The Noncentral Electric Interaction in Alpha-Radioactivity*

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(Received July 24, 1950)†

Apart from the penetrability of the coulomb potential barrier, which determines the order of magnitude of the intensity of α-decay, there are three secondary effects which alter the decay constant: angular momentum, "formation" prohibition as discussed by Perlman, Ghiorso, and Seaborg, and noncentral electric interaction with the product nucleus. The general theory of this last effect is discussed and its magnitude estimated. It is suggested that it may be of considerable importance in determining the intensity of highly forbidden ground state transitions described by Perlman, Ghiorso, and Seaborg. Further numerical work would be desirable.

I. INTRODUCTION

THE standard formula for the decay constant in α-activity is of the form

$$\lambda=(v_i/2r_0)G\exp(-2\omega_0). \quad (A.1)$$

Here v_i is the velocity of the α-particle inside the nucleus of radius r_0, so that the first factor is the frequency of collisions of the α-particle with the nuclear barrier. The last factor, $\exp(-2\omega_0)$, describes the "leak" through the coulomb potential barrier around the nucleus and determines the order of magnitude of λ. (ω_0 will be defined later.) The remaining factor G depends in general on v_i, r_0, and v, the velocity at infinity. The form of G is determined by the model used to describe the forces holding the α-particle in the nucleus. For a rectangular well potential, G can be obtained explicitly.

This basic formula (A.1) applies only to the case in which the α-particle is emitted with zero angular momentum from an even–even nucleus in which the "formation" time of the α-particle is negligible. This point has been demonstrated by the analysis of Perlman, Ghiorso, and Seaborg.[1] In nuclei other than even–even, the α-particle must be formed from the nucleons; and in order to obtain full energy, it must include the unpaired nucleon(s) in the highest energy state. This effect makes the α-decay of such nuclei forbidden to some extent with respect to a hypothetical even–even decay of the same energy and same charge number. Also, in the decay of any one nuclear species, the ground-state transition requires that the α-particle carry away the maximum energy, which must be obtained from the unpaired nucleon(s). Hence, apart from barrier penetration, the low energy lines of a complex α-spectrum might be expected to be more probable than the ground-state transition. In other words, for their respective energies, the ground-state transition is forbidden relative to the other transitions possible. This effect requires a modification of the factor G; the exact theory is clearly complicated.

The formula for λ is also changed if the α-particle has non-zero angular momentum.[2,3]

There is a third mechanism[4] which can alter the value of λ given by the standard formula; namely, the noncentral interaction of the α-particle outside the nucleus with the unsymmetrically distributed protons in the product nucleus. By means of this coupling, the α-particle can take energy from or give energy up to the nucleus and thus alter its own ability to penetrate the barrier. The study of this effect arose from an attempt to understand, theoretically, certain observations[5] of the α-ray spectra of Po and Ra. The connection of the theory with this particular problem has already been given in a paper[6] hereafter referred to as P. In the present paper we propose to discuss this noncentral electric interaction effect in general. In P it was suggested that this mechanism might be important if a ground-state transition could be forbidden; a physical basis for this assumption is provided by the formation prohibition.

II. PERTURBATION SOLUTIONS

The general equations of the theory are given in P. We may summarize them as follows. The total wave function of the system is

$$\Psi=\sum_m \phi_m(\mathbf{r})u_m(\xi), \quad (1.1)$$

where $u_m(\xi)$ are the complete set of normalized orthogonal eigenfunctions of the residual nucleus, ξ referring to all the coordinates which determine a nuclear configuration. The α-particle has position $\mathbf{r}$ and its wave functions are written

$$\phi_n(\mathbf{r})=r^{-1}f_n(r)\sum_{M=-l_n}^{l_n} c_M P_{l_n}^M(\cos\theta)e^{iM\varphi}. \quad (1.2)$$

The potential $\sum(2e^2/|\mathbf{r}-\mathbf{r}_i|)-2Ze^2/r$ is treated as a perturbation. It is expanded in powers of $1/r$ and it is

* Most of the work here described was carried on at The University, Birmingham, England.
† Revised manuscript received January 22, 1951.
[1] Perlman, Ghiorso, and Seaborg, Phys. Rev. **77**, 26 (1950).
[2] G. Gamow and C. L. Critchfield, *Theory of Atomic Nucleus and Nuclear Energy Sources* (Clarendon Press, Oxford, 1949).
[3] M. A. Preston, Phys. Rev. **71**, 865 (1947).
[4] Originally suggested some years ago by Professor Gamow.
[5] W. Y. Chang, Phys. Rev. **69**, 60, and **70**, 632 (1946).
[6] M. A. Preston, Phys. Rev. **75**, 90 (1949). This work suggested that Chang's results were spurious. See W. G. Wadey, Phys. Rev. **74**, 1846 (1948).

deduced that the radial functions f_n satisfy

$$(d^2f_n/dr^2)+\{(2m/\hbar^2)(E_n-V)-l_n(l_n+1)/r^2\}f_n = \sum_{m=0}^{N-1} f_m \sum_{k=|l_m-l_n|}^{l_m+l_n} A_k{}^{nm} r^{-(k+1)}, \quad (1.3)$$

$$k\neq 0,\ n=0,\ 1,\ \cdots,\ N-1.$$

Here

$$V=2Ze^2/r, \quad r>r_0. \quad (1.4)$$

$A_k{}^{nm}$ is a mean transition matrix element averaged over the possible orientations of the angular momentum $k\hbar$ and is of order of magnitude

$$(4\sqrt{2}/2k+1)(me^2/\hbar^2)\int u_n{}^* \textstyle\sum_i r_i{}^k P_k(\cos\theta_i) u_m d\xi,$$

where $(r_i,\ \theta_i,\ \varphi_i)$ are the coordinates of the ith proton in the nucleus and the summation is over all such protons. These A's represent roughly the coupling which changes the angular momentum and energy of the α-particle. The origin is at the center of mass of the residual nucleons, m is the reduced mass, and E_n is the (strictly complex) energy of the nth α-particle group.

To simplify the problem, we shall study the case in which the residual nucleus has one or more excited levels all of the same spin. We assume that the transitions between these levels are unimportant compared with those between any one of them and the ground level; i.e., $A_k{}^{nm}\ll A_k{}^{n0}$ when $m\neq 0$. The case of only one excited level is, of course, included. Also, one of the terms of the summation over k in Eq. (1.3) is much greater than the rest. If the ground state has spin zero, there is only one term; otherwise the term with the least value of k is the important one. Let this value of $k=p-1$.

We now proceed to set up a perturbation calculation by the substitution

$$f_n(r)=w_n(r)\chi_n(k_nr), \quad (1.5)$$

where $\chi_n(x)$ is the confluent hypergeometric function defined by

$$(d^2\chi_n/dx^2)+\{1-(\kappa_n/x)-l_n(l_n+1)/x^2\}\chi_n=0, \quad (1.6)$$

and as $x\to\infty$,

$$\chi_n\to\exp i\{x+\tfrac{1}{4}\pi+\tfrac{1}{2}\kappa_n\ln(\kappa_n/4ex)\}. \quad (1.7)$$

$$\kappa_n=4Ze^2/\hbar v_n, \quad k_n=mv_n/\hbar, \quad E_n=\tfrac{1}{2}mv_n{}^2. \quad (1.8)$$

Thus, χ_n is the wave function of the α-particle in the standard theory which does not take account of the interaction with the individual protons of the nucleus. χ_n is studied in P, Sec. IV. With this substitution and the simplifications indicated in the previous paragraph, Eqs. (1.3) become

$$\chi_0w_0''+2k_0\chi_0'w_0'=\sum_{m\neq 0}\chi_mw_mA_{p-1}{}^{0m}r^{-p}, \quad (1.9a)$$

$$\chi_nw_n''+2k_n\chi_n'w_n'=\chi_0w_0A_{p-1}{}^{n0}r^{-p}, \quad n\neq 0. \quad (1.9b)$$

t he argument of the function, that is, with respect to k_nr for χ_n and with respect to r for w_n.

To solve these equations one might be inclined to use a perturbation technique. At infinity, both χ_0 and χ_n have magnitude unity; the high energy α-particles are always observed to be considerably more intense than those of even slightly lower energy. Since the subscript 0 refers to the ground state, $E_0>E_n$; and hence at infinity $w_n\ll w_0$, and we might therefore assume the usual perturbation conditions; *viz.*, $w_0=a_0$, a constant, and w_n small. Then Eq. (1.9a) is discarded and we obtain

$$w_n(r)=v_{n0}(k_0r)+a_n\exp i\delta, \quad (1.10a)$$

$$w_n(\infty)=a_n\exp i\delta, \quad (1.10b)$$

$$w_0(r)=a_0. \quad (1.10c)$$

Here v_{n0} is given by P (3.17) and P (4.12), *viz.*,

$$v_{n0}(x)=\tfrac{1}{2}a_0\gamma_{0n}\rho_{n0}(\cot\alpha_0\cot\beta_0)^{\frac{1}{2}}\exp(\omega_0-\Omega_0)k_0{}^{p-2} \times A_{p-1}{}^{n0}\{I_{n0}{}^-(p-1)-I_{n0}{}^+(p-1)\}x^{-p}. \quad (1.11)$$

$\gamma_{nj}=k_n/k_j$ and the other functions in Eqs. (1.10) are defined in P, Sec. IV.

This is a straightforward perturbation calculation, which is suggested by the fact that $a_n\ll a_0$. However, in general, this solution is not valid. The value of the ratio a_n/a_0 predicted by a simple theory, based only on the barrier penetrability, is the ratio of the Gamow factors, *viz.*, $\exp(\omega(r_0)-\Omega(r_0))$, where r_0 is the nuclear radius and ω and Ω refer, respectively, to states 0 and n, being defined explicitly in P(4.2) and P(4.10). If a_n/a_0 has this value, then χ_n will equal χ_0a_0 at $r=r_0$, because of the corresponding exponential increase of χ_n/χ_0. Hence, χ_nw_n will be of at least the same order as χ_0w_0 in the region near the nucleus, unless a_n/a_0 is much less than the Gamow ratio. Hence the neglect of the right-hand side of Eq. (1.9a) is not usually justified.

Consequently, this simple perturbation calculation has had to be modified. The method used in P consists essentially of treating as a small perturbation not the functions f_n, but rather the departure of both the f_n's and f_0 from the functions $a_n\exp(i\delta_n)\chi_n$ and $a_0\chi_0$ which would be their forms if there were no interaction of the type we are studying. That is, we have there written

$$f_0=w_0\chi_0=(a_0+\textstyle\sum_n v_{0n})\chi_0, \quad (1.12a)$$

$$f_n=w_n\chi_n=(a_n\exp i\delta_n+v_{n0})\chi_0, \quad (1.12b)$$

and have assumed $|v_{0n}|\ll a_n\chi_n/\chi_0$ and $|v_{n0}|\ll a_0\chi_0/\chi_n$. The right-hand sides of these inequalities are, respectively, the unperturbed values of a_0 and a_n. Substitution in Eq. (1.9) then yields equations for v_{0n} and v_{n0} which are solved in P, with the result that v_{n0} is given as above by Eq. (1.11) and

$$v_{0n}(k_nr)=\tfrac{1}{2}a_n\rho_{0n}(\cot\alpha_0\cot\beta_0)^{\frac{1}{2}}\exp(\Omega_0-\omega_0)k_0{}^{p-2} \times A_{p-1}{}^{n0*}\{I_{0n}{}^-(p-1)-I_{0n}{}^+(p-1)\}x^{-p}. \quad (1.13)$$

The solutions (1.12) have much wider applications than Eqs. (1.10), but there are conditions in which Eqs. (1.12) also do not hold. These are if $|v_{0n}|$ or $|v_{n0}|$

becomes comparable with $a_n\chi_n/\chi_0$ or $a_0\chi_0/\chi_n$, respectively. If we look at the forms of v_{0n} and v_{n0}, we see that the orders of magnitude of all the terms but the integrals I^- are roughly independent of E_0-E_n, but as $E_0-E_n \to 0$, the I^- terms may become large, and the solution (1.12) becomes invalid. In this case, of course, Eq. (1.10) also fails. The quantities $(I^--I^+)x^{-p}$ are largest for points near the nucleus. Hence, we can expect our perturbation calculations to fail first at points near the nucleus as E_0-E_n is decreased.

III. SOLUTION FOR SMALL ENERGY DIFFERENCE

It is possible, however, to give an alternative method of solution which may be useful when the perturbation solution begins to fail. This depends on the fact that, since E_0-E_n is small, χ_0 and χ_n are not very different. In Eq. (1.6) for χ_n, the term κ_n/x arises from the coulomb potential and is numerically the ratio of this potential to the total energy of the particle (about 5 near the nucleus and for representative α-energies). It is much more important than the centrifugal term. For small changes in energy the percent variation of κ_n/x is the same as that of the energy. In other words, changes in energy of 500 kev or less will not have a large effect on the coefficient of χ in the differential equation. Thus, if we renormalize χ_n so that it has the same value as χ_0 at $r=r_0$, it will be a good approximation to replace both χ_0 and χ_n by the same function χ, provided r is not extended too far from r_0. Details as to the meaning of "too far" consistent with any prescribed degree of accuracy are given elsewhere.[7] For example, if $E_0-E_n=250$ kev, the method gives 10 percent accuracy for $r \lesssim 1.6r_0$, and for smaller energy differences or less accuracy the range can be extended. Since, as E_0-E_n is decreased the breakdown of perturbation theory occurs near the nucleus, these ranges appear to be sufficient. (Actually, we shall see that for 250 kev the perturbation method is valid.)

For simplicity, we consider the case where there is significant interaction between the ground state "0" and one excited state "n". Then we use $\chi(kr)$ to represent both $\chi_0(k_0r)$ and $\chi_n(k_nr)\chi_0(k_0r_0)/\chi_n(k_nr_0)$. We define χ exactly by

$$\chi''+(1-\kappa/kr)\chi=0, \quad \chi(kr_0)=\chi_0(k_0r_0). \tag{2.1}$$

Here κ, k, and v are certain mean values.

We now replace $\chi_0(k_0r)$ and $\chi_n(k_nr)$ in Eq. (1.9) by $\chi(kr)$, and after some substitutions find that we can write

$$f_0(r)=w_0\chi_0=\tfrac{1}{2}a_0A[\Gamma_1C_+(r)+\Gamma_2D_+(r)+\Delta_1C_-(r)+\Delta_2D_-(r)]\chi_0(k_0r)/\chi_0(k_0r_0), \tag{2.2a}$$

$$f_n=\tfrac{1}{2}a_0Ae^{i\vartheta}[\Delta_1C_-+\Delta_2D_--\Gamma_1C_+-\Gamma_2D_+]\chi_n/\chi_n(k_nr_0). \tag{2.2b}$$

Here, $A_{p-1}{}^{n0}=Ae^{i\vartheta}$, Γ_1, Γ_2, Δ_1, Δ_2 are constants and χC_+, χD_+, χC_-, χD_- are the independent solutions, corresponding to the two signs of A, of the equation

$$d^2F/dx^2+[1-(\kappa/x)\pm A/x^p]F=0. \tag{2.3}$$

It is only in the region near the nucleus that the solution (2.2) is valid. If the lowest value of r for which the perturbation solution (1.12) is sufficiently accurate is less than the highest value for which Eq. (2.2) is reliable, we can then join the two solutions at some intermediate value r_1. Whether or not this can be done in any particular case can be determined numerically.

It is necessary to fix the four constants Γ_1, Γ_2, Δ_1, Δ_2 in Eq. (2.2) and a_0 and $a_n \exp i\delta$ in Eq. (1.12). These are fixed by boundary conditions. At the nuclear radius we have, apart from the constant of proportionality between f_0 and f_n, a matrix of four complex constants α, β, γ, δ determining the values of df_0/dr and df_n/dr:

$$\text{at } r=r_0 \begin{cases} f_0'=\alpha f_0+\beta f_n & \text{(2.4a)} \\ f_n'=\gamma f_0+\delta f_n. & \text{(2.4b)} \end{cases}$$

Clearly, Γ_1, Γ_2, Δ_1, Δ_2 can be found in terms of α, β, γ, δ. Then at some value of $r=r_1$, the solutions (2.7) and (1.15) are joined by the four conditions

$$(f_0, f_n, df_0/dr, df_n/dr)_{(2.2)} = (f_0, f_n, df_0/dr, df_n/dr)_{(1.12)} \tag{2.5}$$

at $r=r_1$. This implies that the perturbation solutions also involve four undetermined constants, which are thus given in terms of α, β, γ, δ. However, we have only two constants available in Eq. (1.12), *viz.*, a_0 and $a_n \exp i\delta$; knowledge of the other two is equivalent to assuming the presence of only outgoing waves at infinity. Thus, instead of α, β, γ, δ, knowledge of which depends on details of intranuclear phenomena, we use the absence of incoming waves at infinity. The constants which remain are Γ_1, Γ_2, Δ_1, Δ_2, $C=(f_n/f_0)_\infty = a_n \exp i\delta/a_0$ and $\mu=(f_n/f_0)_{r_0}$. Equations (2.5) determine the Γ's and Δ's as functions of C and then Eqs. (2.2) allow us to find the relationship between C and μ. Thus we can express the relative intensity of the two states at infinity in terms of the relative intensity at r_0. We can also find λ, the decay constant, from

$$(f_0f_0^{*\prime}-f_0^*f_0'+f_nf_n^{*\prime}-f_n^*f_n')_{r_0} = (2im\lambda/\hbar)\int_0^{r_0}(f_0f_0^*+f_nf_n^*)dr. \tag{2.6}$$

The integral on the right can be estimated only roughly.

Apart from obtaining explicit forms for the functions $C_\pm$, $D_\pm$ in the solutions of Eq. (2.3), this completes the general solution of the problem. In a given numerical case, it is desirable to discuss the solutions for a range of values of A, since this nuclear parameter cannot be well known *a priori*.

It is to be noted that, although Eqs. (2.5) involve r_1, the value of r at which the two solutions are joined, the final result should not be at all sensitive to this parameter, if it really lies in an interval of r where both solutions are reasonably accurate. In a numerical case, it

[7] M. A. Preston, Thesis, University of Birmingham (1949).

would be wise to repeat the calculations with a slightly different value of r_1, thus obtaining a final check on the over-all accuracy of the various approximations.

IV. EXPLICIT FORMS FOR $C_\pm$, $D_\pm$

We have still to solve explicitly

$$(d^2F/dx^2)+[1-(\kappa/x)\pm A/x^p]F=0. \tag{2.3}$$

If we are dealing with a dipole transition, $p=2$, and the solutions of Eq. (2.3) are confluent hypergometric functions. We base the solution of Eq. (2.3) on the known expressions [P(4.1) and (4.6)] for the solutions of

$$d^2\chi/dx^2+(1-\kappa/x-l(l+1)/x^2)\chi=0. \tag{3.1}$$

Since these solutions are known for the first few integral values of l, we can express the solutions F of Eq. (2.3) in terms of the solutions χ and the difference between A and the nearest value of $l(l+1)$.

Now by P, Eq. (5.1),

$$A_1{}^{n0}=4\sqrt{2}me^2R/3\hbar^2\sim 2.53 \times 10^{12}\left(\int u_n{}^*\Sigma r_i \cos\theta_i u_0 d\xi \text{ cm}\right). \tag{3.2}$$

It is difficult to estimate the matrix element R, but as it refers to a dipole transition it will be at most a few times 10^{-13} cm and may be as small as 10^{-14} cm. Hence, for the dimensionless quantity A, we have the approximate limits

$$1\gtrsim A\gtrsim 10^{-2}. \tag{3.3}$$

Thus, if $\lambda=l(l+1)\pm A$, Eq. (3.3) shows that λ has its smallest value when $l=0$. Therefore, we shall study the equation

$$(d^2F/dx^2)+[1-(\kappa/x)+\lambda/x^2]F=0 \tag{3.4}$$

and shall make use of the functions χ which satisfy

$$(d^2\chi/dx^2)+(1-\kappa/x)\chi=0. \tag{3.5}$$

For higher multipoles, similar considerations show that near the nucleus $A_p x^{-p}$ is small compared with $(1-\kappa/x)$; and therefore we express solutions of

$$(d^2F/dx^2)+[1-(\kappa/x)+\lambda/x^p]F=0 \tag{3.6}$$

in terms of the solutions χ of Eq. (3.5).

Our procedure is to put

$$F(x)=\chi(x)\sigma(x) \tag{3.7}$$

and express σ as a power series in the parameter λ, *viz.*:

$$\sigma=\sum_0^\infty c_n(x)\lambda^n. \tag{3.8}$$

We want two independent solutions of Eq. (3.6). One is obtained by making F almost the same as the solution

$$\chi=(\cot\alpha)^{\frac{1}{2}}e^{\omega}; \qquad [\text{P (4.1)}]$$

that is, we take $F=\chi$, $F'=\chi'$ at the nuclear boundary. The other independent solution of Eq. (3.6) is obtained in the same way, using for χ the other solution of Eq. (3.5), *viz.*, $(\cot\alpha)^{\frac{1}{2}}e^{-\omega}$.

For the second solution we write

$$\sigma=\sum_0^\infty \gamma_n\lambda^n. \tag{3.9}$$

Then,

$$C_+=\sum c_nA^n, \quad D_+=\sum \gamma_nA^n, \quad C_-=\sum c_n(-A)^n, \quad D_-=\sum \gamma_n(-A)^n. \tag{3.10}$$

Recursion formulas for c_n and γ_n have been obtained.[7] These involve numerical integration, but usually c_n and γ_n are required for only a few low values of n.

V. NUMERICAL RESULTS FOR A DIPOLE CASE

Numerical calculations have been made for a typical case. For E_0 we have taken 5.303 Mev, simply because some of the quantities had already been obtained for this energy, which is that of the main line of polonium. For the small energy difference, we have taken $E_n-E_0=250$ kev, so that the calculations of Sec. II on the validity of replacing χ_0 and χ_n by the same function apply. We have assumed a dipole transition; i.e., $p=2$. However, the methods of this section are applicable to higher multipoles.

For $E=5.30$ Mev, and $r_0=8.27\times10^{-13}$ cm, it is necessary in calculating $C_\pm$, $D_\pm$ to take the power series (3.10) only to the terms in A^2.

We now proceed to solve (2.5) and use (2.2) to obtain the relationship between $C=(f_n/f_0)_\infty$ and $\mu=(f_n/f_0)_{r_0}$. It is found that terms in A^2 can be neglected; to this order the result is

$$\mu=\frac{\Pi C+Ae^{i\vartheta}\{-[(\gamma_1'-c_1')/\gamma_0']-(\Pi\psi I^+\gamma_0/\gamma_0')+\frac{1}{2}\Pi\tau(I_{n0}{}^--I^+)\}}{1+ACe^{-i\vartheta}\{-[\Pi(\gamma_1'-c_1')/\gamma_0']-(\phi I^+\gamma_0/\gamma_0')+\frac{1}{2}\rho(I_{n0}{}^--I^+)\}}. \tag{4.1}$$

The following abbreviations have been used.

$$\Pi=(\cot\beta_0\tan\alpha_0)^{\frac{1}{2}}\exp(\Omega_0-\omega_0). \tag{4.2}$$

Π^2 is the ratio of the barrier penetrabilities at the two energies.

$$\rho=(\cot\alpha_1\cot\beta_1)^{\frac{1}{2}}\exp(\Omega_1-\omega_1)(kr_1)^{-2}. \tag{4.3}$$

$$\tau=\gamma_{0n}(\cot\alpha_1\cot\beta_1)^{\frac{1}{2}}\exp(\omega_1-\Omega_1)(kr_1)^{-2}. \tag{4.4}$$

$$\phi=\kappa\sin2\tilde{a}_1\rho\tan\alpha_1. \tag{4.5}$$

$$\psi=\kappa\sin2\tilde{a}_1\tau\tan\beta_1. \tag{4.6}$$

$$I^+=I_{0n}{}^+(r_1)=I_{n0}{}^+(r_1); \quad I_{0n}{}^-=I_{0n}{}^-(r_1). \tag{4.7}$$

In these equations the subscript 0 refers to values at $r=r_0$, the nuclear radius; subscript 1 refers to $r=r_1$, the point where the solution of Sec. III is joined to the perturbation solution; $\tilde{a}=\arccos(kr/\kappa)^{\frac{1}{2}}$; α and β are

the corresponding quantities for E_0 and E_n, defined exactly in P Eqs. (4.1), (4.11). $' \equiv d/d\tilde{\alpha}$.

Let us compare this result with those of the perturbation methods. If the perturbation solution (1.12) is carried right down to $r=r_0$, instead of using the alternative method near the nucleus, it can be seen that

$$\mu=\frac{\Pi C+Ae^{i\theta}\tfrac{1}{2}\Pi\tau_0(I_{n0}^{-}-I^{+})_{r0}}{1+ACe^{-i\theta}\tfrac{1}{2}\rho_0(I_{0n}^{-}-I^{+})_{r0}}, \tag{4.8}$$

where τ_0 and ρ_0 are τ and ρ with r_1, replaced by r_0. Also the more restricted perturbation solution (1.10), if extended to $r=r_0$, gives

$$\mu=\Pi C+Ae^{i\theta}\tfrac{1}{2}\Pi\tau_0(I_{n0}^{-}-I^{+})_{r0}. \tag{4.9}$$

In our numerical case, Eq. (4.1) gives

$$\mu=\frac{\Pi C+Ae^{i\theta}(7.9_6+6.1_8i)\times10^{-2}}{1+CAe^{-i\theta}(1.6_4-0.08_4i)\times10^{-1}} \tag{4.1a}$$

and Eq. (4.8) gives

$$\mu=\frac{\Pi C+Ae^{i\theta}(8.3_6+6.4_9i)\times10^{-2}}{1+CAe^{-i\theta}(1.5_1-0.08_0i)\times10^{-1}} \tag{4.8a}$$

and Eq. (4.9), of course, gives

$$\mu=\Pi C+Ae^{i\theta}(8.3_6+6.4_9i)\times10^{-2},\quad(\Pi=4.84). \tag{4.9a}$$

In these expressions we have taken $r_1=14.72\times10^{-13}$ cm. If we had taken $r_1=16.81\times10^{-13}$ cm, the coefficient in the numerator of Eq. (4.1a) would have been $(7.8_5+6.1_8i)\times10^{-2}$ and in the denominator it would have been $(1.6_7-0.08_4i)\times10^{-1}$. Thus, the result is not sensitive to the value of r_1. The calculations for I^+ and I^- are given elsewhere.[7]

The observed relative intensity of the α-particles of the lower energy E_n to that of particles of energy E_0 is $|C|^2$. When $A=0$, that is, when the effect of the electrostatic interaction is not taken into account, we have $|C|^2=\mu^2/\Pi^2$, which simply expresses the result of the standard theory, Π^2 being the ratio of the penetrabilities of the coulomb potential barrier at the two energies.

The results of the perturbation calculation as given in Eq. (4.8a) are clearly very close to those of the second method (4.1a). This is not surprising, however, for in this case the criteria stated in Sec. I for the validity of Eq. (1.12) imply $0.11A\ll1$, which is certainly satisfied. Thus, even with the energy difference of 250 kev, the improved perturbation method is valid.

The original perturbation method, corresponding to Eq. (4.9a) is valid if $|v_{n0}+a_n\exp i\delta|\,\chi_n\ll a_0\chi_0$ for all r. This holds only if C is very small and then, of course, there is no real difference between Eqs. (4.8a) and (4.9a).

Let us now consider the magnitude of the interaction effect we have been studying. Rewrite Eq. (4.1) in the form

$$C=\frac{(\mu/\Pi)-Ae^{i\theta}\{-[(\gamma_1'-c_1')/\gamma_0'\Pi]-(\psi I^{+}\gamma_0/\gamma_0')+\tfrac{1}{2}\tau(I_{n0}^{-}-I^{+})\}}{1-\mu Ae^{-i\theta}\{-[(\gamma_1'-c_1')/\gamma_0']-(\phi I^{+}\gamma_0/\gamma_0'\Pi)+\tfrac{1}{2}\rho(I_{0n}^{-}-I^{+})/\Pi\}}. \tag{4.10}$$

Firstly, we take the case $|\mu|=1$. This expresses that, in the absence of the potential barrier, decay with either energy E_0 or E_n is equally probable. Since the energies are close together, this seems a reasonable assumption in the absence of any special restrictive conditions. Then, if $\mu=\exp i\Theta$, we obtain, to order A,

$$|C|^2=\Pi^{-2}-\Pi^{-1}A3.11\times10^{-2}\cos(\theta-\Theta-\tan^{-1}1.32).$$

For the energies chosen, $\Pi^{-2}=4.269\times10^{-2}$; and this is the relative intensity at infinity on the standard theory. Thus, the second term measures the interaction effect; and, numerically,

$$|C|^2=4.269\times10^{-2}\{1-0.15A\cos(\theta-\Theta-\tan^{-1}1.32)\}.$$

Thus, the effect produces at most a 10 percent change in the relative intensities at infinity, and with the more reasonable values $0.2>A>0.01$, the effect is less than 3 percent.

The above is for $|\mu|=1$. Next consider $\mu=0$. This means that there is no direct α-emission with energy E_n but that all the rays with this energy are due to the interaction. Then

$$|C|^2=4.4\times10^{-4}A^2.$$

Thus, the maximum relative intensity (of the lower energy particles to the greater) due to the interaction alone is about 4×10^{-4}; and more probable values corresponding to smaller values of A are 10^{-5} down to 10^{-8}.

We can also put $|\mu|=\infty$, i.e., no direct emission of the higher energy particles. Then the intensity of the higher energy particles is less than that of the lower energy ones and the ratio is

$$|1/C|^2=2.3\times10^{-2}A^2.$$

Thus, this effect is 100 times as great as the reverse; i.e., it is about 100 times more probable that the α-particle will absorb the energy of a nucleus left in an excited state than that it will excite a nucleus left in its ground state. Both these processes of course give weaker lines than the corresponding direct decay with $|\mu|=1$.

The case $|\mu|=\infty$ is the extreme limit of the forbidden ground-state transition suggested by Perlman, Ghiorso, and Seaborg. Their model for non-even–even nuclei would suggest values of $|\mu|$ between one and infinity. It is seen that in such cases the effect here studied might be of some importance. The numerical values above are for $E_0=5.30$ Mev, $E_0-E_n=250$ kev. From the calculations in P with $E_0-E_n=1.2$ Mev, we find that for $|\mu|=\infty$, $|1/C|^2\sim10^{-3}A^2$. It would be

interesting to repeat the calculations for energy difference in the region where perturbation methods really fail and a larger value of $|1/C|^2$ might occur. The value $E_0-E_n\sim 50$ kev should provide such a case and would be interesting in view of the two lines of Bi^{212}(ThC), which differ by 40 kev.[8] The greatest difficulty in such a calculation would be the numerical evaluation of I_{n0}^- and I_{0n}^-.

In addition to Bi^{212}, there are a number of other cases[1] where this effect might be of some importance. These are shown in Table I. It is difficult to separate the contributions of the various phenomena which affect the intensities. Usually, neither the angular momentum of the α-particle nor the degree of prohibition (value of μ) can be estimated with any certainty. (The penetrability of the coulomb barrier is, of course, the factor which fixes the order of magnitude of the intensities.) In any given experimental case, we would like the difference between the observed partial decay constant and that predicted by standard theory [Eq. (A.1)] to be apportioned among the three effects: angular momentum, "formation" prohibition, and noncentral electric interaction. At present, this does not seem possible, but there appears to be evidence that all three play some role.

TABLE I. Relatively forbidden transitions in which the electric interaction may be important. For Ra^{223}, the figures refer to lines I and II.

Nucleus	E_0-E_n kev	$\lvert C\rvert^{-2}$
U^{235}	160	0.11
Th^{229}	100	0.5
Ra^{223}	80	0.40
Bi^{212}	40	0.39
Bi^{214}	60	0.82

I am much indebted to Professor R. E. Peierls for stimulating discussion and to my wife for help with the rather heavy numerical calculations.

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PHYSICAL REVIEW VOLUME 82, NUMBER 4 MAY 15, 1951

Conductivity of Cold-Worked Metals*

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(Received November 10, 1950)

In the vicinity of an edge type dislocation, the density of electrons, and therefore the width of the filled portion of the conduction band is not uniform. To keep the electrons in equilibrium, an electrostatic potential is required. The scattering caused by this potential is used to calculate the increased resistance of isotropically cold-worked copper.

I. INTRODUCTION

COLD working produces an increase in the resistance of metals. This increase has been attributed to the formation of dislocations by Koehler.[1,2] Koehler's calculations have been refined by Mackenzie and Sondheimer,[3] who give a very elegant treatment of the problem. In particular, they calculated the change in resistance produced by cold-working, at high temperatures, whereas Koehler calculated the resistance arising from dislocations at absolute zero. These two quantities need not be equal.[4–6] Both treatments attribute a screened coulomb potential to each ion of the crystal. The perturbation that scatters the conduction electrons arises from the fact that these ions, and consequently their screened coulomb fields, are displaced from their normal lattice positions.

In the earlier treatments it is assumed, therefore, that the screening electrons which are associated with a particular ion stay with that ion as the latter is displaced from its position. No large scale redistribution of electronic charge is permitted.

Actually, however, the electrons can redistribute themselves. An edge dislocation contains an extra plane of ions, above or below the slip plane. Consider the case in which the extra plane is above the slip plane. If each ion were to keep its screening electrons, the Fermi energy above the slip plane would be larger than below, and, as a result, some electrons would go from the upper to the lower region. This in turn gives an unbalance of charge and an electric field. Equilibrium will be attained when the Fermi levels in the different regions have come together.[7] If the number of electrons which are displaced to establish the field is small, the effect of this change of electron density on the Fermi energy can be neglected. The purpose of this paper is to calculate the incremental resistance per dislocation due to this electrostatic field. It will turn out to be

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