears that this combination also cannot fit the experimental data.

It is perhaps interesting to note that the curvature of the ordinary allowed Fermi plot of the data of the high energy Cs^{137} group is similar to that observed for the spectrum of RaE.[28–30]

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IV. CONCLUSION

The shapes of the beta-spectra of allowed transitions can be described by any one of the five possible invariant forms of the Fermi theory, *viz.* scalar, vector, tensor, axial vector, or pseudoscalar. Also, the linear combinations S–A–P and V–T should be considered.

The shapes of beta-spectra resulting from transitions involving a spin change of one unit greater than the degree of forbiddenness (e.g., once-forbidden $\Delta I=2$, twice-forbidden $\Delta I=3$) can be described by *either* the axial vector *or* the tensor interactions, but not by the scalar, vector, or pseudoscalar forms alone.

The high energy transition between the $g_{7/2}$ ground state of Cs^{137} and the $d_{3/2}$ ground level of Ba^{137} results in a beta-spectrum whose measured shape can be explained by the "forbidden factor," C_{2T}, arising from the tensor form of interaction. Furthermore, neither the axial vector form of interaction or the linear combination S–A–P are capable of accounting for the experimental data. The linear combination V–T remains as a possibility. However, the addition of the vector term does not help explain anything which is not already adequately described by the tensor form alone.

It is concluded, therefore, that the law of beta-decay is such that when subject to a Lorentz transformation the product of the electron and neutrino wave functions, $\psi_e\psi_\nu$, behaves like a tensor.