Energy Level Perturbation of a Particle in a Spheroidal Potential Well

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RECENT papers[1-3] have drawn attention to the problem of the motion of a nucleon in a spheroidal potential well. Since so far this problem has been treated for P_2 deformations, i.e., for an ellipsoidal well, it may be of some interest to point out a method which is applicable to spheroidal deformations of a more general type:

$$R(\mu)=R_0[1+\sum_{i=1}^{\infty}\alpha_i P_i(\mu)], \quad (1)$$

μ being the direction cosine of the nucleon position vector, relative to the core symmetry axis, and R_0 the renormalized radius of the core.

The spheroidal potential well of depth D may be represented in the following way:

$$U(r,\mu)=D\mathbf{1}[r-R(\mu)]\simeq D\mathbf{1}(r-R_0)-D\delta(r-R_0)\Delta R(\mu), \quad (2)$$

where

$$\Delta R(\mu)=R_0\sum_i \alpha_i P_i(\mu)$$

and

$$\mathbf{1}(r-R_0)=\begin{cases}0 & \text{for } r\leqslant R_0\\ 1 & \text{for } r>R_0\end{cases}$$

In spite of the singular character of the perturbation energy term $-D\delta(r-R_0)\Delta R(\mu)$, standard perturbation methods yield correct results as far as small deformations are concerned.

The energy shift relative to the unperturbed eigenvalue $E_{n,l,m}$ is given by:

$$\Delta E_{n,l,m}=-DR_0^3|\rho_{n,l}(R_0)|^2\sum_i \alpha_i\int|Y_{l,m}(\mu,\varphi)|^2P_i(\mu)d\mu d\varphi, \quad (3)$$

where $\rho_{n,l}$ and $Y_{l,m}$ are the normalized radial and angular parts of the unperturbed eigenfunctions.

The integral in formula (3), which we denote by $I_{l,m,i}$, has the following values in the cases $i=2$ and $i=4$:

$$I_{l,m,2}=\frac{l(l+1)-3m^2}{(2l-1)(2l+3)}, \quad (4)$$

$$I_{l,m,4}=\frac{3}{4}\frac{35m^4+5[5-6l(l+1)]m^2+3l(l^2-1)(l+2)}{(2l-3)(2l-1)(2l+3)(2l+5)}. \quad (5)$$

Formula (3) is in agreement with Feenberg and Hammack's particular result; in fact, for an ellipsoidal deformation ($\Delta R=R_0\alpha_2P_2(\mu)$) we have $\alpha_2=\frac{2}{3}e$, where e is the eccentricity introduced by these authors.

The case of an impenetrable spheroidal box is obtained by evaluation of the limit:

$$\lim_{D\to\infty}D|\rho_{n,l}(R_0)|^2=\hbar^2x_{n,l}^2/MR_0^5,$$

where $x_{n,l}$ is zth zero of a bessel function of order $l+\frac{1}{2}$.

Formulas (3) and (5) together with the well-known expression of the surface and electrostatic deformation energy may be used for a variational calculation of the P_4 contribution to the total energy shift. The equilibrium shape of the nucleus up to P_4 deformations may be calculated in the same way; an equatorial neck may thus possibly appear in some cases.

Formula (5) shows that the P_4 degree of freedom remains unexcited for s and p states of the nucleon.

A more detailed discussion will soon be published in *Nuovo Cimento*.

[1] J. Rainwater, Phys. Rev. **79**, 432 (1950).
[2] A. Bohr, Phys. Rev. **81**, 134 (1951).
[3] E. Feenberg and K. C. Hammack, Phys. Rev. **81**, 285 (1951).

The Production and Detection of a Non-Equilibrium Number of Vacancies in a Metal*

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THE mechanism of diffusion in substitutional alloys and of self-diffusion in metals has never been determined satisfactorily. The experiments described below seem to be the most direct evidence, to date, that diffusion in close-packed metals occurs, predominantly, through the movement of vacancies.

It has been found[1,2] that anelastic effects occur in alloys when the two components differ in size. These effects, first observed in the case of brass,[1] have been attributed[3] to stress-induced reorientation of pairs of solute atoms. Inasmuch as the mechanism of this reorientation must be the same as that responsible for macroscopic diffusion, the writer[2] has begun a series of investigations to utilize this effect in the study of atomic mobility in alloys. In contrast to diffusion experiments, the time required for a measurement of the relaxation effects is of the order of magnitude of the mean time of stay of an atom. The alloys used are α solid solutions of Ag-Zn; it has been found, for example, that the total relaxation in an alloy of 29 atomic percent Zn is about 0.18.

If the mechanism of atomic movement is predominantly through the movement of vacancies, it should be possible to quench a non-equilibrium number of vacancies into a specimen; the presence of the excess of vacancies may be detected by the effect they have in lowering the relaxation time of the specimen at any given temperature. Such effects have been found. A specimen in the form of a wire 0.03 inch in diameter, is held at 400°C in a N_2 atmosphere for several minutes and then rapidly quenched into water at room temperature. Subsequent measurements of elastic aftereffect at 50°C, in which the wire is twisted for 15 minutes, then released, shows an initial time of relaxation of about 15 minutes (the first curve of Fig. 1). The magnitude of the relaxation is in agreement

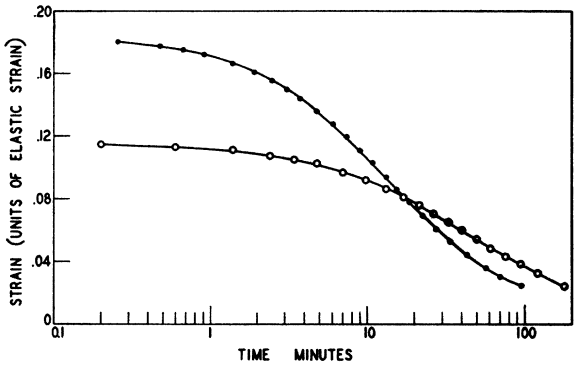

FIG. 1. Elastic aftereffect at 50°C of a 29 atomic percent Ag-Zn wire quenched from 400°C. Curve 2 follows curve 1 directly; each curve was obtained after a 15-minute twist of the wire.

with that obtained under equilibrium conditions at higher temperatures. However, under equilibrium conditions, the time of relaxation at 50°C, as obtained by extrapolating the data from higher temperatures, is about 3.4×10^5 minutes. The time of relaxation of the quenched specimen slowly increases, as shown by the second curve of Fig. 1, which indicates that the number of vacancies is gradually approaching the equilibrium number. None of the effects described appear when the same specimen is slowly cooled from 400°C.

The only alternate possibility to the explanation of these experiments in terms of vacancies is to assume that the observed relaxation effects result from quenching strains in the wire. In view of the small diameter of the specimen and the fairly rapid