Calculation of Matrix Elements

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ANY line in a Feynman-Dyson[1,2] graph gives a factor in the integrand of the matrix element. If k denotes a linear combination of the internal energy-momenta over which we subsequently integrate, scalar bosons have (apart from constant factors) a propagation factor $(k^2+\kappa^2)^{-1}$; and fermions have a factor[3]

$$\left[-\kappa-(i\gamma/2)(\partial/\partial p)\int_{\kappa^2}^{\infty}d\sigma\right]\{(p-k)^2+\sigma\}^{-1}. \qquad (1)$$

p is either an external momentum or a "false external momentum" introduced when closed loops of fermion lines occur, and eventually equated to zero. A different symbol p is used for each line.

Thus, the internal momenta always occur as in a scalar boson factor (this can include scalar meson-photon interaction). These factors, combined by Feynman's methods, result in an integral of the type

$$\int_{-\infty}^{\infty}d^4k\cdots\int_{-\infty}^{\infty}d^4k^{(n)}[Q(k,k',\cdots,k^{(n)})], \qquad (2)$$

where Q is a quadratic function of the internal momenta. The integral is invariant under orthogonal transformations and simple translations of the k's with jacobian $+1$, and Q is reducible to diagonal form with positive coefficients. Evaluating this integral by Feynman's methods, and by using invariance properties, it follows that Eq. (2) is a constant multiple of $\Lambda^{r-2n-2}\chi^{2n-r}$, where χ is the discriminant of Q, and Λ is the discriminant of the purely quadratic part of Q; both are invariants. Λ is independent of p's and σ's and so commutes with the square bracket in Eq. (1). Therefore, we need simply consider the effect of a product of these brackets (perhaps interspersed by γ-matrices) on χ^{-m}, $(m=r-2n)$.

If a graph contains self-energy (s.e.) parts, Eq. (2) is divergent in some k variables; and following Ward,[4] we differentiate with respect to some σ in the s.e. parts. Then Eq. (2) converges and is a constant multiple of $\Lambda^{(r+1)-2n-2}\chi^{2n-(r+1)}$. The divergences appearing later, partly caused by reintegration with respect to σ, are all eliminated by subtraction procedure. Apart from subtraction, the same formula for Eq. (2) holds in every case.

χ is linear in the σ's and quadratic in the p's; $\partial\chi/\partial\sigma=\chi_\sigma$ is independent of p's and σ's, and the "derivative" of χ defined by $\chi_p=(2\chi_\sigma)^{-1}\partial\chi/\partial p$ is easily found. The "second derivative," $\chi_{pp'}=(4\chi_\sigma\chi_{\sigma'})^{-1}\partial^2\chi/\partial p\partial p'$, is a multiple of the unit (metric) tensor and has diagonal components, say, $\Omega_{pp'}$. Defining $\Omega_p=-\kappa+i\gamma\chi_p$, all matrix elements are expressible in terms of Ω_p, $\Omega_{pp'}$, χ, Λ, and γ-matrices.

For example, in considering a product of fermion operators occurring in Eq. (1), we use the formula

$$\left[-\kappa-(i\gamma/2)(\partial/\partial p)\int_{\kappa^2}^{\infty}d\sigma\right]F(\chi)\chi^{-l}$$
$$=[\Omega_p+I(\chi)(2\chi_\sigma)^{-1}\partial/\partial p](\chi^{-l};F(\chi)). \qquad (3)$$

In Eq. (3), $F(\chi)$ is a function of the derivatives of χ (all independent of the σ's); $I(\chi)$ is an operator defined by

$$I(\chi)\chi^{-l}=(1-l)^{-1}\left[\int_{\infty}^{\beta}d\alpha(\chi+\alpha)^{-l}\right]_{\beta=0}.$$

$I(\chi)$ operates on χ^{-l} and $\partial/\partial p$ operates on $F(\chi)$ only; the resulting factors are multiplied together. Hence, a term

$$\left[-\kappa-(i\gamma/2)(\partial/\partial p)\int_{\kappa^2}^{\infty}d\sigma\right]\gamma_\mu\left[-\kappa-(i\gamma/2)(\partial/\partial p')\int_{\kappa^2}^{\infty}d\sigma'\right]\gamma_\nu\cdots$$
$$\times\left[-\kappa-(i\gamma/2)(\partial/\partial p^{(n)})\int_{\kappa^2}^{\infty}d\sigma^{(n)}\right]\chi^{-m} \qquad (4)$$

is evaluated as follows. The "basic term" is

$$\Omega_p\gamma_\mu\Omega_{p'}\gamma_\nu\cdots\Omega_{p^{(n)}}\chi^{-m}. \qquad (5)$$

We can substitute for any pair $\Omega_{p^{(i)}}\cdots\Omega_{p^{(j)}}$ the terms $\gamma_\alpha\cdots\times\gamma_\alpha\Omega_{p^{(i)}p^{(j)}}I(\chi)$, retaining the original order of the γ-matrices. The value of Eq. (4) is Eq. (5) plus all possible terms derived by any number of these substitutions. Occurrence of several $I(\chi)$ implies repeated integration of χ^{-m}.

Difficulty arises when $I(\chi)\chi^{-1}$ and $I(\chi)I(\chi)\chi^{-1}$ appear. $I(\chi)\chi^{-1}$ terms enter in vertex (v.) and fermion self-energy (f.s.e.) parts, and $I(\chi)I(\chi)\chi^{-1}$ occurs in boson self-energy (b.s.e.) parts in electrodynamics. These are just the parts involving subtraction procedure. For s.e. parts, the earlier differentiation with respect to one σ in that part is balanced by the introduction of another factor $I(\chi)$. Subtractions, made successively, starting with the "innermost" divergence, are performed before applying $I(\chi)$, so that we have to subtract from terms containing only inverse powers of χ. If an unsubtracted term is $T=T(p,p',\cdots)$ and T_f is T with the p's put equal to free particle momenta, then, for v. and f.s.e. parts, the term after subtraction is $T-T_f$. For b.s.e. parts, we subtract the first *two* terms of the Taylor expansion of T. Subtraction eliminates all difficulties in interpreting $I(\chi)$. Both the original term and those subtracted are derived directly by the general method.

The problem of integration over Feynman auxiliary variables has not yet been undertaken.

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Distribution of Energy Loss of Electrons in Aluminum

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THE conversion electrons of Ba^{137} have been examined before and after passing through a thin foil (13.4 mg/cm² of Al) by means of a beta-spectrograph similar to that described by DuMond.[1,2] The results are shown in Fig. 1, where the ordinates

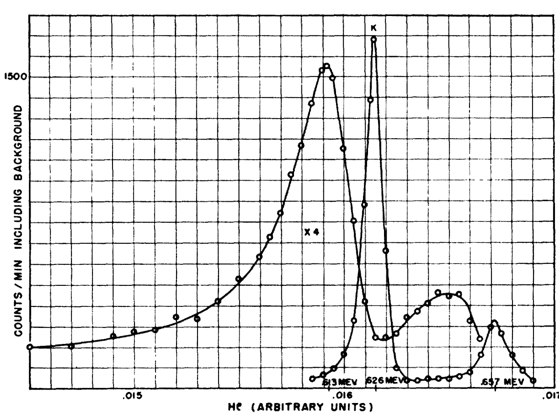

FIG. 1. Conversion lines of 0.663-Mev γ-ray from Ba^{137} with and without 13.4-mg/cm² aluminum absorber.

for the electrons which have traversed the foil have been multiplied by four. The separation of the K and L conversion lines and the large decrease in the number of counts at the maximum of the K line when a foil is introduced are indicative of the high resolution obtainable with this type of instrument. Calculations of the areas under the K lines with and without the foil indicate a loss of about 23 percent of the incident electrons due to scattering out of the beam.

Previous experiments have been characterized by a spread in incident energy of the same order as the spread due to straggling in the foil,[3-5] or by the use of a sharp beta-spectrum superimposed on a continuous beta-spectrum, or both.[6] It was the purpose of this experiment to make unnecessary the corrections which have had to be applied to these experiments in order to compare with energy loss theory.

In Fig. 2 the experimental counts less the background of 45 cpm have been plotted together with the theories of Landau[7]-Williams[8]

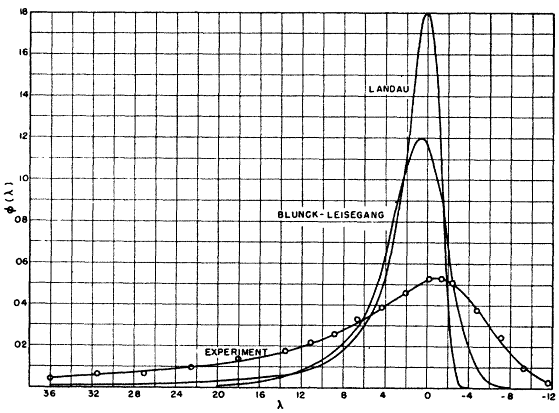

FIG. 2. Normalized energy loss distribution according to Landau, Blunck-Leisegang, and experiment.

and Blunck-Leisegang.[9] The variation in the most probable energy loss among the theories and the experiment of about 1.5 lambda-units is the same order as the experimental error and hence may or may not be significant. The width of the energy loss distribution is considerably greater than predicted by either theory, a result noted previously in the work of White and Millington[6] with mica foils. Here (in the notation of Landau[7]) λ is a dimensionless variable proportional to the energy loss, and $\Phi(\lambda)d\lambda$ is the normalized probability for an energy loss between λ and $\lambda+d\lambda$. The experimental curve has been normalized to unity in order to facilitate comparison with the theory. The electron energy before passing through the foil lies at $\lambda=-11.6$.

When completed, the spectrograph should have a resolution superior to that obtained here of about 0.5 percent without sacrificing the 1 percent of the solid angle from the source. Further experiments with the spectrograph and an electrostatic accelerating arrangement will permit measurement of electron straggling at any energy up to 2 Mev. A description of the spectrograph will appear in the literature in the near future.

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Radioactivity of Cerium

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A SEARCH for further isomers at the end of the 5th nuclear shell has led to a reinvestigation of the activities of $_{58}Ce_{79}{}^{137}$ and $_{58}Ce_{81}{}^{139}$. Within the experimental limitation of a lifetime greater than about a day, no isomeric activity was observed. This would be consistent with level trends discernible[1] from neighboring isomers of barium, xenon, and tellurium.

Normal and enriched samples of cerium isotopes were bombarded by neutrons in the piles at the Oak Ridge and Argonne National Laboratories. The compositions of the samples were:[2]

Sample 1: 0.19 percent (136), 0.25 percent (138), 88.48 percent (140), 11.07 percent (142);
Sample 2: 0.10 percent (136), 4.42 percent (138), 92.00 percent (140), 3.48 percent (142);
Sample 3: 8.94 percent (136), 0.81 percent (138), 84.98 percent (140), 5.27 percent (142).

All bombarded samples showed strongly the 30-day activity of Ce^{141}. A γ-transition of 145 ± 0.5 kev was found to be associated with this activity. The transition is converted in praseodymium, and the N_K/N_L ratio is approximately seven. These observations are in good agreement with the latest reported values of Freedman and Engelkemeir.[3]

Sample 2 showed lines in the electron spectrum from a 165.5 ±0.5-kev γ-transition, most probably converted in lanthanum. These lines, when compared with those of the 145-kev transition in Ce^{141}, decayed with a long half-life of between 100 and 400 days. They are undoubtedly to be attributed to the 140-day Ce^{139} activity discovered by Pool and Krisberg.[4] Owing to the weakness of the activity, arising from only a 4 percent abundance of the capture isotope Ce^{138}, an approximate value of $\geqslant 4$ could only be determined for the N_K/N_L ratio.

The electron spectrum from sample 3 exhibited lines from a 257 ± 1-kev γ-transition, decaying with a half-life of approximately 2 days. These lines were not evident in the spectra from samples 1 and 2; and, therefore, the 257-kev transition cannot be ascribed to the activity of Ce^{143}, of similar lifetime. It is rather to be identified with the 280-kev γ-transition of 36-hour Ce^{137} discovered by Chubbuck and Perlman.[5] The activity in the present experiments was again too weak to obtain either an accurate determination of the position of the L conversion line, for an assignment of its conversion atom, or an accurate measurement of the N_K/N_L ratio, which was estimated to be ~ 4.

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Radiations from I^{126}

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IODINE 126 is known to decay with a half-life of 13 days, emitting negative beta-particles and gamma-rays.[1] However, the yields reported for the reactions $I^{127}(\gamma, n)I^{126}$,[2] $Sb^{123}(\alpha, n)I^{126}$,[3] and $Bi^{209}(d, \text{fission})$,[4] as determined from the I^{126} beta-activity, seem low. An investigation of the decay scheme of I^{126} has therefore been undertaken, and preliminary results are reported here.

The I^{126} was produced at MIT by the reaction $I^{127}(n, 2n)I^{126}$, and it was concentrated by a Szilard-Chalmers separation from the irradiated solid potassium iodate. The activity, with some inactive iodine carrier, was purified and finally converted to aqueous I_2 solution which was placed over a copper foil. The activity deposited itself onto the foil as a result of the reaction $2Cu+I_2\rightarrow 2CuI$.[5]

Tellurium K x-rays were observed in the decay of I^{126} by use of a proportional counter[6] and pulse height analyzer.[7] The counter had a 107-mg/cm² beryllium window, and it was filled to three atmospheres pressure with a mixture of 97 percent krypton and 3 percent ethane. The counting efficiency for tellurium K x-rays was about 90 percent. Figure 1 shows a typical I^{126} pulse height