interesting to repeat the calculations for energy difference in the region where perturbation methods really fail and a larger value of $|1/C|^2$ might occur. The value $E_0-E_n\sim 50$ kev should provide such a case and would be interesting in view of the two lines of Bi^{212}(ThC), which differ by 40 kev.[8] The greatest difficulty in such a calculation would be the numerical evaluation of I_{n0}^- and I_{0n}^-.

In addition to Bi^{212}, there are a number of other cases[1] where this effect might be of some importance. These are shown in Table I. It is difficult to separate the contributions of the various phenomena which affect the intensities. Usually, neither the angular momentum of the α-particle nor the degree of prohibition (value of μ) can be estimated with any certainty. (The penetrability of the coulomb barrier is, of course, the factor which fixes the order of magnitude of the intensities.) In any given experimental case, we would like the difference between the observed partial decay constant and that predicted by standard theory [Eq. (A.1)] to be apportioned among the three effects: angular momentum, "formation" prohibition, and noncentral electric interaction. At present, this does not seem possible, but there appears to be evidence that all three play some role.

TABLE I. Relatively forbidden transitions in which the electric interaction may be important. For Ra^{223}, the figures refer to lines I and II.

Nucleus	E_0-E_n kev	$\lvert C\rvert^{-2}$
U^{235}	160	0.11
Th^{229}	100	0.5
Ra^{223}	80	0.40
Bi^{212}	40	0.39
Bi^{214}	60	0.82

I am much indebted to Professor R. E. Peierls for stimulating discussion and to my wife for help with the rather heavy numerical calculations.

[8] G. T. Seaborg and I. Perlman, Revs. Modern Phys. **20**, 585 (1948).

PHYSICAL REVIEW VOLUME 82, NUMBER 4 MAY 15, 1951

Conductivity of Cold-Worked Metals*

ROLF LANDAUER
Lewis Laboratory, NACA, Cleveland, Ohio
(Received November 10, 1950)

In the vicinity of an edge type dislocation, the density of electrons, and therefore the width of the filled portion of the conduction band is not uniform. To keep the electrons in equilibrium, an electrostatic potential is required. The scattering caused by this potential is used to calculate the increased resistance of isotropically cold-worked copper.

I. INTRODUCTION

COLD working produces an increase in the resistance of metals. This increase has been attributed to the formation of dislocations by Koehler.[1,2] Koehler's calculations have been refined by Mackenzie and Sondheimer,[3] who give a very elegant treatment of the problem. In particular, they calculated the change in resistance produced by cold-working, at high temperatures, whereas Koehler calculated the resistance arising from dislocations at absolute zero. These two quantities need not be equal.[4–6] Both treatments attribute a screened coulomb potential to each ion of the crystal. The perturbation that scatters the conduction electrons arises from the fact that these ions, and consequently their screened coulomb fields, are displaced from their normal lattice positions.

In the earlier treatments it is assumed, therefore, that the screening electrons which are associated with a particular ion stay with that ion as the latter is displaced from its position. No large scale redistribution of electronic charge is permitted.

Actually, however, the electrons can redistribute themselves. An edge dislocation contains an extra plane of ions, above or below the slip plane. Consider the case in which the extra plane is above the slip plane. If each ion were to keep its screening electrons, the Fermi energy above the slip plane would be larger than below, and, as a result, some electrons would go from the upper to the lower region. This in turn gives an unbalance of charge and an electric field. Equilibrium will be attained when the Fermi levels in the different regions have come together.[7] If the number of electrons which are displaced to establish the field is small, the effect of this change of electron density on the Fermi energy can be neglected. The purpose of this paper is to calculate the incremental resistance per dislocation due to this electrostatic field. It will turn out to be

[1] J. S. Koehler, Phys. Rev. **75**, 106 (1949).
[2] J. S. Koehler, Am. J. Phys. **10**, 275 (1942).
[3] J. K. Mackenzie and E. H. Sondheimer, Phys. Rev. **77**, 264 (1950).
[4] E. H. Sondheimer and A. H. Wilson, Proc. Roy. Soc. (London) **A190**, 435 (1947).
[5] M. Kohler, Z. Physik **126**, 495 (1949).
[6] J. W. Rutter and J. Reekie, Phys. Rev. **78**, 70 (1950).
[7] J. Bardeen and W. Shockley, Phys. Rev. **80**, 72 (1950).

considerably larger than the resistance Mackenzie and Sondheimer found.

II. THEORY

The stresses surrounding an edge dislocation have been given by Koehler.[8] According to these, the relative change in ionic density about a dislocation is given by

$$\frac{\Delta n}{n}=\frac{a}{2\pi}\frac{(1-\nu-2\nu^2)}{(1-\nu^2)}\frac{\sin\theta}{r}. \tag{1}$$

Here, a is the lattice spacing of the simple cubic lattice on which this picture is based, and ν is Poisson's ratio. The usual polar coordinate system is used in which the z-axis is taken to be the axis of the dislocation and the angle θ is measured with respect to the slip plane. Now let us assume, for the moment, that the electronic density changes likewise. The width of the filled portion of the conduction band is[9]

$$E=(h^2/2m^*)(3n/8\pi)^{\frac{2}{3}}, \tag{2}$$

where m^* is the effective mass, and n is the density of electrons. The change in the width due to the stresses will then be

$$\Delta E=\tfrac{2}{3}E\Delta n/n, \tag{3}$$

or

$$\Delta E=\frac{a}{3\pi}\frac{(1-\nu-2\nu^2)}{(1-\nu)^2}\frac{\sin\theta}{r}. \tag{4}$$

This neglects a possible change in the effective mass.

In equilibrium, the top of the filled portion of the conduction band must be at the same level everywhere. Therefore, since the width varies, the bottom of the conduction band must deviate from its normal level by an amount opposite to that given in Eq. (3). The electrons will, therefore, act as if moving in an electrostatic potential V, which satisfies the relation

$$eV=\Delta E, \tag{5}$$

where e is the absolute magnitude of the electronic charge. Hence,

$$V=\frac{a}{3\pi e}\frac{(1-\nu-2\nu^2)}{(1-\nu^2)}\frac{\sin\theta}{r}. \tag{6}$$

This is just the potential of a line dipole, located along the dislocation axis. Actually, not all of the potential in Eq. (6) is really an electrostatic potential. Part of it arises from the fact that the bottom of the band is itself sensitive to lattice spacing. It is, however, the deviation of the bottom of the band that must be used as an effective potential.[10]

The calculation of the incremental resistance can proceed from this perturbing potential, exactly as Mackenzie and Sondheimer did. This computation gives, for the change of resistivity,

$$\frac{\Delta\rho}{\rho}=\frac{16}{27}E^2a^2\left[\frac{1-\nu-2\nu^2}{1-\nu^2}\right]^2\frac{\tau m^*k_0}{\hbar^3 n}N. \tag{7}$$

Here N is the number of dislocations per cm^2, k_0 is the wave vector at the top of the Fermi distribution, and τ is the mean free time between electron collisions. This last quantity can be found from the experimental value of conductivity by using the condition

$$\sigma=ne^2\tau/m^*. \tag{8}$$

We shall use the values for copper, obtained from specific heat measurements,[9] $E=4.78$ ev, $m^*/m=1.47$. Furthermore, taking $k_0=1.37\times10^8$/cm, $n=0.85\times10^{23}$/cm^3, $\nu=0.34$, and $a=2.55\times10^{-8}$ cm, we obtain

$$\Delta\rho/\rho=2.5\times10^{-14}N. \tag{9}$$

The experimental value[1] of $\Delta\rho/\rho$ for heavily cold-worked copper is 2 percent. This gives $N=8\times10^{11}$. Koehler[8] estimated on the basis of energy storage measurements that $N=6\times10^{11}$.

It must be kept in mind that actually dislocations occur in the form of half-dislocations in a face centered cubic crystal.[11] Furthermore, some of the stored energy may come from screw dislocations, or the screw components of dislocation rings. The resistance of screw dislocation is presumably much smaller than our result, since no volume compression is associated with them.

The author is indebted to Dr. Sondheimer for his introduction to this problem.

[8] J. S. Koehler, Phys. Rev. **60**, 398 (1941).

[9] F. Seitz, *Modern Theory of Solids* (McGraw-Hill Book Company, Inc., New York, 1940), chapter IV.

[10] J. C. Slater, Phys. Rev. **76**, 1592 (1949). We should use the effective mass wave equation derived by Bardeen and Shockley in reference 7, which covers the case of elastic deformations as well as electrostatic disturbances. This equation is:

$$[(\hbar^2/2m^*)\nabla^2+\delta U_s]A(r)=EA(r),$$

where δU_s represents the height of the energy level above the bottom of the band. $A(r)$ is a function which modulates $\psi_0(r)$, which in turn is the wave function corresponding to the band edge. The variation of m^* in the $\hbar^2/2m^*\nabla^2$ term compensates exactly for the variation of m^* in (2) which was neglected. Hence, the treatment given in this paper, gives the correct answer.

[11] R. D. Heidenreich and W. Shockley, *The Strength of Solids* (Physical Society, London, 1948), p. 1.