

sufficient aluminum absorber to cut out the uranium alpha-particles. The U^{237} counting rate was determined by using the least squares method to extrapolate the linear growth curve of the uranium daughters to an accurately controlled zero time (time of chemical purification). Such extrapolations with similar unirradiated uranium gave a background of 2 c/min (above counter background). The counting rate of U^{237} for each sample varied from 6 to 100 c/min. This activity decayed with a 6.6-day half-life.

The excitation function could be approximated by a parabola; hence it was possible to estimate the threshold from a linear extrapolation of the square root of the U^{237} yield plotted as a function of x-ray energy. The results are shown in Fig. 1, where

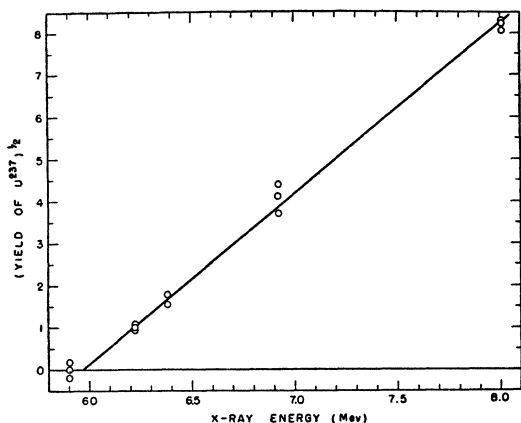


FIG. 1. Square root of the U^{237} yield plotted as a function of x-ray energy.

the yield is expressed in c/min of U^{237} for 50 mg of U_3O_8 given constant bombardment. Analysis of the data gives a threshold of 5.97 ± 0.10 Mev for the $U^{238}(\gamma, n)U^{237}$ reaction.

British workers⁴ obtained a neutron emission threshold of 5.85 ± 0.15 Mev for natural uranium by an indirect measurement of the neutrons produced in photodisintegration. As Parsons and Collie⁴ point out, their method of measurement does not yield a clearly defined threshold since the end product is not identified, and the source of the neutrons is uncertain. However, the close agreement between the two values leads one to believe that they were measuring the neutrons from the $U^{238}(\gamma, n)U^{237}$ reaction.

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¹ J. A. Harvey, Phys. Rev. **79**, 241 (1950); H. Palevsky and A. O. Hanson, Phys. Rev. **79**, 242 (1950); Kinsey, Bartholomew, and Walker, Phys. Rev. **78**, 77 (1950).

² Huizenga, Magnusson, Simpson, and Winslow, Phys. Rev. **79**, 908 (1950).

³ McElhinney, Hanson, Becker, Duffield, and Diven, Phys. Rev. **75**, 542 (1949).

⁴ R. W. Parsons and C. H. Collie, Proc. Phys. Soc. (London) **63A**, 839 (1950).

Lithium Ammonium Tartrate Monohydrate, A New Ferroelectric Crystal*

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IN an investigation of the electrical properties of a series of tartrates we have found that $LiNH_4C_4H_4O_6 \cdot H_2O$ is ferroelectric. (In a private communication to the author, Matthias has reported his parallel discovery of the ferroelectric behavior of this crystal.¹) It appears that isomorphous salts such as $LiRb$ - and $LiTl$ -tartrate $\cdot H_2O$ are also ferroelectric. The electrical behavior of the $LiNH_4$ salt is reported in the present letter.

The crystals are orthorhombic, and the dielectric constants in

all three axial directions are small (between 8 and 10) at room temperature. Upon lowering the temperature, ϵ_a and ϵ_c remain essentially constant, whereas ϵ_b shows a sudden and sharp peak at about 106°K (Fig. 1). Below this temperature we find a spon-

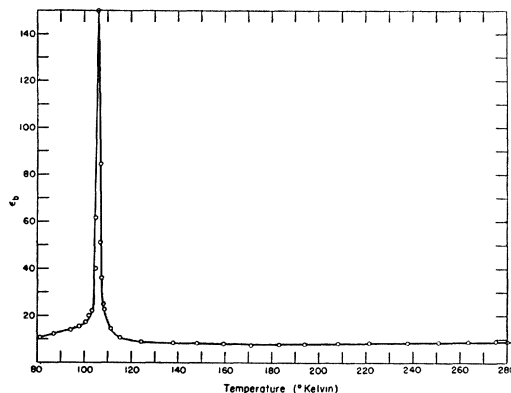


FIG. 1. Dielectric constant ϵ_b versus temperature.

taneous electric polarization which establishes this transition as a Curie point (Θ).

The hysteresis loops (Fig. 2) show the onset of a spontaneous

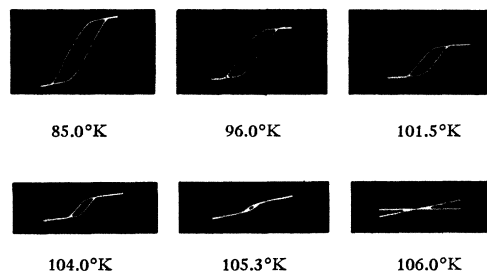


FIG. 2. Hysteresis loops versus temperature.

polarization P_s at the Curie point and its increase when the temperature is lowered. The saturated value at low temperatures is about $P_s = 0.21 \times 10^{-6}$ coulomb/cm².

The crystal is strongly piezoelectric, with a modulus d_{25} of about 20×10^{-8} cgs at room temperature. The piezoelectric resonance frequencies show the usual sharp minimum at the Curie point (Fig. 3). Using these measurements of the dielectric constant

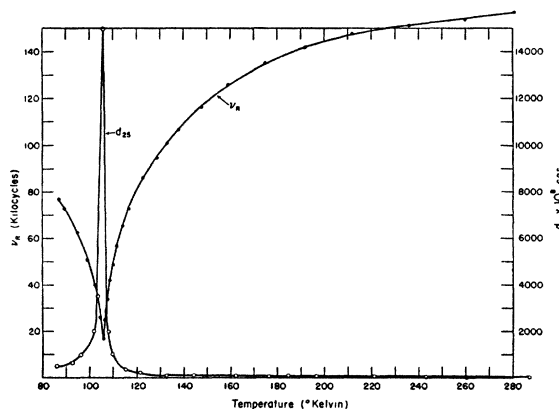


FIG. 3. Resonance frequency and piezo constant d_{25} versus temperature.

ϵ_0 and the resonance frequency ν_R , the calculations for the piezoelectric modulus d_{26} show a sharp increase at low temperature, with a peak value at Θ of about $10,000$ to $20,000 \times 10^{-8}$ cgs (Fig. 3).

Though $\text{LiNH}_4\text{C}_4\text{H}_4\text{O}_6 \cdot \text{H}_2\text{O}$ is related crystallographically and chemically to rochelle salt ($\text{KNaC}_4\text{H}_4\text{O}_6 \cdot 4\text{H}_2\text{O}$) and shows typical ferroelectric properties, we observe the following principal differences. The ferroelectric direction is along the b -axis in the LiNH_4 salt, whereas in rochelle salt it is along the a -axis. There is just one molecule of water of crystallization, instead of the four as in rochelle salt. It appears that there is no lower Curie point in the LiNH_4 salt, again in contrast with the behavior of rochelle salt.

An x-ray analysis of the LiNH_4 salt is in progress, in order to establish the dependence of the ferroelectric direction upon the molecular arrangement, the position and role of the water molecule, and if possible the nature of the structural transition at the Curie point. Crystallographic and electrical properties of other members of this group of tartrates are also under examination.

The author wishes to thank Professor Ray Pepinsky and members of his solid-state group for support of this program.

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Energy Levels of a Vector Particle in a Pure Coulomb Field*

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IN a recent paper¹ one of the authors showed that the Schrödinger equation for a hamiltonian with a singular potential can be made to yield a complete orthonormal set of solutions. In addition to the usual condition of quadratic integrability, the wave function is required to have a fixed behavior at the origin. This behavior is conveniently described by prescribing a phase β . Since the choice of β is rather arbitrary, it is of interest to know how sensitively the resulting spectrum depends on this parameter.

In a relativistic problem there is always some value of β which will give a bound state corresponding to any fixed energy between $\pm mc^2$. One really wants to know if it is only for a small range of phases that the spectrum varies considerably. This has been found true¹ for Dirac and Klein-Gordon particles in fields such that $\alpha Z \gtrsim 1$ or $\frac{1}{2}$, respectively. To investigate this further we have considered the energy levels for a vector particle in a pure coulomb field. It has been shown¹ that the phase method does apply to this problem.

Since the exact numerical determination of the energy levels for a given β would be extremely difficult in the vector case, we have attempted to obtain approximate values for the deepest levels and their dependence on phase by means of the variation principle

$$\delta M = 0; \quad M = (\Psi | \tau_3 H | \Psi) / (\Psi | \tau_3 | \Psi). \quad (1)$$

We will use the same notation and units as in the paper referred to above. The stationary values of M are the desired eigenvalues. While the method has only been used for a weak coulomb field ($\alpha Z \ll 1$), no practical restriction results since the finite size of the nucleus will certainly eliminate the singularity for any other case. The best trial functions independent of β were found to be:

$$F_1 = (q+1)r^s \exp(-pr/2), \quad G_2 = (q-1)r^s \exp(-pr/2), \\ F_2 = \frac{(\epsilon r^2 + \alpha Z r)F_1 - rG_2'}{1+r^2/j(j+1)}, \quad G_1 = \frac{(\epsilon r^2 + \alpha Z r)G_2 - rF_1' - F_1}{1+r^2/j(j+1)}, \quad (2)$$

depending on the parameters ϵ , p , s , q .

Varying with respect to ϵ gives $\epsilon = M$. Assuming p to be of order αZ and expanding M in powers of αZ makes it possible to extremise with respect to p , s , q .

It was found that there are two values of q for which M is

extremal. Denoting these by q_{\min} and q_{\max} the results may be expressed as:

$$q_{\min} = 2j(j+1) - (2j+1); \quad s = j; \quad p = 2\alpha Z/j; \\ M_{\min} \cong 1 - (\alpha Z)^2/2j^2, \quad (3)$$

$$q_{\max} = 2j(j+1) + (2j+1); \quad s = j+2; \quad p = 2\alpha Z/(j+2); \\ M_{\max} \cong 1 - (\alpha Z)^2/2(j+2)^2. \quad (4)$$

Comparing with the behavior of the exact wave function we see,

(a) Since $\epsilon = M$, the connection of F_2 , G_1 , with F_1 , G_2 is correct.

(b) The exponential decrease [$\exp(-1-M^2r)$] at infinity is correct.

(c) The behavior at the origin is wrong since the exact wave function is either strongly vanishing ($\sim \psi = \exp[-\lambda/r^{\frac{1}{2}}]$) or strongly oscillatory ($\sim \varphi = \cos[\lambda/r^{\frac{1}{2}} + \beta]$).

Improving the wave function by inserting a factor of ψ or φ into the expression for F_1 and G_2 gave: (1) with ψ the same result (3) and (4) as above; (2) with φ the same result (3) and (4) as above, except for $\beta = \pi/2 + O(\alpha Z)$. For $\beta = \pi/2$, deeper levels were obtained.

These results seem to indicate that, except for a small range of β of order αZ , the energy levels of a vector particle in a coulomb field are to order αZ the same as those given by the nonrelativistic theory. The shift of levels due to magnetic and relativistic effects is of higher order² in αZ just as in the case of spin 0 and $\frac{1}{2}$ particles.

To check these conclusions the relative amounts of $l = j+1$ and $l = j-1$ states present in the bound states were computed. With the above trial functions the lowest state corresponding to M_{\min} was found to be almost entirely $l = j-1$, while M_{\max} corresponded to a predominant $l = j+1$ state.

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¹ K. M. Case, Phys. Rev. **80**, 797 (1950).

² Compare with A. Simon, Phys. Rev. **76**, 1877 (1949).

On the Phase Transition of Tungsten Trioxide

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IN a previous letter,¹ we described the optical and x-ray studies on the domain structure of tungsten trioxide, WO_3 . Although WO_3 behaves like a ferroelectric, having high dielectric constants and domain structure similar to that of BaTiO_3 , it was unknown at that time whether WO_3 shows a ferroelectric transition at a certain temperature (the Curie point) or not.

The present authors proceeded with the x-ray study on the lattice transformation of this crystal at higher temperatures, and found that it transforms into the tetragonal lattice from the orthorhombic one between 700°C and 750°C (Fig. 1). This

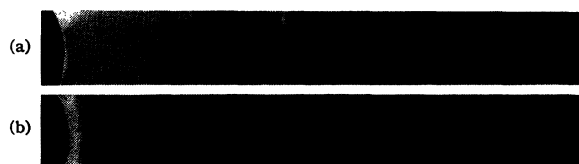


FIG. 1. X-ray powder photographs of WO_3 using $\text{Cu } K\alpha$ radiation. (a) Orthorhombic lattice at 700°C ; (b) tetragonal lattice at 750°C .

transformation seems to correspond to the disappearance of the domain patterns at nearly the same temperature, which were recently observed microscopically by Sawada.²

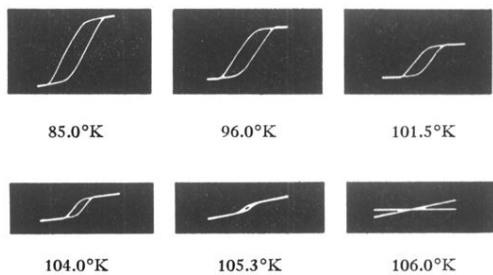


FIG. 2. Hysteresis loops *versus* temperature.