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A Precise Mechanical Measurement of the Gyromagnetic Ratio of Iron

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In this experiment the gyromagnetic ratio of a sample of very pure iron was determined. The sample was wound with a magnetizing coil and supported as a torsional pendulum in an evacuated space almost completely free of residual magnetic fields. Changes in pendulum amplitude were brought about by repeated and synchronized reversals of the magnetizing current. These changes were measured along with corresponding changes in magnetic moment of the sample. The average value obtained for the gyromagnetic ratio was 1.0278±0.0014 times the mass-to-charge ratio of the electron.

I. INTRODUCTION

IN 1908, it was predicted, by Richardson,[1] that the magnetization of an object should produce an angular momentum about its magnetic axis. He showed that the ratio of this angular momentum to the corresponding magnetic moment should be $2m/e$ for orbital electron motion.

This gyromagnetic ratio has been determined mechanically by several investigators,[2] and, in the case of iron, is known to be very nearly $\frac{1}{2}$ the amount predicted by Richardson. This is now satisfactorily explained by considering the major part of the magnetic moment of the ultimate magnetic particle to be caused by a spinning electron.

Barnett[3] gives, in his most recent publications, a value of 1.031 m/e for the gyromagnetic ratio of iron. Most other investigators have obtained values lower than this but still definitely greater than m/e.

Recently, it has become possible to obtain a measure of the gyromagnetic ratio by means of microwave resonance absorption techniques. By this method, most investigations have resulted in values considerably less than m/e. Kittel[4] has attempted to explain this discrepancy between the values obtained by microwave techniques and those obtained mechanically.

Because of this discrepancy, and because of the wide variation of results obtained by the various investigators who have determined the gyromagnetic ratio mechanically, it was felt that a precise determination by a very direct mechanical method would be of interest.

Since apparatus was available at the General Motors Research Laboratories, which was ideally suited for making mechanical measurements of this type, it was decided to utilize it for obtaining a precise determination of the gyromagnetic ratio of iron.

A cylinder of high purity iron was obtained. This cylinder was wound with a uniform magnetizing coil and supported as a torsional pendulum rotating about its magnetic axis with a long period and a very low damping coefficient.

II. EXPERIMENTAL FACTORS INVOLVED

If the angular momentum of an undamped torsional pendulum is changed by an amount Δz as the pendulum is crossing the center of its swing, the change in amplitude will be $\Delta\theta = \Delta z P/(2\pi I)$, where P is the period of the pendulum and I its moment of inertia. $\Delta\theta$, which is the change in amplitude in radians, was observed in these experiments by measuring the change in double amplitude d on a scale at a distance X from the rotating element mirror. Therefore, $\Delta\theta = d/(4X)$, and $\Delta z = \pi I d/(2PX)$. This equation assumes that the angular momentum impulse is applied exactly as the pendulum is crossing the center of its swing, or at a zero phase angle. If a slight error, either positive or negative, in phase angle is made, the measured value of d will be too small. In this case, the value d in the above equation would be replaced by d/k, where k is the cosine of the phase angle error.

In these experiments the current in the element is reversed periodically, and the resulting changes in angular momentum and magnetic moment are measured. Also, a large number of readings are taken throughout a small range in values of the magnetizing current. For this condition a linear relationship exists between the element current i_e and the magnetic moment, so that $M = ai_e + b$. The change in magnetic moment when the current is reversed is $\Delta M = 2(ai_e + b)$.

These values of Δz and ΔM are the measured values of the total changes in angular momentum and magnetic moment. It is necessary to subtract from these the changes in angular momentum and magnetic moment developed by the change in flow of electrons in the magnetizing winding in order to obtain the values for the iron alone.

If the summation of the areas of all the turns in the element winding is ΣA_e, and if the current is changed through this winding by an amount Δi_e, then the change in magnetic moment of the winding is $\Delta i_e \Sigma A_e$, and the corresponding change in angular momentum is $(2m/e)\Delta i_e \Sigma A_e$.

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[1] O. W. Richardson, Phys. Rev. **66**, 248 (1908).

[2] A. P. Chattock and L. F. Bates, Trans. Roy. Soc. (London) 22, 257 (1923); W. Sucksmith and L. F. Bates, Proc. Roy. Soc. (London) **104**, 499 (1923); L. F. Bates, Nature **134**, 50 (1934); F. Coeterier, Helv. Phys. Acta **8**, 522 (1935); A. J. P. Meyer, Compt. rend. **228**, 1934 (1949).

[3] S. J. Barnett, Proc. Amer. Acad. Arts Sci. **73**, 401 (1940); S. J. Barnett, Phys. Rev. **66**, 224 (1944).

[4] C. Kittel, Phys. Rev. **76**, 743 (1949).

Therefore, for the iron alone,

$$\Delta z = \pi Id/(2PXk) - (2m/e)\Delta i_e \Sigma A_e,$$

and

$$\Delta M = 2(ai_e + b) - \Delta i_e \Sigma A_e.$$

The ratio between these two quantities is the gyromagnetic ratio of the iron.

Since the element current is always reversed, $\Delta i_e = 2i_e$. Making this substitution, and taking the ratio $\Delta z/\Delta M$,

$$G = \frac{\pi Id/(2PXk) - 4(m/e)i_e \Sigma A_e}{2(ai_e + b) - 2i_e \Sigma A_e}.$$

This is the equation which is used for determining the gyromagnetic ratio from the experimental data.

III. INSTRUMENTATION AND EXPERIMENTAL PROCEDURE

The instrumentation herein described was set up at a test station located in a region free from all artificial magnetic and mechanical disturbances.

Readings on the gyromagnetic element were taken in an evacuated instrument mounted on a pier having its foundation ten feet below the ground level. Surrounding magnetic fields were almost completely eliminated. A light beam about 52 ft in length was used for determining the changes in angular momentum. The various necessary pieces of equipment will be described separately.

(A) Field Neutralizing System

Neutralization of the earth's magnetic field was accomplished by a system of three mutually perpendicular Helmholtz coils having a common center and with axes in the NS, EW, and vertical directions.

The forms for both the NS and vertical components were accurately machined on massive aluminum structures. The NS coils were 8 ft in diameter; the vertical coils were 9 ft in diameter.

The EW system was made of rolled aluminum channel and was 4 ft in diameter. All systems had a winding cross section one inch square.

The NS coil was oriented so that its magnetic axis was horizontal and in the mean direction of the earth's horizontal component. The EW coil was set very nearly at right angles to this direction. The currents in both the NS and EW coils were controlled by a system of variometers so as to follow the naturally occurring changes in the horizontal component of the earth's field. No variometer was used on the vertical system. The proper current was determined for these coils so as to neutralize the mean value of the earth's vertical component.

(B) The NS Variometer

The variometer used to determine the proper value for the main NS Helmholtz coil current was located in a small auxiliary building 135 ft north of the primary system. This variometer included a 4 ft diameter Helmholtz coil with its axis aligned, as nearly as possible, with the direction of the earth's total field. At the center of this Helmholtz coil was located an airtight case containing a suspended feeler coil having its magnetic axis horizontal and in an east-west direction. The current in this coil was reversed by means of a manually operated switch located at the variometer control station, which was in the main building. Also located inside of this airtight case was a photoelectric multiplying system for electrically transferring information concerning the feeler coil position back to the central control point.

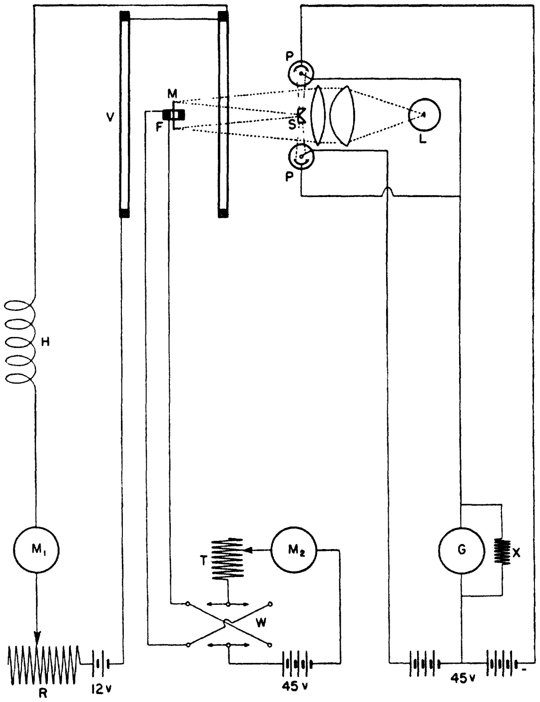

FIG. 1. North south variometer system. P, Type SR 53 vacuum photocells; S, light splitting prism; L, lamp; M, mirror carried by feeler coil; F, feeler coil; V, four-foot diameter Helmholtz coils; H, main NS Helmholtz coil; M_1, M_2, indicating ammeters; G, galvanometer to indicate feeler coil motion; X, damping resistance; R, field control rheostats; W, reversing switch.

If any horizontal north-south field were present at the center of this variometer system, a reversal of the feeler coil current would be accompanied by a corresponding motion of the feeler coil.

An operator continually checked the field value by reversing the feeler coil current and adjusting the Helmholtz current to obtain zero motion.

This variometer Helmholtz coil had its horizontal NS component matched to the main NS Helmholtz system and was then electrically connected in series with it. A diagram of the NS variometer system is shown in Fig. 1.

(C) The EW Variometer System

The variometer used to determine the proper current for the main EW Helmholtz coil was located 15 ft directly north of the primary system. It consisted of a magnetized rod of alnico, hung in a suitable container by a very fine filament. This magnetized rod carried a mirror and was oil damped.

On the outside of the case was a coil having its magnetic axis east and west. This coil was matched to the main EW Helmholtz coil and connected in series with it; a control rheostat was located at the variometer control point. By means of a high quality projector, a beam of light was directed onto the mirror which, in turn, focused a cross hair on a scale located at the variometer control center, which was 54 ft away. A zero point for the mean position of the horizontal component of the earth's field was located on this scale. When the field varied from this position, it was returned by the variometer operator, who moved a field rheostat to keep the cross hair always on zero, thus compensating

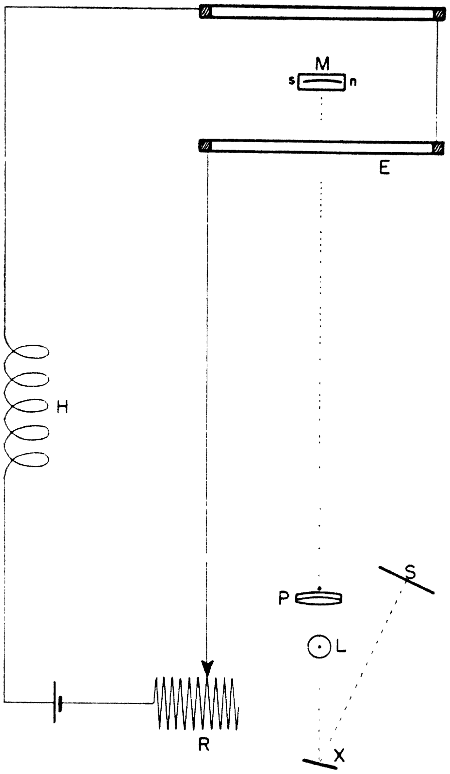

Fig. 2. East-west variometer system. ns, magnetized alnico rod; M, mirror; E, EW coil system; H, main EW Helmholtz coil; P, projecting lens; L, Western Union concentrated arc lamp; X, plane mirror; S, scale; R, field control rheostats.

for the field changes. A diagram of the EW variometer system is shown in Fig. 2.

(D) Precision of Field Neutralization

Throughout a set of gyromagnetic readings, which would usually last about 6 hours, the above variometer systems could be operated so as to prevent variations in the residual horizontal magnetic field of more than 1 gamma (1 gamma $=10^{-5}$ oersteds). The homogeneity of the large Helmholtz coil fields was such that a field difference between center and ends of the gyromagnetic element of about $\frac{1}{2}$ gamma could be expected.

(E) The Gyromagnetic Element

The material used to measure the gyromagnetic ratio of iron consisted of an accurately ground and annealed cylinder 1.5 cm in diameter and 22 cm long.

Table I. Analysis of iron of gyromagnetic element. All elements present to 0.001 percent or greater shown.

Iron	99.94%
Oxygen	0.04%
Carbon	0.005%
Nitrogen	0.004%
Sulfur	0.003%
Nickel	0.0015%
Cobalt	0.001%
Phosphorous	0.001%
Silicon	0.001%
All others	0.0035%

An analysis of the sample is given in Table I. All elements present to greater than 0.001 percent are shown. The cylinder was wound carefully with a magnetizing winding consisting of 14 layers of No. 41 formex covered wire. At the lower end, a small ratchet-operated reversing switch was located so that the current through the element could be reversed with respect to that flowing in the suspension system. When the element was in position, this reversing switch could be operated by a bellows system located in the bottom of the instrument. It was found in previous work on electron inertia[5] that systematic errors could be introduced by the suspension system if a reversing switch was not used on the element. At the upper end a tube carrying the suspension and mirror systems was located. The mirror consisted of four optically flat faces so that readings on the element could be taken at four 90° azimuth positions.

Adjusting screws were provided at the necessary locations so that the element could be brought into both static and dynamic balance about its axis of rotation. This was necessary to prevent transverse disturbances from producing rotational motion. When properly adjusted, the element had an extremely small horizontal magnetic moment, in addition to having mechanical symmetry. A diagram of the element is shown in Fig. 3.

(F) Optical System

The general optical arrangement was the same as that used in a conventional galvanometer projection system. However, in this case, a much longer optical arm was used, requiring very precise optical components throughout. It was also necessary to enclose the optical path in a tunnel to prevent the disturbing effects of air currents. The distance from the element mirror to the scale was about 52 ft. The width of the cross-hair image on the scale was about $\frac{1}{5}$ mm. The scale was observed with a low power magnifying glass, and it was possible to estimate readings to 0.1 mm.

(G) Instrumentation for the Measurement of Magnetic Moment

The change in magnetic moment accompanying a given change in the gyromagnetic element current was measured concurrently with the measurements of the changes in angular momentum. This was done by means of an astatic magnetometer and standard coil system. The magnetometer was located at a point halfway between the element and the standard coil, and the circuits were so arranged that the two currents could be reversed simultaneously. The current in the standard coil was adjusted so that its field annulled the field produced by the standard coil at the magnetometer position. The magnetometer was fitted with a photo electric multiplying device similar to that used on the NS variometer. It was thus a null device of high sensitivity.

It was found that a magnetometer system such as this had certain errors which made it impossible to obtain sufficiently precise values for the changes in magnetic moment. The most important of these were,

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[5] C. F. Kettering and G. G. Scott, Phys. Rev. 66, 257 (1944).

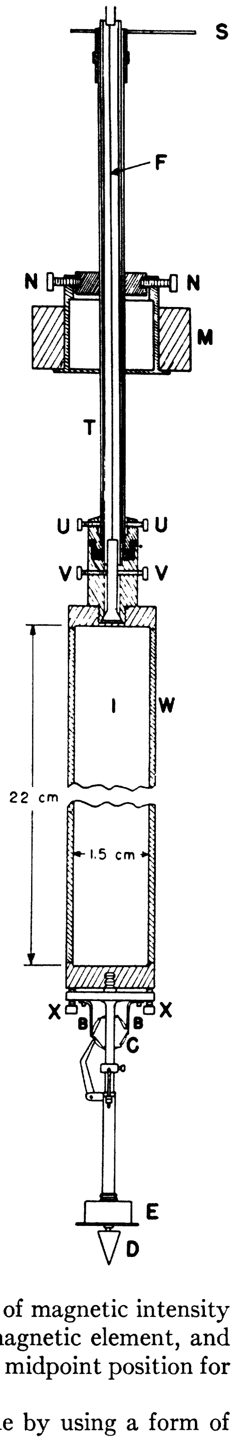

FIG. 3. Gyromagnetic element. S, return current spiral; F, supporting filament; N, mirror adjusting screws; M, four-sided, optically flat element mirror; T, tube; U, tube adjusting screws; V, suspension centering screws; I, iron sample; W, winding; X, reversing switch centering screws; B, brushes; C, commutator; E, disk which is raised by bellows arrangement to operate reversing switch; D, damping cone.

uncertainty as to the distribution of magnetic intensity throughout the iron of the gyromagnetic element, and difficulty in determining the exact midpoint position for the magnetometer.

These difficulties were overcome by using a form of torsion balance working in the very uniform fields generated by the large Helmholtz coil system. This torsional system was used to calibrate the magnetometer. The gyromagnetic element, with its axis in a horizontal EW position, was suspended on a critically damped torsional system rotating about a vertical axis. The element could be quickly replaced in this system by an air core standard accurately made for this purpose. This standard coil consisted of 28 layers of No. 34 double formex insulated wire wound on a $\frac{3}{16}$ inch diameter brass rod 22 cm long. In winding this coil, a micrometer, reading to 0.0001 inch, was used to determine the inside and outside diameter of each layer, and two counters were used to eliminate any possibility of obtaining the wrong count of the number of turns. Over 1000 micrometer readings were averaged for the entire coil. The ΣA_S value, as computed, was 28131 cm^2.

In taking readings, the current in the large NS Helmholtz system was so adjusted in each instance that exactly the same torque was obtained for both the standard and the gyromagnetic element. The magnetic moments then had the same ratio as the field values required to produce this common torque.

Data obtained by this method are plotted in Fig. 4. This plot shows the relationship between the magnetic moment of the gyromagnetic element and the current flowing in it. In Fig. 5, a plot of the same two variables, obtained by the approximate magnetometer system previously described is shown. These magnetometer values were found to be in error by 1.9 percent. This error is largely caused by differences in distribution of magnetic intensity between element and standard coil.

(H) Measurement of Moment of Inertia

The moment of inertia of the element was measured by a comparison of its period, on the same suspension, with the periods of two brass cylinders accurately made for the purpose. The moment of inertia of one of the cylinders was about equal to that of the element. The other was about 4 times as great. The mass of each

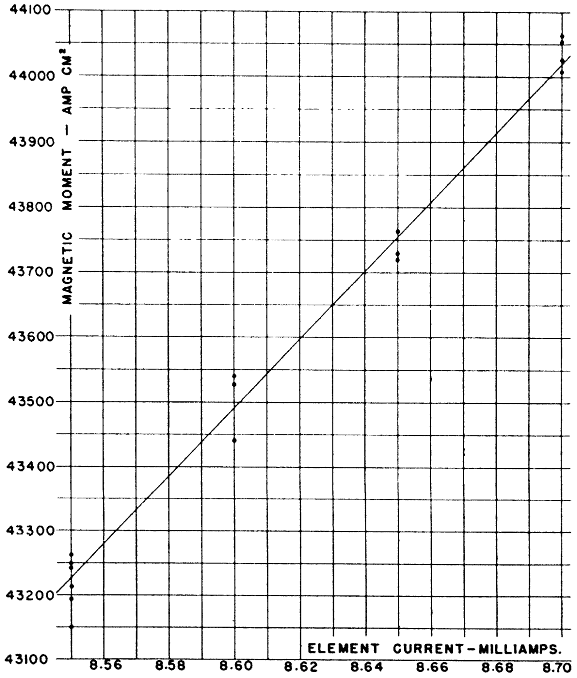

FIG. 4. Plot of element current *vs* magnetic moment—torque method. M = magnetic moment, amp cm^2; $M=5.2832\times 10^6\, i_e - 1944.2$; i_e = element current, amp.

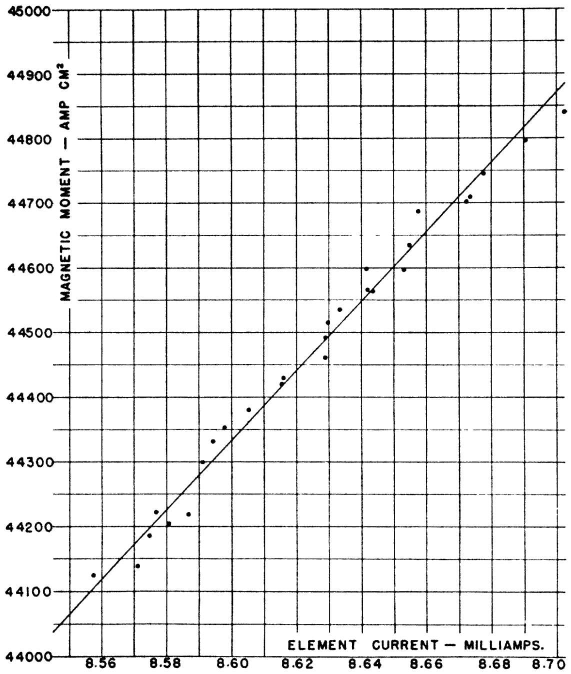

FIG. 5. Plot of element current *vs* magnetic moment magnetometer method. These values were taken concurrently with the readings to determine changes in angular momentum.

cylinder was made very nearly equal to that of the element. This was done to eliminate any possible difference in restoring force due to a difference in tension in the suspension ribbon.

(I) Measurement of the Changes in Angular Momentum

The most difficult part of this experiment involves the measurement of the amplitude changes associated with the changes in angular momentum of the gyromagnetic element. With the very small horizontal component of magnetic moment of the element, and with the high degree of neutralization of the earth's magnetic field, no shift in the center of the element swing could be noted upon reversal of the element current. All angular momentum changes were produced by reversing the magnetizing current when the gyromagnetic element was crossing the center of its swing. Since the period was about 26 sec, sufficient time was available between reversals for the operator to record the extremity of the swing and to determine the center position. The amplitude was first built up for 60 current reversals and then cut down for 60 current reversals. Since the damping coefficient was very small, results which were very nearly linear were obtained. All of the amplitude values were recorded and the statistically best straight line determined for each run. The amplitude change per reversal was determined from the slopes of these straight lines, and damping corrections were made where required. For one complete set of data, the amplitude was built up and cut down for each of two 180° azimuth positions of the element and for both positions of the small reversing switch located at the lower end of the element. These two 180° azimuth positions were varied throughout the course of the experiments to reduce further the possibility of any systematic error that might be caused by an inhomogeneity in the residual magnetic field. The value of the magnetizing current was determined immediately before and after each increase or decrease run. The average of these two current values was used to determine the magnetic moment for the run.

Whenever the azimuth position was changed, or whenever the small reversing switch was operated, it was necessary to remove lateral vibrations of the element caused by these changes. To do this, a small damping pot containing a heavy oil was located on the bellows which actuated the switch mechanism. The arrangement was such that raising the bellows brought the oil up and around a small cone located below the reversing switch; lowering the bellows left the element free. This quickly eliminated lateral vibrations.

IV. RESULTS

In Table II, a condensation of all data obtained for the various days on which readings were taken is given. Systematic errors caused by small residual fields and by any suspension effects have been eliminated from these readings. These values were used to compute the gyromagnetic ratio for the day. Table III gives the gyro-

TABLE II. Condensed data for the determination of the gyromagnetic ratio of iron.

Date	Double amplitude change per reversal-cm d^a	Element current amperes $i_e{}^b$	Element magnetic moment amp cm² $M_e{}^c = ai_e + b$	Period seconds P
		First set-up		
12-14-49	0.046684	0.0058472	29039	26.340
		Second set-up		
1-12-50	0.071195	0.0087514	44291	26.270
1-13-50	0.071135	0.0087006	44023	26.265
1-16-50	0.070902	0.0086935	43985	26.245
1-17-50	0.070644	0.0086931	43983	26.245
1-18-50	0.071057	0.0086996	44017	26.250
1-19-50	0.070606	0.0086878	43955	26.255
1-20-50	0.070910	0.0086835	43932	26.260
		Third set-up		
2-22-50	0.070190	0.0086245	43622	26.315
2-24-50	0.070696	0.0086706	43864	26.310
2-27-50	0.070594	0.0086681	43851	26.310
3-1-50	0.070373	0.0086219	43607	26.315
3-2-50	0.070174	0.0086068	43527	26.305
3-3-50	0.070098	0.0086286	43642	26.300
3-6-50	0.070476	0.0086436	43722	26.320

Factors common to all sets of data

Moment of inertia	$I = 195.52$ gram cm²
Scale distance	$X = 1589.5$ cm
Mass-charge ratio of electron	$m/e = 5.6844 \times 10^{-9}$ grams per coulomb
Area summation of element winding	$\Sigma A_e = 73715$ cm²
Phase angle correction factor	$k = 0.99941$

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[a] Each value of d obtained from 240 separate amplitude readings.
[b] Each value of i_e obtained from 16 separate potentiometer readings.
[c] Values of M_e obtained from plot shown in Fig. 7. k determined from a separate oscilliograph study in which the cosines of approximately 350 phase angle errors were averaged.

TABLE III. Value of gyromagnetic ratio of iron computed from Table II.

Date	Gyromagnetic ratio in terms of m/e of the electrons
12-14-49	1.0235
1-12-50	1.0264
1-13-50	1.0322
1-16-50	1.0304
1-17-50	1.0266
1-18-50	1.0318
1-19-50	1.0263
1-20-50	1.0312
2-22-50	1.0257
2-24-50	1.0276
2-27-50	1.0264
3-1-50	1.0288
3-2-50	1.0281
3-3-50	1.0244
3-6-50	1.0274
Average	1.0278±0.0014 m/e

magnetic ratio for the 15 days on which readings were taken.

(A) Estimation of the Probable Accidental Error

The quantities contributing in a major manner to accidental error are listed in Table IV, along with the values of the probable errors assigned.

Repeated readings of a quantity were taken, and the probable error was computed from the variations of the individual values from the mean. The error in $d/(ai_e+b)$ was obtained by using the variations from the mean in the final results shown in Table III. In the case of ΣA_S, two standard coils, each made with equal precision, were compared, to obtain an idea of the error in this quantity.

V. EARLIER DETERMINATIONS OF GYROMAGNETIC RATIO AT THIS LABORATORY

There have been two earlier determinations of the gyromagnetic ratio of this same sample of iron made at the General Motors magnetic laboratory. These determinations were made before the 8 and 9 ft diameter Helmholtz systems were installed. The neutralizing system at this time consisted of three Helmholtz coils, about 4 ft in diameter, which were not nearly as accurately made.

TABLE IV. Probable accidental errors.

Moment of inertia	I	±0.041%
Double amplitude change per reversal per unit of magnetic moment	$d/(ai_e+b)$	±0.044%
Period	P	±0.005%
Scale distance	X	±0.016%
Area summation of standard coil	ΣA_S	±0.032%
0.138% of 1.0278=0.0014	Total	±0.138%

In these earlier measurements the magnetic moments were determined by the magnetometer system alone. However, the distance between the magnetometer and the element was twice as great, a factor which would considerably decrease errors due to differences in the distribution of magnetic intensity between element and standard coils. Also, in these earlier measurements, smaller element currents were used, and the sensitivity of the optical system was greater.

Between January 15 and February 16, 1948, readings were taken on 11 different days, using an element current of 1.61 mamp. The average value for the gyromagnetic ratio for this set of data was 1.031 m/e. Also, between November 16 and November 24, 1948, readings were taken on 7 different days, using an element current of 2.00 mamp. This set of data produced a gyromagnetic ratio of 1.028 m/e.

VI. CONCLUSIONS

The final value obtained for the gyromagnetic ratio of iron is 1.0278±0.0014 times the reciprocal of the specific electronic charge. This value is in good agreement with the value given by Barnett for this ratio for iron.

The author wishes to express his appreciation to Dr. E. J. Martin and other members of the General Motors Research Staff who cooperated in this experiment. Also to Dr. S. R. Williams of Amherst College who has recently been a research consultant at this laboratory. Thanks are also due Dr. G. M. Rassweiler for help in preparation of the manuscript. Special thanks are due Dr. C. F. Kettering whose many valuable suggestions and continued interest made this work possible.