Dyson[3] terminology, to a given order term there corresponds a complicated graph. This can be reduced to a simpler graph by formally substituting into the latter "effective" causal functions and "effective" coupling operators for the true functions and operators. Only if the degree of linearity in momentum space of these effective quantities is (aside from a logarithmic term) of no higher order than the degree of the true quantities will this complicated graph not introduce additional interaction densities. The arbitrary constants reduce, therefore, to a renormalization of the (arbitrary) constants (coupling parameters) of a finite number of interaction densities.

A sufficient condition for this reduction is the following inequality between the degrees of linearity $c, -s, -i$, in momentum space of $\Omega(p, \cdots)\to p^c$, $\Delta^{(c)AB}(p)\to p^{-s}$, $D^{(c)\alpha\beta}(p)\to p^{-i}$, and the multilinearities m of the charged field u and $u^\dagger$, and k of A^α in (1):

$$0\leqslant c\leqslant m(s-4)+\tfrac{1}{2}k(i-4)+4. \qquad (2)$$

This condition (2) is also necessary because we can show that if the highest powers of the contributions from two or more different graphs compensate each other, the total contribution is zero identically. This particular property of the graphs has the consequence that the contributions arising from the terms $\sim\mu^{-2}$ in $D^{(c)\alpha\beta}$ for photons of non-zero rest mass μ compensate each other.[4] Therefore, we can put $i=2$ (instead of $i=0$).

If we apply this condition to charged particles of spin $0, \frac{1}{2}, 1$, we see at once that the method used cannot be applied to the interaction of vector mesons ($s=0$) with the photon field.[5] For Dirac electrons ($s=1$), only the coupling of zero order is possible (charge-potential interaction). The interaction between magnetic moment and field (Pauli term) cannot be introduced. For scalar mesons ($s=2$), the coupling of order one allows the charge-potential interaction. Furthermore, the quadrilinear couplings $u^\dagger uA^\alpha A_\alpha$ (necessary for gauge invariance) and $u^\dagger u^\dagger uu$ (found by Rohrlich[6]) have to be introduced. In every case, the bilinear mass renormalization interactions are necessary and the non-gauge-invariant quadrilinear term $(A^\alpha A_\alpha)^2$ is also[7] compatible with (2). However, no other interactions can be introduced.

* Assisted by the Swiss Atomic Energy Commission.

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Maximum Rate of Wave Function Amplitude Change

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(Received March 28, 1951)

THE WKB approximation supplies us with an estimate of how fast the amplitude of wave functions can change. Near a turning point, where $E=V$, the WKB approximation changes amplitude with distance at a far greater rate than the actual wave function does. Far away from turning points, it is, however, possible for the actual wave function to change its amplitude at a greater rate than the WKB approximation does. An example is an electron suffering a Bragg reflection in a one-dimensional crystal. The wave function for such an electron has an envelope which decreases exponentially into the crystal. The WKB approximation would predict an amplitude which is the same in each cell of the crystal. The purpose of this note is to give an upper bound to the rate at which the wave function envelope can change its amplitude.

The author has recently given an expansion[1] for the wave function in terms of successive reflections. An upper bound for the wave function can be obtained by summing the absolute values of all the wavelets which contribute to the wave function and ignoring their phase differences. Let the correct wave function, $\psi(x)$, have a vanishing derivative at $x=a$. Then our summation gives us a factor $\sigma(x)$ such that

$$|\psi(x)|\leq\sigma(x)|\psi(a)|.$$

The factor $\sigma(x)$ has the value unity at $x=a$. As we move away from $x=a$, we will go through two types of regions: those in which $E-V$ is increasing as we move away from $x=a$, and those in which it is decreasing. In the regions in which $E-V$ increases, $\sigma(x)$ remains constant. In the regions in which $E-V$ decreases $\sigma(x)$ varies as $1/(E-V)^{\frac{1}{2}}$. In going from one type of region to the other, $\sigma(x)$ is continuous. These statements, which completely determine $\sigma(x)$, apply only to regions throughout which $E-V$ remains positive.

If, instead of taking a point at which the derivative vanishes, we are given the value of the wave function and its derivative, a maximum can also be found. A more complicated expression results in this case.

In the particular case of the periodic potential, consisting of a series of simple troughs where $(E-V)^{\frac{1}{2}}$ has the maximum value p_a and the minimum value p_b, the wave function increases at most by a factor p_a/p_b, in going from cell to cell.

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Influence of the Atomic Shell on Nuclear Angular Correlation

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(Received April 5, 1951)

THE influence of the atomic shell on the angular correlation of successive nuclear radiation has been discussed by several authors.[1-4]

In general, one would expect a reduction of the angular correlation when the energy of interaction between the nucleus and the electron shell (hfs splitting) is greater than that between the nucleus and the radiation field. In most cases, then, a reduction of correlation by the magnetic field of the shell would be expected in those transitions in which the lifetime of the intermediate nuclear state is of the order of 10^{-11} sec or larger.

There exist, however, a number of isotopes (Table I) which have rather large half-lives of the intermediate level (10^{-12} or longer) and which nevertheless exhibit a strong angular correlation.

In each case of Table I, the magnetic moment of the atom in its ground state is zero. The same is true for the normally occurring doubly ionized ions (e.g., Cd^{++}). Thus, there is no magnetic interaction between shell and nucleus, and therefore no perturbation of the angular correlation occurs.

A strong perturbation can be expected, however, if the decaying

TABLE I. Angular correlation and atomic shell.

Isotope	Cd^{111}	Hg^{197}	Pb^{204}	Mg^{24}	Sr^{88}	Pd^{106}	Ba^{134}
Cascade	$\gamma-\gamma$	e^--e^-	$\gamma-\gamma$	$\gamma-\gamma$	$\gamma-\gamma$	$\gamma-\gamma$	$\gamma-\gamma$
Lifetime of the intermediate state in sec.	8×10^{-8}	7×10^{-9}	3×10^{-7}	—	—	—	—
Anisotropy $[K(180°)/K(90°)]-1$	−0.06	+0.24	+0.22	+0.17	−0.06	+0.50	+0.12
Ground state of the atom	$^1S_0$	$^1S_0$	$^3P_0$	$^1S_0$	$^1S_0$	$^1S_0$	$^1S_0$
Magnetic moment in Bohr magnetons	0	0	0	0	0	0	0
References	a, b	c	d	e	e	e	e

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[b] Reference 4. [c] Reference 7. [d] Reference 3. [e] Reference 2.

atom is, for a time comparable with the lifetime of the intermediate nuclear level, in a state with an appreciable magnetic moment. This occurs for the elements of Table I, if the atom is suitably excited (e.g., Cd $^1P_1$, $^3P_1$) or suitably ionized (e.g., Cd^+, Cd^{+++}). Therefore, it should be possible to reduce the angular correlation by exciting or ionizing the decaying atom and preventing it for a sufficiently long time from being de-excited or neutralized. Most of the nuclear events are accompanied by processes which may lead to excitation or ionization (e.g., beta-decay or K-capture with subsequent rearrangement of the shell,[5] internal conversion, Auger effect after K-capture or internal conversion, recoil after the decay).

We consider first the case of excitation. If the decaying atom is embedded in a metal, the transition from the excited to the ground state is very fast[6] compared with the lifetime of the intermediate nuclear level, and we may assume that the measured correlation is "true." But if the atom forms a lattice defect in an ionic crystal or a semiconductor, the lifetime of the excited state may be so long that its associated magnetic moment may partially destroy the angular correlation.

A similar consideration may be valid for ionized atoms. The free electrons of a metal neutralize the ion in a very short time. In an insulator, however, the time for the neutralization may be long compared with the half-life of the nuclear level.

Thus, there may be a possibility of observing an influence of the atomic shell by comparing the angular correlation of metal sources with that of ionic crystal sources.

Usually, angular correlation is measured with the radioactive element in a chemical compound. This is not very suitable for the investigation of the mentioned effects, because it is very difficult to avoid contaminations. Furthermore, it is not possible to vary the type of binding and the nature of the surroundings to a very large extent.

An excellent method for varying the environment of the atom is through the use of evaporated "double stream" sources.[7,8] The substance of the second vapor stream may be varied for different sources from metal to ionic and molecular crystal. Such evaporated sources would give perhaps the best approximation to the ideal arrangement of isolated atoms or ions in a homogeneous medium.

A second proposal for embedding the source in a suitable medium might be the thermal diffusion of the radioactive isotope into the medium.

Experiments of this type should also give insight into the mechanism of the rearrangement after the decay.

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Influence of the Atomic Shell on Nuclear Angular Correlation in Cd^{111}

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(Received April 5, 1951)

THE previous letter[1] discusses possibilities of influencing the angular correlation of successive nuclear radiations. We have succeeded in establishing a definite effect upon the angular correlation of the γ-rays in Cd^{111}. Cd^{111} results from the K-capture of In^{111}, and seemed especially promising because of the long lifetime of the intermediate state and the properties of the electron shell.[1]

The resolving time of the coincidence circuit† (2.2×10^{-7} sec) was so great that practically all transitions were measured regardless of the lifetime of the intermediate level. Both crystals were shielded with lead against scattered quanta.

The sources were prepared with a "double stream" method.[2] The cadmium, containing the radioactive In^{111} (from bombardment by 7-Mev protons), was placed in a carbon crucible. By slowly heating the crucible in a vacuum, all cadmium was evaporated away, most of the radioactive indium remaining in the crucible. Finally, the crucible was heated to 900°C for 1 minute and the (carrier-free) indium, accompanied by an intense vapor stream from a second crucible, was evaporated onto a thin aluminum foil. As a result of this procedure In^{111} was embedded free from contaminations within a suitable medium. After the evaporation the foil was rolled and placed in a small tube with very thin walls, so that the scattering of the γ-rays in the whole source was negligible. Four to six different sources were prepared in this way from each cyclotron irradiation.

The anisotropy was determined by measuring the coincidence rate at 90° and 180° alternatingly. The different sources were frequently interchanged. The different measurements were always consistent statistically. All results were corrected for the finite angular resolution.

The results for Cd^{111} are summarized in Table I. These values provide two important conclusions:

1. We have to assume that the value for "thick" metal sources ($>10^3$A) is the best approximation to the true angular correlation for the atom in the ground state. Therefore our new value, $A=-0.16\pm0.01$, is considerably higher than the hitherto reported values[3,4] $A=-0.07\pm0.04$ and $A=-0.06\pm0.02$.

It seems possible that the values found for the anisotropy in several other cases (e.g., Sr^{88}, Pd^{106}, Te^{124}) may similarly be in error and should be remeasured with a metallic embedding medium. In addition, several $\beta-\gamma$-correlations (e.g., Cd^{115}), which until now have shown an isotropic distribution, may become anisotropic when the source is embedded in a suitable medium. The wide discrepancy in the measured anisotropy of Rb^{86} may also be due to the way in which the source was prepared.

TABLE I. Angular correlation of Cd^{111} in different media.

Embedding medium	Approx. thickness in A	$A=\frac{K(180°)}{K(90°)}-1$, in %	Mean statistical error, in %
LiF	10^5	+1.6	±2.6
AgCl	5×10^4	+0.9	±1.4
SiO 1	10^5	−1.1	±1.9
SiO 2	5×10^4	+0.3	±2.0
Se (red)[a]	10^5	−5.8	±2.2
Au 1	$<10^3$	−5.8	±2.2
Au 2	$<10^3$	−6.5	±1.7
Ag 1	$<10^2$	−5.0	±2.1
Ag 2	$<10^3$	−10.6	±1.9
Ag 3	$\sim10^3$	−17.6	±1.3
Ag 4	3×10^3	−16.8	±2.1
Ag 5	5×10^3	−15.7	±2.3
Ag 6	10^4	−15.6	±1.7
Ag 7	10^4	−17.0	±1.4
Ag 8	2×10^4	−14.0	±1.2
Chemical compound $In(OH)_3$, 4 different sources		−3.0 to −7.5	±2.0 ±2.0

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2. A variation of the correlation with the thickness of the metal film is obvious from Table I. This is not surprising, because very thin metal films show strong anomalies in their electric and magnetic behavior. One is therefore required to use relatively thick metal sources in order to obtain the "true" angular correlation. In measurements involving low energy electrons, the scattering in the source will thus necessarily be large.

We thank Professor P. Scherrer for his continued interest, and H. Albers-Schönberg and H. Huber for their assistance during the measurements.

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