

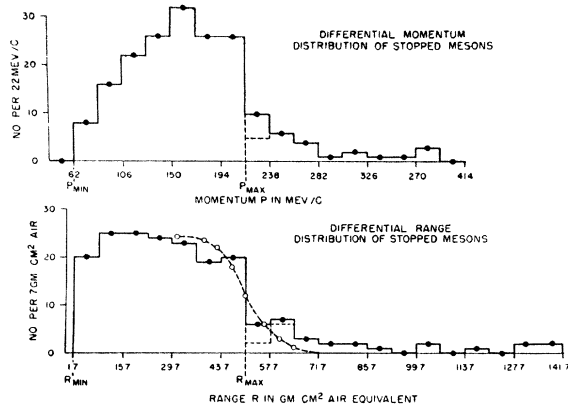


FIG. 2. Momentum and range distribution of stopped mesons. The range distribution was obtained by transforming the momentum distribution under the assumption of a mu-meson mass of 216 electron masses.

errors were difficult to evaluate, all that can be said at present is that they are probably protons. If the technique can be significantly improved, a similar experiment will be undertaken in order to obtain more accurate mass determinations.

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¹ B. Rossi, *Revs. Modern Phys.* **20**, 537 (1948).

² W. L. Kraushaar, *Phys. Rev.* **76**, 1045 (1949).

³ L. Germain, *Phys. Rev.* **80**, 616 (1950).

⁴ H. P. Koenig, *Phys. Rev.* **69**, 590 (1946).

Theory of Antiferromagnetic Resonance

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WE calculate the frequencies associated with magnetic dipole transitions in an antiferromagnetic crystal, extending to dynamic effects the Van Vleck theory of antiferromagnetism, which works remarkably well for static effects. The model is that of two sublattices of magnetization $\mathbf{M}_1$, $\mathbf{M}_2$ oppositely directed and each of magnitude M_s . The calculation given here is classical; a quantum calculation following the method used by Van Vleck¹ in ferromagnetic resonance has been made and will be discussed in a subsequent publication.

The exchange forces are treated as molecular fields $\mathbf{H}_1 = -\lambda\mathbf{M}_2$, $\mathbf{H}_2 = -\lambda\mathbf{M}_1$ acting on the sublattices 1 and 2. It is known that there are preferred directions of orientation in antiferromagnetic crystals, and in analogy to ferromagnetism we introduce an anisotropy energy density constant K to describe the energy involved in turning both spin systems together relative to the crystal lattice. We may then, for small deflections, say that there is an anisotropy field $H_A = K/M_s$ acting on each sublattice. We take the static field H_0 and the preferred axis to be in the z -direction. We suppose that the crystal consists of a single domain and is spherical in shape, so that demagnetizing effects do not enter.

The equations of motion with a transverse rf field are

$$\begin{aligned} d\mathbf{M}_1/dt &= \gamma\mathbf{M}_1 \times [(\mathbf{H}^x - \lambda\mathbf{M}_2^x)\mathbf{i} \\ &\quad + (\mathbf{H}^y - \lambda\mathbf{M}_2^y)\mathbf{j} + (\mathbf{H}_0 + H_A + H_E)\mathbf{k}]; \\ d\mathbf{M}_2/dt &= \gamma\mathbf{M}_2 \times [(\mathbf{H}^x - \lambda\mathbf{M}_1^x)\mathbf{i} \\ &\quad + (\mathbf{H}^y - \lambda\mathbf{M}_1^y)\mathbf{j} + (\mathbf{H}_0 - H_A - H_E)\mathbf{k}]; \end{aligned}$$

here $H_E = \lambda M_s = \lambda M_1^z = -\lambda M_2^z$. Defining $M^- = M_x - jM_y$, $H^- = H_x - jH_y$, we solve for the susceptibility:

$$\begin{aligned} \chi^- &= M^-/H^- = 2\gamma^2 M_s H_A / (\omega - \omega_0)^2; \\ \omega_0/\gamma &= H_0 \pm [H_A(H_A + 2H_E)]^{1/2}. \end{aligned} \quad (1)$$

Note that H_E enters only if $H_A \neq 0$. For $H_0 = 0$ and $H_A \ll H_E$, the

zero field splitting is $\omega_0 \cong \gamma(2H_A H_E)^{1/2} = \gamma(2K\lambda)^{1/2}$. The two frequencies in Eq. (1) correspond to different directions of circular polarization, and a linearly polarized rf field will excite both precessional modes. In polycrystalline specimens the line widths will be at least of the order of H_0 , as only the component of the static field parallel to the domain axis is fully effective.

In the common antiferromagnetics such as MnO, MnF₂, FeO, and Cr₂O₃, we may estimate $H_A \sim 10^3$ oersteds and $H_E \sim 10^6$ oersteds. Then $(2H_A H_E)^{1/2} \sim 5 \times 10^4$ oersteds and $\omega_0 \sim 5$ cm⁻¹, which is higher than the experimental frequency 0.3 cm⁻¹ used by Maxwell² *et al.* and Hutchison.³ The low experimental frequency is the reason we suggest to explain the observed extinction of the spin resonance absorption lines in antiferromagnetic crystals on cooling below the Curie temperature. It would be valuable to work in intense magnetic fields to pull one of the frequencies into the usual range, or else to work at millimeter wavelengths.

We note that the anisotropy energy may be quite high even in cubic antiferromagnetics containing Mn²⁺ ions in a ⁶S ground state. Consider a simple-minded model: the Kramers superexchange interaction⁴ connecting Mn ions in MnO depends on the overlap of the wave functions of electrons on the Mn and O ions. With an admixture of orbital moment the overlap depends on the spin direction. The order of magnitude of the anisotropy energy per ion on this model may be estimated as $\sim |g - 2|k\Theta$; for $\Delta g \sim 10^{-2}$ and $k\Theta \sim 5 \times 10^2$ cm⁻¹ the anisotropy field can be as high as 10^4 oersteds.

With ferrite-type ferromagnetism, according to the Néel theory, we may set $M_1^z = M_s$; $M_2^z = -(1 - \eta)M_s$. The observed saturation magnetization is $M_1^z + M_2^z = \eta M_s$. We let $H_E = \lambda M_s$, and suppose for simplicity that H_A has the same value for both sublattices. We find

$$\omega/\gamma = H_0 - (\eta H_E/2) \pm [(\eta H_E/2)^2 + H_E H_A (2 - \eta) + H_A^2]^{1/2}.$$

The over-all anisotropy field deduced from static deflections at small angles will differ in ferrites from that deduced from microwave resonance experiments by a fractional amount $\propto H_A/H_E\eta$, which may usually be neglected. The difference should be detectable in weakly magnetic zinc ferrites.

¹ J. H. Van Vleck, *Phys. Rev.* **78**, 266 (1950).

² Trounson, Bleil, and Maxwell, *Phys. Rev.* **79**, 226 (1950); L. R. Maxwell, talk at Pittsburgh meeting APS, March 1951.

³ C. A. Hutchison, private communication.

⁴ P. W. Anderson, *Phys. Rev.* **79**, 350 (1950). Because of superexchange the value of g used may refer to an excited Mn²⁺ state.

Fractional Transition Probabilities of the First Positive Band System ($B^3\Pi \rightarrow A^3\Sigma$) of Molecular Nitrogen

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AS part of a computational program of fractional vibrational transition probabilities for molecular band systems occurring in upper atmospheric radiations, the results for the first positive system of molecular nitrogen are here presented in Table I.

The method of calculation employed is similar to that used in previously reported results,¹ and involves a computation of the overlap integral of suitably modified hermite wave functions.² It will be noted that the fractional transitional probability $f(v', v'')$ is so defined that in any v' progression

$$\sum_{v''} f(v', v'') = 1.$$

From a 3-dimensional presentation of Table I it will also be noted that what was formerly thought of as the primary Condon parabola in the (v', v'') array of reported intensities of the first positive bands³ is in fact the limbs corresponding to the smaller v'' values of the primary and a subsidiary parabola. The limb of the