ϵ_b and the resonance frequency ν_R, the calculations for the piezoelectric modulus d_{25} show a sharp increase at low temperature, with a peak value at Θ of about $10{,}000$ to $20{,}000\times10^{-8}$ cgs (Fig. 3).

Though $LiNH_4C_4H_4O_6\cdot H_2O$ is related crystallographically and chemically to rochelle salt ($KNaC_4H_4O_6\cdot 4H_2O$) and shows typical ferroelectric properties, we observe the following principal differences. The ferroelectric direction is along the *b*-axis in the $LiNH_4$ salt, whereas in rochelle salt it is along the *a*-axis. There is just one molecule of water of crystallization, instead of the four as in rochelle salt. It appears that there is no lower Curie point in the $LiNH_4$ salt, again in contrast with the behavior of rochelle salt.

An x-ray analysis of the $LiNH_4$ salt is in progress, in order to establish the dependence of the ferroelectric direction upon the molecular arrangement, the position and role of the water molecule, and if possible the nature of the structural transition at the Curie point. Crystallographic and electrical properties of other members of this group of tartrates are also under examination.

The author wishes to thank Professor Ray Pepinsky and members of his solid-state group for support of this program.

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[1] B. T. Matthias and J. K. Hulm, Phys. Rev. **82**, June 1 (1951).

Energy Levels of a Vector Particle in a Pure Coulomb Field*

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In a recent paper[1] one of the authors showed that the Schrödinger equation for a hamiltonian with a singular potential can be made to yield a complete orthonormal set of solutions. In addition to the usual condition of quadratic integrability, the wave function is required to have a fixed behavior at the origin. This behavior is conveniently described by prescribing a phase β. Since the choice of β is rather arbitrary, it is of interest to know how sensitively the resulting spectrum depends on this parameter.

In a relativistic problem there is always some value of β which will give a bound state corresponding to any fixed energy between $\pm mc^2$. One really wants to know if it is only for a small range of phases that the spectrum varies considerably. This has been found true[1] for Dirac and Klein-Gordon particles in fields such that $\alpha Z\gtrsim 1$ or $\frac{1}{2}$, respectively. To investigate this further we have considered the energy levels for a vector particle in a pure coulomb field. It has been shown[1] that the phase method does apply to this problem.

Since the exact numerical determination of the energy levels for a given β would be extremely difficult in the vector case, we have attempted to obtain approximate values for the deepest levels and their dependence on phase by means of the variation principle

$$\delta M=0;\quad M=(\Psi|\tau_3H|\Psi)/(\Psi|\tau_3|\Psi). \tag{1}$$

We will use the same notation and units as in the paper referred to above. The stationary values of M are the desired eigenvalues. While the method has only been used for a weak coulomb field ($\alpha Z\ll 1$), no practical restriction results since the finite size of the nucleus will certainly eliminate the singularity for any other case. The best trial functions independent of β were found to be:

$$F_1=(q+1)r^s\exp(-pr/2),\quad G_2=(q-1)r^s\exp(-pr/2),$$
$$F_2=\frac{(\epsilon r^2+\alpha Zr)F_1-rG_2'}{1+r^2/j(j+1)},\quad G_1=\frac{(\epsilon r^2+\alpha Zr)G_2-rF_1'-F_1}{1+r^2/j(j+1)}, \tag{2}$$

depending on the parameters ϵ, p, s, q.

Varying with respect to ϵ gives $\epsilon=M$. Assuming p to be of order αZ and expanding M in powers of αZ makes it possible to extremise with respect to p, s, q.

It was found that there are two values of q for which M is extremal. Denoting these by $q_{\min}$ and $q_{\max}$ the results may be expressed as:

$$q_{\min}=2j(j+1)-(2j+1);\quad s=j;\quad p=2\alpha Z/j;\quad M_{\min}\cong 1-(\alpha Z)^2/2j^2, \tag{3}$$

$$q_{\max}=2j(j+1)+(2j+1);\quad s=j+2;\quad p=2\alpha Z/(j+2);\quad M_{\max}\cong 1-(\alpha Z)^2/2(j+2)^2. \tag{4}$$

Comparing with the behavior of the exact wave function we see,

(a) Since $\epsilon=M$, the connection of F_2, G_1, with F_1, G_2 is correct.

(b) The exponential decrease $[\exp-(1-M^2r)^{\frac{1}{2}}]$ at infinity is correct.

(c) The behavior at the origin is wrong since the exact wave function is either strongly vanishing ($\sim\psi=\exp[-\lambda/r^{\frac{1}{2}}]$) or strongly oscillatory ($\sim\varphi=\cos[\lambda/r^{\frac{1}{2}}+\beta]$).

Improving the wave function by inserting a factor of ψ or φ into the expression for F_1 and G_2 gave: (1) with ψ the same result (3) and (4) as above; (2) with φ the same result (3) and (4) as above, except for $\beta=\pi/2+O(\alpha Z)$. For $\beta=\pi/2$, deeper levels were obtained.

These results seem to indicate that, except for a small range of β of order αZ, the energy levels of a vector particle in a coulomb field are to order αZ the same as those given by the nonrelativistic theory. The shift of levels due to magnetic and relativistic effects is of higher order[2] in αZ just as in the case of spin 0 and $\frac{1}{2}$ particles.

To check these conclusions the relative amounts of $l=j+1$ and $l=j-1$ states present in the bound states were computed. With the above trial functions the lowest state corresponding to $M_{\min}$ was found to be almost entirely $l=j-1$, while $M_{\max}$ corresponded to a predominant $l=j+1$ state.

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† Now at the Randall Laboratory of Physics, University of Michigan, Ann Arbor, Michigan.

[1] K. M. Case, Phys. Rev. **80**, 797 (1950).

[2] Compare with A. Simon, Phys. Rev. **76**, 1877 (1949).

On the Phase Transition of Tungsten Trioxide

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(Received March 26, 1951)

In a previous letter,[1] we described the optical and x-ray studies on the domain structure of tungsten trioxide, WO_3. Although WO_3 behaves like a ferroelectric, having high dielectric constants and domain structure similar to that of $BaTiO_3$, it was unknown at that time whether WO_3 shows a ferroelectric transition at a certain temperature (the Curie point) or not.

The present authors proceeded with the x-ray study on the lattice transformation of this crystal at higher temperatures, and found that it transforms into the tetragonal lattice from the orthorhombic one between 700°C and 750°C (Fig. 1). This

(a)
(b)

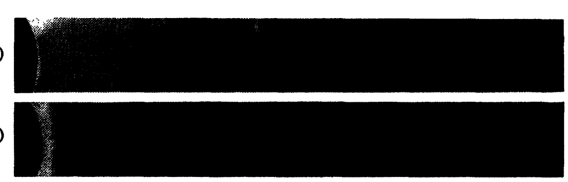

FIG. 1. X-ray powder photographs of WO_3 using Cu K_α radiation. (a) Orthorhombic lattice at 700°C; (b) tetragonal lattice at 750°C.

transformation seems to correspond to the disappearance of the domain patterns at nearly the same temperature, which were recently observed microscopically by Sawada.[2]

The lattice constants of WO_3 calculated from Debye lines are plotted in Fig. 2, in which it is seen that a and b expand linearly with temperature in the orthorhombic region up to about 700°C, where an abrupt phase change occurs and the crystal becomes tetragonal, resulting in $a=b$. The lattice constant c shows a similar linearity in both phases, accompanying a discontinuous change at the transition temperature, which is, however, considerably smaller than the others. Above the transition temperature, the tetragonal lattice persists up to about 1100°C. It is uncertain at the present stage, however, whether or not WO_3 has another phase change (tetragonal⇄cubic) above this temperature.

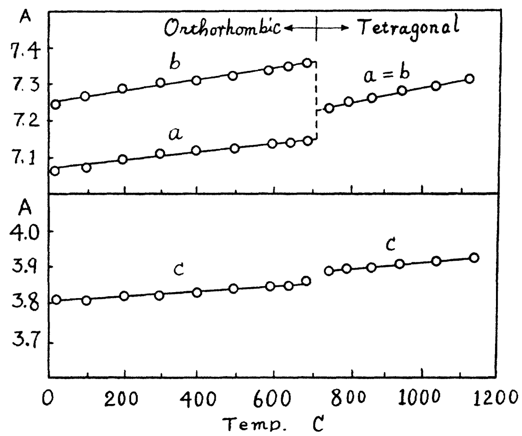

FIG. 2. Lattice constants *vs* temperature curve of WO_3.

It has been observed, by the dilatometer and specific heat measurements recently performed by us, that there exists a sharp contraction in volume and a fairly large energy change at about 720°C. Detailed x-ray studies to detect the minute change in the neighborhood of the transitional region by means of a back reflection method are now in progress.

The disappearance of the domain patterns at about 700°C when viewed parallel to the c-axis under the microscope may be explained easily by considering that twinning planes {110} disappear, since the lattice parameters a and b coincide with each other. Dielectric studies will be required to reveal the ferroelectric transition of this crystal at about 720°C, but they will be exceedingly difficult on account of its semiconductive properties at higher temperatures.

The details of the investigation will be published elsewhere.

The authors wish to express their sincere thanks to Professor S. Miyake for his encouragement in the course of the study, and also to Mr. S. Sawada for his kind information.

[1] R. Ueda and T. Ichinokawa, Phys. Rev. **80**, 1106 (1950).
[2] S. Sawada (unpublished).

Cloud-Chamber Analysis of Stopped Particles in the Sea-Level Cosmic Radiation*

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(Received March 26, 1951)

A WILSON cloud chamber beneath a filter of 119 g cm^{-2} lead plus 21 g cm^{-2} lead-equivalent and above an absorber of 81 g cm^{-2} lead plus 7 g cm^{-2} lead-equivalent was expanded by an anticoincidence arrangement which selected particles stopping in the absorber (see Fig. 1). The momentum was determined from the track curvature in a magnetic field of about 4300 gauss. The specific ionization was estimated from the density of droplets along the track as compared with minimum ionization tracks. From these two quantities, particles occurring singly were identified as electrons, mesons, or protons. Although pi-mesons were indistinguishable from mu-mesons in this experiment, as is seen below, the momentum cutoff indicates that the mesons are predominantly mu-mesons, as is expected.

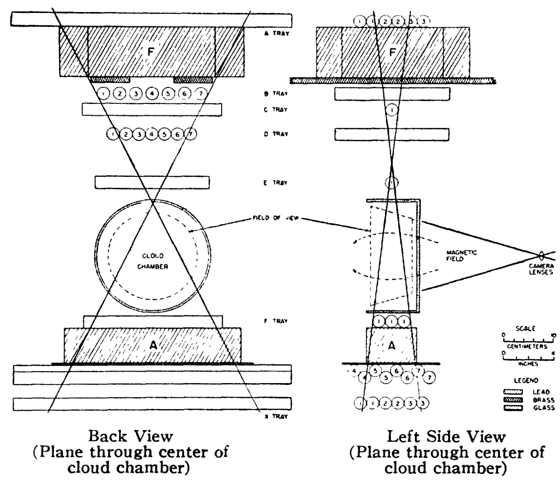

FIG. 1. Experimental arrangement. The cloud chamber was expanded by the event ($A\ B_n\ C\ D_n\ E\ F$-X).

A total of 348 pictures were taken at an average rate of 0.0159 count/min. Of these, 227 were mu-mesons, 10 were dense tracks with momenta greater than 250 Mev/c, 23 were energetic particles which failed to trigger the anticoincidence tray beneath the absorber, 15 were single electrons with momenta less than 70 Mev/c, and 73 were other electronic events such as showers and knock-on processes which triggered the proper sequence of counters.

By assuming that the telescope counting rate of (0.503±0.003) count/min is due entirely to the hard component,[1] the absolute intensity of mesons stopping within the differential range interval of this experiment was found to be $(3.26\pm0.23)\times10^{-6}$ g^{-1} sec^{-1} sterad^{-1} (air-equivalent) for mesons with a range of 115 g cm^{-2} (air-equivalent).[2,3] The intensity has been corrected by +7 percent for mesons lost by scattering and magnetic deflection, by +3 percent for the difference in angular distribution between the hard component and the slow mesons,[2] by −10 percent for the effective increase of the lead absorber as a result of multiple scattering,[4] and by −8 percent for the inclusion of single electrons which could not be distinguished from mu-mesons in the momentum interval 70 Mev/c to 160 Mev/c.

The ratio of positive to negative mesons stopping in the absorber was 0.94±0.07.

The observed differential distribution in momentum of the stopped mesons (see Fig. 2) had a low intensity tail extending beyond the expected vertical cutoff of 216 Mev/c to about 400 Mev/c probably as a result of multiple scattering in the absorber. The upper cutoff in momentum as determined from this distribution gave a meson mass of 220±12 electron masses. A mass determination was not made from the lower cutoff in momentum because the thickness of material penetrated was not known to better than 10 percent.

The 10 particles which formed dense tracks were all positively charged. One of them was identified as an alpha-particle (or heavier nucleus). The remaining nine particles had mass values which ranged from 900 to 2000 electron masses. However, since the

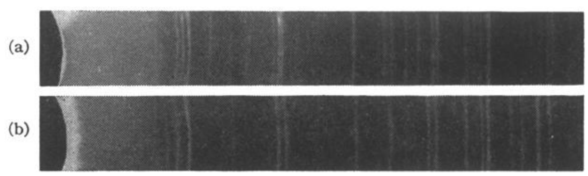

FIG. 1. X-ray powder photographs of WO_3 using Cu K_α radiation. (a) Orthorhombic lattice at 700°C; (b) tetragonal lattice at 750°C.