Some New Atomic Mass Measurements and Remarks on the Mass Evidence for Magic Numbers*

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New mass measurements are given for the nuclei Ti^{48}, Sr^{86}, Sr^{88}, Mo^{98}, Sn^{117}, Sn^{120}, Pt^{194}, Pt^{196}, and Pb^{208}. The evidence given for the theory of magic numbers in nuclear structure by nuclear mass measurements is discussed.

INTRODUCTION

MASS spectrographic mass measurements made in this laboratory have been extended to include Ti^{48}, Sr^{86}, Sr^{88}, Mo^{98}, Sn^{117}, Sn^{120}, Pt^{194}, Pt^{196}, and Pb^{208}. These measurements are briefly described below.

I. Ti^{48}

With a spark between two titanium electrodes, doublets were photographed at mass 16, formed by singly-charged O^{16} and triply-charged Ti^{48}. From nine doublets, the packing fraction of Ti^{48} is $f=-7.49\pm0.04$. In this measurement the mass scale was provided by the $Ti^{47}-Ti^{48}$ separation, which is assumed to be 0.99711 ±0.00043, the value recently obtained by Harvey.[1] Previous measurements of the Ti^{48} packing fraction have been -6.99 ± 0.15 (Aston, 1938),[2] -7.22 ± 0.1 (Dempster, 1938),[3] -7.64 ± 0.10 (Okuda and Ogata, 1941),[4] and -7.60 ± 0.07 (Duckworth, 1942).[5]

II. MASSES OF Sr^{86} AND Sr^{88}

With a spark between a platinum electrode and an electrode consisting of a nickel tube packed with strontium chloride, doublets were photographed at mass 44, formed by singly-charged CO_2 and doubly-charged Sr^{88}. At the same time triplets were photographed at mass 43, formed by singly-charged C_3H_7 and C_2OH_3 and doubly-charged Sr^{86}. From four photographs of the CO_2-Sr^{88} doublet, the packing fraction difference is $\delta f=8.41\pm0.04$. From six photographs, the $C_3H_7-Sr^{86}$ packing fraction difference is $\delta f=23.48$ ±0.06, while from two photographs, the value for $C_2OH_3-Sr^{86}$ of $\delta f=14.89\pm0.09$. In all cases, the $Sr^{86}-Sr^{88}$ separation, assumed to be integral, served as the mass scale. Using Bainbridge's recommended values[6] for the masses of C^{12} and H^1, the packing fractions of CO_2, C_3H_7 and C_2OH_3 are computed to be 0.88 ± 0.01, 15.92 ± 0.015, and 7.46 ± 0.015, respectively. These values, when combined with the above packing fraction differences, give the packing fraction of Sr^{88} as $f=-7.53\pm0.05$, and that of Sr^{86} as $f=-7.56\pm0.06$, from the $C_3H_7-Sr^{86}$ comparison, and $f=-7.43\pm0.09$, from the $C_2OH_3-Sr^{86}$ comparison. The packing fraction of Sr^{88} has not been measured previously; the only previous measurement of Sr^{86} was made by Mattauch[7] in 1937 who obtained, from measurement of the $Si^{29}F_3-Sr^{86}$ doublet, the preliminary value of -9.1 ± 0.40.

III. MASSES OF Sn^{117} AND Sn^{120}

With a spark between a tin electrode and a platinum electrode, the $C_3H_3-Sn^{117}$ doublet was photographed at mass 39. From six photographs, the $C_3H_3-Sn^{117}$ packing fraction difference is $\delta f=14.17\pm0.04$. This result, combined with a packing fraction for C_3H_3 of 9.22 ± 0.015, gives, for Sn^{117}, the value of $f=-4.95$ ±0.04. No previous measurement has been made of the mass of this nuclide.

The $Ni^{60}-Sn^{120}$, $Ni^{61}-Sn^{122}$, and $Ni^{62}-Sn^{124}$ doublets were photographed at masses 60, 61, and 62, using a spark between a nickel and a tin electrode. From twelve photographs, the $Sn^{120}-Ni^{60}$ packing fraction difference is $\delta f=3.61\pm0.03$. This result, combined with a previous value[8] for Ni^{60} of $f=-8.60\pm0.05$, gives, for Sn^{120}, $f=-4.99\pm0.06$. From one photograph of the $Sn^{122}-Ni^{61}$ doublet, $\delta f=3.71\pm0.10$, and from four photographs of the $Sn^{124}-Ni^{62}$ doublet, $\delta f=4.23\pm0.05$.

On many of the plates on which the $Ni^{60}-Sn^{120}$ doublets were photographed, good $Ni^{58}-Sn^{116}$ doublets were present. From nine photographs, the $Sn^{116}-Ni^{58}$ packing fraction difference is $\delta f=2.76\pm0.02$. This doublet had been studied previously in this laboratory at which time the packing fraction difference was found[9] to be $\delta f=2.66\pm0.01$. Since the reason for this discrepancy is not known, the value of 2.71 ± 0.05 will be used in recomputing the mass of Sn^{116}

IV. MASSES OF Mo^{98}, Pt^{194}, AND Pt^{196}

With one electrode of platinum and the other of molybdenum, the $Pt^{194}-Mo^{97}$ and $Pt^{196}-Mo^{98}$ doublets

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† Now in the United States Navy.
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[7] J. Mattauch, Naturwiss. **25**, 170 (1937).
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[9] H. E. Duckworth and R. S. Preston, Phys. Rev. **79**, 402 (1950).

TABLE I. New mass values.

Nuclide	Packing fraction $\times 10^4$	Mass
Ti^{48}	-7.49 ± 0.04	47.96405 ± 0.00019
Sr^{86}	-7.52 ± 0.05	85.93533 ± 0.00043
Sr^{88}	-7.53 ± 0.06	87.93374 ± 0.00053
Mo^{98}	-6.52 ± 0.04	97.93610 ± 0.00040
Sn^{116}	-5.30 ± 0.06	115.93852 ± 0.00070
Sn^{117}	-4.95 ± 0.04	116.94208 ± 0.00047
Sn^{120}	-4.99 ± 0.06	119.94012 ± 0.00072
Pt^{194}	1.32 ± 0.07	$194.0256\ \pm0.0014$
Pt^{196}	1.40 ± 0.03	196.02744 ± 0.00060
Pb^{208}	2.03 ± 0.07	$208.0422\ \pm0.0015$

were photographed at mass numbers 97 and 98. From nine doublets, the $Pt^{194}-Mo^{97}$ packing fraction difference is $\delta f=7.78\pm0.02$, and from a similar number of $Pt^{196}-Mo^{98}$ doublets, $\delta f=7.92\pm0.03$. These doublets were measured by Dempster who found packing fraction differences of 7.7 ± 0.2 and 7.68 ± 0.2, respectively.[10] The $Mo^{97}(\gamma,n)Mo^{96}$ threshold has been measured by Hanson *et al.*[11] and found to be 7.10 ± 0.30 Mev. This result can be combined with a previously reported[8] mass value for Mo^{96} to obtain for Mo^{97} a packing fraction $f=-6.46\pm0.06$. The packing fraction of Pt^{194} can then be computed, from the above result, to be $f=+1.32\pm0.07$.

Harvey has found, from a study of the $Pt^{194}(d,p)Pt^{195}$ and $Pt^{195}(d,p)Pt^{196}$ reactions, that the $Pt^{195}-Pt^{194}$ and $Pt^{196}-Pt^{195}$ mass differences are 1.00239 ± 0.00021 and 1.00038 ± 0.00021, respectively.[1] These mass differences can be used, together with the above packing fraction for Pt^{194} and a previously reported one[12] for Pt^{195}, to deduce two independent values for the packing fraction of Pt^{196}. These are $f=+1.45\pm0.07$ and $f=+1.37\pm0.04$, respectively. Adopting a value for the packing fraction of Pt^{196} of $f=1.40\pm0.03$, one can compute, from the above $Pt^{196}-Mo^{98}$ result, that of Mo^{98} to be $f=-6.52\pm0.04$.

V. MASS OF Pb^{208}

With one palladium electrode and one lead electrode, the $Pb^{208}-Pd^{104}$ doublet was photographed at mass 104 with exposure times of 30 to 60 sec. The $Pd^{104}-Pd^{106}$ separation, assumed to be integral, was used as the mass scale. Measurements of seven doublets give $\delta f=8.15\pm0.04$. This result, combined with a previously reported[12] packing fraction for Pd^{104} of -6.12 ± 0.05, gives, for Pb^{208}, $f=2.03\pm0.07$. This doublet was studied previously by Dempster,[10] who found $\delta f=7.96\pm0.15$.

VI. SUMMARY OF RESULTS

These new mass values have been collected in Table I.

VII. DISCUSSION OF RESULTS

These mass values, plus some which have been reported earlier and others which can be computed from the mass spectrographic values by means of transmutation data,[12a] give some information concerning the effect on the atomic mass of some of the magic numbers. For this purpose, they are plotted in Fig. 1 on a packing fraction curve.

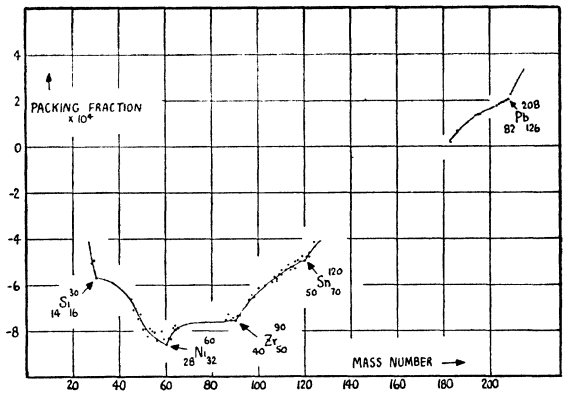

FIG. 1. A packing fraction curve representing recent atomic mass measurements.

It can be seen in Fig. 1 that there is a pronounced break in the packing fraction curve in the region of mass 90; this almost certainly marks the completion of the 50-neutron shell. It is likely that the break is located at $_{40}Zr^{90}$ and may indicate that the 40-proton configuration is making a significant contribution to the stability of this nuclide. It will be recalled that each of the recently proposed single-particle nuclear energy level schemes[13–16] predicts that 40 will be a magic number, marking the completion of the $3p$ shell. If it be true that $_{40}Zr^{90}$ is thus doubly magic, this break in the packing fraction curve corresponds to the well-known break occurring at the position of another doubly-magic nuclide, namely $_{82}Pb^{208}$.

In the 50 proton region, the packing fraction curve shows a slightly decreasing slope which may be testimony to the stabilizing influence of the 50-proton configuration. There is some evidence that the curve changes slope at $_{50}Sn^{120}$, a nuclide which is likely doubly magic according to the scheme of Maria Mayer, although not according to any of the others. Mention should also be made of $_{50}Sn^{114}$, $_{50}Sn^{116}$, and $_{50}Sn^{118}$, the first of which is doubly magic by the schemes of Mayer and Haxel *et al.*, the second of which is doubly magic by the scheme of Feenberg and is likely so according to Mayer, and the last of which is doubly magic according to the schemes of Nordheim and Haxel *et al.* The data in Fig. 1 suggest that $_{50}Sn^{116}$ possesses extra stability, that $_{50}Sn^{114}$ possibly does, but that $_{50}Sn^{118}$ does not.

The data shown in Fig. 1 may give some help in

[10] A. J. Dempster, Phys. Rev. **53**, 64 (1938).
[11] Hanson, Duffield, Knight, Diven, and Palevsky, Phys. Rev. **76**, 578 (1949).
[12] Duckworth, Woodcock, and Preston, Phys. Rev. **78**, 479 (1950).

[12a] The authors are grateful to Dr. B. B. Kinsey, Dr. J. A. Harvey, and to Dr. J. Halpern for making available to them prepublication data which were useful in computing many of the packing fractions plotted in Fig. 1.
[13] Maria Mayer, Phys. Rev. **75**, 1969 (1949); **78**, 17 (1950).
[14] E. Feenberg and K. C. Hammack, Phys. Rev. **75**, 1877 (1949).
[15] L. W. Nordheim, Phys. Rev. **75**, 1894 (1949).
[16] Haxel, Jensen, and Suess, Phys. Rev. **75**, 1766 (1949).

identifying magic numbers in the region $50<A<65$. The level scheme of Maria Mayer suggests that 28 and 32 are magic numbers in this region, whereas Haxel *et al.* suggest 28 and 34. Nordheim suggests 34, and Feenberg and Hammack hint that 32 and/or 34 may be magic. The minimum in the packing fraction curve in Fig. 1 is located at $_{28}Ni^{60}$.§ This coincides, according to the Mayer scheme, with a doubly-magic nuclide.

The heaviest stable isotope of silicon, $_{14}Si^{30}$, is of interest because of its exceptionally low packing fraction. Maria Mayer has shown that strong spin-orbit coupling can lead to a reversal of the $2s$, $3d_{5/2}$ level order with the result that 14 nucleons complete the $3d_{5/2}$ shell and constitute a particularly stable configuration. The 16-nucleon configuration, representing the completion of the $2s$ shell, should also be very stable. Thus, it may be that $_{14}Si^{30}$ owes its exceptional stability to a doubly-magic configuration. Its doubly-magic brother, $_{14}Si^{28}$, is somewhat less closely knit. The increase in stability resulting from an excess of neutrons over protons, which is so pronounced in the heavier nuclides, is presumably responsible for this difference.

It seems likely that additional mass measurements can be of considerable use in the identification of the ground states of nuclei, particularly in the case of even-even nuclei, where no information has so far been derived from spin measurements. In conclusion, one may venture to say that the mass evidence to date gives general support to the level scheme of Maria Mayer.

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§ *Note added in proof.*—Some recent experiments suggest that Ni^{62} may mark the minimum of the packing fraction curve. These experiments are being continued.

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Properties of Dirac Wave Functions in a Central Field*

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The boundary conditions for a Dirac particle in a central scalar field are discussed for both bound and continuum states. In this development, the methods employed are considerably different from those customarily used for the corresponding nonrelativistic case.

I. INTRODUCTION

WHILE the Dirac wave functions for a particle in a central field possess properties which are, in many instances, qualitatively similar to those exhibited by the corresponding solutions to the Schroedinger problem, there are several essential differences which have apparently not been considered very completely or stated explicitly in previous investigations. An important point of difference is concerned with the boundary conditions which, in the relativistic and nonrelativistic problems, must be discussed quite independently. Intimately connected with the formulation of boundary conditions is the question of which potential functions are admissible from a physical point of view. Here, the radically different answer provided by the relativistic problem is, in part, traceable to the energy doubling (existence of positive and negative energy states) and, in part, arises from the spin properties. A third point of interest is the study of the nodal properties (oscillation theorems, etc.) in the case of bound states. Despite the fact that the Dirac equations do not form a Sturm-Liouville system, several of the theorems concerning nodal properties are applicable.

The following is devoted primarily to a discussion of the three aforementioned problems: (1) boundary conditions, (2) admissible potentials, and (3) nodal properties. In connection with the study of nodal properties, we have found it necessary to develop methods somewhat different from those generally used in the treatment of Sturm-Liouville systems. Since these methods are also applicable to such systems, they may be of interest for classes of problems other than the one discussed here (Sec. VII).

The desirability of such a study was encountered in our program for computation of L-shell internal conversion coefficients, which required extensive numerical calculation of Dirac wave functions in a central noncoulomb field.[1] Some considerations which may be

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† Revised manuscript received January 18, 1951.

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