nuclear matrix elements due to the jump of the principal quantum number. As was pointed out by many authors, this does not contradict Mayer's shell model, but leads to some difficulties with Feenberg's model, with respect to the $3p$ level.

(2) A series of spectacular experiments started by Langer,[9] Wu,[10] and others, led to the discovery of many of "a"-type spectra concerned with the first-forbidden transitions. It may naturally be asked in this connection why no other type of spectrum associated with the first-forbidden formulas of Konopinski and Uhlenbeck,[11] for instance, with $\int \mathbf{r}$, has so far been discovered. We will try to indicate here that this question may be answered satisfactorily by the use of Mayer's shell model. The selection rule for the matrix element $\int \mathbf{r}$ is $\Delta J = \pm 1, 0$, parity change yes. However, in Mayer's level scheme, nuclei with the same principal quantum number have the same parity. Therefore, it naturally follows from this that, if parity changes in a β-transition, the principal quantum number will also jump. For example, transitions $(4f_{7/2} \leftrightarrow 3d_{3/2})$, $(2s_{1/2} \rightarrow 3p_{3/2})$, etc., in spectroscopic notation, are such cases. Of these, the transitions $(2s_{1/2} \rightarrow 3p_{3/2})$ are actually found to be less frequent than the transitions $(4f_{7/2} \leftrightarrow 3d_{3/2})$ in Mayer's shell model.[1] Moreover, for the case $\Delta J = \pm 1, 0$, $\int \boldsymbol{\alpha}$, which represents an allowed spectrum, may happen to mask the contribution of $\int \mathbf{r}$. This provides a basis for explanation of the fact that the case with $\int \mathbf{r}$ is far less frequent than the case of "a"-type. On the other hand, in the shell model where the parity and the principal quantum number may be taken independently, the explanation would require a more complicated basis.[5,6]

(3) Two divergent opinions have so far been advanced as regards the classification of the bulk of β-emitters with ft-values $10^5 \sim 10^6$; the one favoring their classification as first-forbidden, and the other proposing to classify them as allowed unfavored transitions (as seen in 1A in Table II of Konopinski).[12] If we try to make the calculation by the first-forbidden formula,[8] we will obtain considerably dispersed values for ft. However, a conventional calculation on the basis of the allowed formula gives more distinct values. If we do not regard this result as fortuitous, but take this as a result of failure to observe the seventh-power law, it follows, then, as stated by Wu[3] and also by Feenberg and Trigg,[4] that it is better to classify these transitions as allowed but unfavored.

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Convergence of the Chapman-Enskog Method for a Completely Ionized Gas*

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THE theory of transport phenomena in an ionized gas has recently been reconsidered by Cohen, Spitzer, and McR. Routly,[1] in an approximation which is specially adapted to inverse square forces. In this connection some results on the convergence of the Chapman-Enskog method which came up in the work reported in an earlier paper[2] may be of some interest. These results refer to the case $H=0$, where the matrix Δ of Eq. (34) is real. In this case the determinants can be evaluated with much less labor and the approximation was carried to 5×5 matrices, i.e., 2 steps beyond the approximation of reference 2. The results are given in Table I. The velocity distribution of electrons in presence of an electric field is, in the notation of reference 2, given by

TABLE I. Successive approximations of the determinant ratios.

i	$Z=1$	$Z=2$	$Z=3$	$Z=\infty$
		$\Delta_{00}{}^{(i)}/\Delta^{(i)}$		
2	1.9320	1.1590	0.8430	$3.25/Z$
3	1.9498	1.1605	0.8430	$3.3906/Z$
4	1.9616	1.1628	0.8435	$3.3945/Z$
5	1.9657	1.1634	0.8436	$3.3950/Z$
∞	—	—	—	$3.3953/Z$
		$\Delta_{01}{}^{(i)}/\Delta^{(i)}$		
2	0.6213	0.4393	0.3398	$1.5/Z$
3	0.5545	0.4201	0.3425	$2.0625/Z$
4	0.55983	0.4212	0.3429	$2.0391/Z$
5	0.5583	0.4207	0.3428	$2.0377/Z$
∞	—	—	—	$2.0372/Z$
		$\Delta_{11}{}^{(i)}/\Delta^{(i)}$		
2	0.4142	0.2929	0.2265	$1/Z$
3	0.6636	0.5433	0.4625	$3.25/Z$
4	0.6660	0.5439	0.4628	$3.3906/Z$
5	0.6665	0.5443	0.4630	$3.3945/Z$
∞	—	—	—	$3.3953/Z$

$$\phi = f\left(1 - \frac{e(\mathbf{v}\cdot\mathbf{E})}{\nu kT} \sum_{0}^{i-1} (-1)^r \frac{\Delta_{0r}{}^{(i)}}{\Delta^{(i)}} L_r\right).$$

For $Z=1$, one obtains

$$\Delta_{02}{}^{(3)}/\Delta^{(3)} = -0.0630, \quad \Delta_{02}{}^{(4)}/\Delta^{(4)} = -0.018365,$$

and

$$\Delta_{03}{}^{(4)}/\Delta^{(4)} = 0.039955.$$

Finally, I should like to point out some corrections to reference 2. In Eqs. (21) delete q in the third equation; in Eq. (30) replace β^2 by β^3; in Eq. (62) replace $\sqrt{2}$ by $(2\pi)^{\frac{1}{2}}$; in Eq. (65) replace $H_{rs}{}^2$ by $H_{rs}{}^i$.

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Elastic and Electromechanical Coupling Coefficients of Single-Crystal Barium Titanate

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SMALL single crystals of barium titanate have been available to us for some time through the work of Matthias.[1] Although suitable for qualitative studies, these crystals were too small for accurate measurements of elastic and piezoelectric constants. Through the cooperation of the Signal Corps Engineering Laboratory, we have obtained larger single crystals of barium titanate produced by the Harshaw Chemical Company under Signal Corps contracts.[2] These were large multi-domain single crystals, but by being polarized with a high electric field the domains could be oriented with their ferroelectric axes predominantly in the direction of the thickness of the crystal plate.

The elastic constants were measured by using the ultrasonic pulse method shown by Fig. 1. Longitudinal or transverse waves are generated or received in fused quartz rods by X- or Y-cut quartz crystals soldered to the rods. The sample is connected to the fused quartz rods by means of a quarter-wave thickness of polystyrene, which has a very low loss and the mechanical impedances of 0.45×10^5 and 0.165×10^5 mechanical ohms per cm² for longitudinal and shear waves, when connected to the fused quartz rods. The velocity and attenuation can be measured by observing the ratio between the incident and transmitted waves.

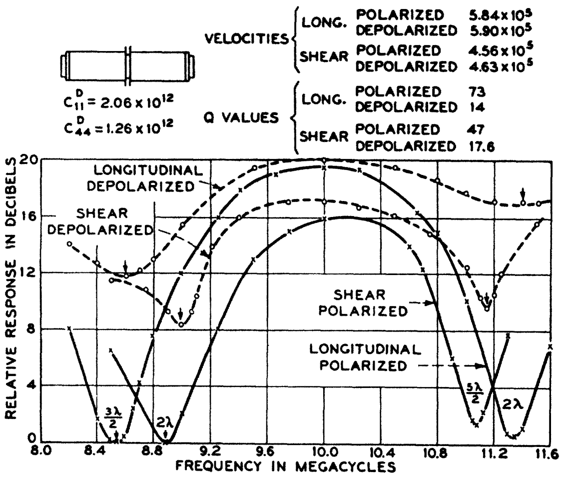

FIG. 1. Relative response as a function of frequency.

The transmitted waves are greatest at half-wavelength frequencies; and, by obtaining two of these, the velocity can be determined. If R is the ratio of the amplitude at odd quarter-wavelengths to the amplitude at half-wavelength frequencies, the attenuation can be calculated from the formula

$$\tanh Al=\left\{R\left(\frac{Z_s^2+Z_0^2}{2Z_sZ_0}\right)-1\right\}\Big/\left\{\frac{Z_s^2+Z_0^2}{2Z_sZ_0}-R\right\}, \qquad (1)$$

where Al is the attenuation in nepers, Bl the phase shift in radians, Z_s the sending and receiving impedances on either side of the sample, and Z_0 the characteristic impedance of the sample equal to the density ρ times the velocity. Figure 1 shows measurements for longitudinal and shear waves for a sample 1 cm square and 1.029 mm thick. The solid lines are for the sample polarized in the direction of the thickness, while the dotted curves are for the sample unpolarized. From these it is seen that the constant electric displacement elastic constants do not differ by more than 3 percent, which indicates that $c_{11}^D=c_{33}^D$ and $c_{44}^D=c_{66}^D$ within several percent. A much higher loss for the depolarized condition indicates a microhysteresis effect. The elastic constants are given by Fig. 1.

Measurements were also made of resonant and antiresonant frequencies of a series of orientations which were polarized along the thickness which lies along a cubic axis. The results are shown in Table I. From these we obtain the values of the constant field elastic constants:

$$\begin{aligned}&s_{11}^E=1.12\times10^{-12}\ \text{cm}^2/\text{dyne};\\&\tfrac{1}{2}s_{11}^E+\tfrac{1}{4}(2s_{12}^E+s_{66}^E)=0.69\times10^{-12};\\&c_{33}^E=1.13\times10^{12}\ \text{dynes/cm}^2.\end{aligned} \qquad (2)$$

For the square crystal the dielectric constant was 2890 at low frequencies, 2740 at frequencies below the thickness resonance, and 1500 above the resonance. From the relation between the dielectric constant above the resonance ϵ_{33}^S and the dielectric constant ϵ_{33}^T below the resonance, which can be written in the form

$$\epsilon_{33}^S/\epsilon_{33}^T=1-k^2, \qquad (3)$$

we find $k=0.67$, which is considerably higher than that found in a polarized ceramic of barium titanate. A similar relation between the elastic constants measured for constant displacement and constant field is

$$c_{33}^E/c_{33}^D=1-k^2 \qquad (4)$$

or $k=0.675$.

TABLE I. Resonant and antiresonant frequencies for various orientations.

Orientation	Dimensions in mm L	W	T	Resonant frequency	Antiresonant frequency	Electromechanical coupling
Length along a cubic axis	10.13	1.15	0.5	189,973	192,950	0.192
Length 45° from a cubic axis	9.90	1.02	0.5	247,650		
Square plate	10.0	10.0	1.0	2,170,000		

From the dielectric constant, the coupling constants 0.192 and 0.67 for the longitudinal length and thickness modes, respectively, and the elastic constants one can calculate the "effective" piezoelectric constants for this degree of poling as

$$\begin{aligned}d_{31}&=-310\times10^{-8}\ \text{stat coulombs/dyne},\\d_{33}&=+950\times10^{-8}\ \text{stat coulombs/dyne}.\end{aligned} \qquad (5)$$

The elastic constants for constant electric displacement are

$$\begin{aligned}c_{11}^D=2.06\pm0.05\times10^{12}\ \text{dynes/cm}^2;\quad c_{12}^D=1.40\pm0.2\times10^{12};\\c_{44}^D=1.26\pm0.05\times10^{12}.\end{aligned} \qquad (6)$$

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Nuclear Magnetic Resonance Absorption in $NaSbF_6$

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PROCTOR and Yu[1] have demonstrated an interesting fine structure in the nuclear magnetic resonance absorption line for Sb^{121} and Sb^{123} in an aqueous solution of sodium hexafluoantimonate and hydrofluoric acid. The structure consists of an equally spaced symmetrical set of seven lines whose over-all width of ~10 gauss is unusually great for a liquid resonance. The second moment for the Sb^{121} line, computed from their illustrated experimental curve, is 5.5 gauss². If the resonance had been obtained in a polycrystalline solid, a second moment of 15.3 gauss² would have been expected.[2,3] In solution the second moment should be less than this, since there should be random reorientation of the octahedral $[SbF_6]^-$ ion, with a frequency much greater than the frequency line width, unless the liquid has an unusually high viscosity. In fact, for equal probability of all spatial orientations, the line should be very narrow, since the average values of the terms $(3\cos^2\theta_{jk}-1)$, which occur in the interaction hamiltonian, are zero.[4]

A high viscosity could reduce the ionic reorientation frequency such that the line width is only partially reduced. It is, however, difficult to see why such a reduction caused by random reorientation should give the well-resolved fine structure. Moreover, it requires the viscosity to be $\sim2\times10^3$ poise.[5]

Suppose, however, that rapid ionic reorientation takes place about one axis only. For either uniform or random reorientation about a tetrad axis of the octahedron the second moment would be reduced to 7.2 gauss²; about a diad axis the value is 1.9 gauss², and about a triad axis it is zero. Rotation about one tetrad axis thus seems the most promising assumption. On this basis the line shape has been calculated by classical computation of the local field due to the six octahedral F^{19} nuclei at the central Sb^{121} nucleus; this procedure appears to be justifiable with this symmetry and motion for non-identical perturbing nuclei of spin $\frac{1}{2}$, since it gives the correct second moment for all ionic orientations. Calculation is made for the 2^6 equally probable configurations of the six F^{19} nuclei; for each the local field is averaged for random reorientation about one tetrad axis. Finally, the line shape shown in Fig. 1(a) is obtained for a randomly oriented assembly of ions and is seen to exhibit an equally spaced symmetrical set of nine lines. When multiplied by a broadening function, a curve is obtained whose derivative is shown in Fig. 1(b). This curve agrees