Letters to the Editor

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Adiabatic Oscillations in Liquid Helium*

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IF a container of liquid helium is connected by a very fine slit or a capillary with a helium bath, the meniscus in the container reaches the level of the bath eventually by way of very slow and very slightly damped oscillations. Allen and Misener,[1] who first observed this phenomenon, assumed that the frequency of the oscillations is determined by the restoring force of gravitation, $Ag\rho x$, where A is the area of the liquid surface in the container, ρ is the density of liquid helium, x is the level difference, and $g=980$ cm/sec². The potential energy is then

$$E_{\text{pot}}=\tfrac{1}{2}Ag\rho x^2.$$

To all intents and purposes the inertia of the liquid can be attributed entirely to the mass of the liquid moving in the capillary. Then, assuming that only the superfluid [velocity $v_s=(A/\sigma)(\rho/\rho_s)\dot{x}$] can pass through the capillary of length l and cross section σ, the kinetic energy is given by

$$E_{\text{kin}}=\tfrac{1}{2}l\sigma\rho_s v_s^2=\tfrac{1}{2}\rho^2 lA^2\dot{x}^2/\rho_s\sigma.$$

Hence, the system represents an harmonic oscillator of frequency

$$\omega_i=2\pi\nu_i=(\rho_s g\sigma/\rho lA)^{\frac{1}{2}}. \tag{1}$$

Atkins[2] has observed similar oscillations in a vessel connected with the bath by the Rollin film, and has used measurements of the frequency to determine the thickness of the film. In measuring these oscillations, particular care must be taken to avoid swamping them by the fountain effect. In Allen and Misener's and in Atkins' experiments, constant temperature was insured by a very good heat contact between container and bath.

If, however, the two containers are well isolated from each other, it is mainly the fountain effect and not gravitation which determines the frequency of the oscillations. We have the equations of motion[3] for the superfluid:

$$\langle\dot{V}_s\rangle+\frac{1}{l}\left\{gx+\frac{1}{\rho}(p-p_0)-\frac{\rho_n}{\rho}(S_n-S_s)(T-T_0)\right\}=0 \tag{2}$$

and for the normal fluid:

$$\langle\dot{V}_n\rangle+\frac{1}{l}\left\{gx+\frac{1}{\rho}(p-p_0)+\frac{\rho_s}{\rho}(S_n-S_s)(T-T_0)\right\}+\frac{8\pi\eta_n}{\rho_n\sigma}\langle V_n\rangle=0, \tag{3}$$

where p_0 and T_0 are the vapor pressure and temperature in the container, S_n and S_s are the entropies per gram of helium attributed to the normal and superfluid components, η_n is the viscosity of the normal fluid, and $\langle V_n\rangle$ the average of V_n over the cross section of the capillary. S_s is generally understood to be zero, or practically zero. We express the balance of entropy by

$$\rho_s\sigma(S-S_s)T_0\langle V_s-V_n\rangle+\rho A(h+x)C_p\dot{T}+\kappa(T-T_0)=0, \tag{4}$$

where C_p is the specific heat per gram, h is the distance from the bottom of the container to the bath level, and κ is an assumed heat leakage between bath and container. We will assume the capillary or slit so narrow that the normal fluid is immobilized by its viscosity. We may then put $\langle V_n\rangle=0$, $\rho_s\sigma\langle V_s\rangle=\rho A\dot{x}$, and dispense with Eq. (3), writing (2) and (4) as

$$(\rho Al\ddot{x}/\rho_s\sigma)+gx+(1/\rho)(p-p_0)-(S-S_s)(T-T_0)=0, \tag{2'}$$

$$\rho A\{T_0(S-S_s)\dot{x}+(h+x)C_p\dot{T}\}+\kappa(T-T_0)=0. \tag{4'}$$

The quantity $p-p_0$ can be expressed by the temperature difference and the derivative of the vapor pressure curve,

$$p-p_0=(T-T_0)(dp/dT)_{\text{vap}}.$$

For x small compared with h, (2′) and (4′) are linear, homogeneous equations of the form:

$$\ddot{x}+\omega_i^2x-\omega_i^2\alpha\tau=0, \tag{2''}$$

$$\dot{x}+\dot{\tau}+L\tau=0, \tag{4''}$$

where

$$\tau\equiv(T-T_0)hC_p/[T_0(S-S_s)].$$

$$\alpha\equiv\frac{T_0(S-S_s)^2}{ghC_p}\left\{1-\frac{1}{\rho(S-S_s)}\left(\frac{dp}{dT}\right)_{\text{vap}}\right\},$$

$$L\equiv\kappa/(\rho AhC_p).$$

Equations (2″) and (4″) have solutions of the form

$$x=B'e^{\lambda t},\quad \tau=B''e^{\lambda t}, \tag{5}$$

where λ is determined by the secular equation:

$$\lambda^3+L\lambda^2+\omega_i^2(1+\alpha)\lambda+L\omega_i^2=0. \tag{6}$$

For $L\rightarrow\infty$ (complete heat exchange), we have $\lambda=i\omega_i$. These are the isothermal oscillations mentioned above.

On the other hand, for $L=0$ (complete insulation), we have a much higher frequency:

$$\omega_a=\omega_i(1+\alpha)^{\frac{1}{2}}=\omega_0[\rho_s(1+\alpha)/\rho]^{\frac{1}{2}}, \tag{7}$$

where ω_0 is the frequency at the absolute zero. Furthermore, there is a static solution $\lambda=0$ and, from (2″), $x_0=\alpha\tau_0$ which is the well-known equilibrium condition for the fountain pressure. The general solutions are vibrations of frequency ω_a around an equilibrium position x_0 given by the temperature difference. For $1.3°<T<T_\lambda$ the entropy is roughly proportional to $T^{5.6}$, and a crude estimate gives $\omega_a/\omega_i=(82/h^{\frac{1}{2}})(T/T_\lambda)^{3.3}$. Figure 1 displays

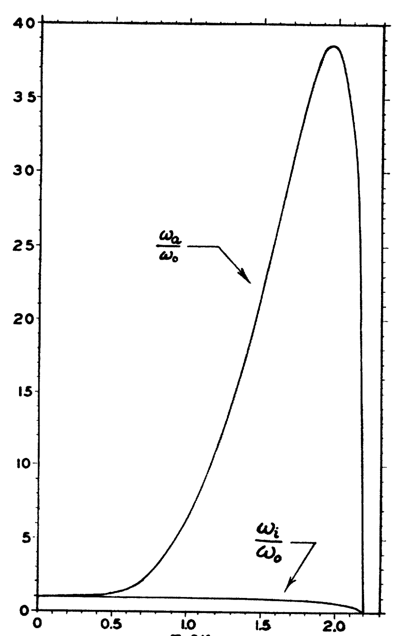

FIG. 1. Undamped adiabatic (ω_a) and isothermal (ω_i) oscillations, calculated for $h=1$ cm. Frequencies as functions of temperature.

the temperature dependence of ω_i/ω_0 and ω_a/ω_0. (In the calculation it was assumed that $S_s=0$ and $S\sim T^{5.6}$ over the entire range.)

Examination of the discriminant Δ of (6), given by

$$27\Delta=1-\tfrac{1}{4}\left\{1+18\left(\frac{\omega_i}{\omega_a}\right)^2-27\left(\frac{\omega_i}{\omega_a}\right)^4\right\}\left(\frac{L}{\omega_a}\right)^2+\left(\frac{\omega_i}{\omega_a}\right)^2\left(\frac{L}{\omega_a}\right)^4$$

reveals that for finite L and T, three distinct situations may obtain. For $L/\omega_a<2$ we have the damped adiabatic vibrations. For $L/\omega_a>\frac{1}{2}\omega_a/\omega_i$ we have the damped isothermal vibrations. Between these values, for $2<L/\omega_a<\frac{1}{2}\omega_a/\omega_i$, the three solutions of (6) are all aperiodic.

We substitute $\lambda=ia-b$, and equate real and imaginary parts of (6) to zero. The two equations so obtained yield an aperiodic solution $a=0$, and two more relations between a, b, and L from which we can derive a relationship between a and b and then calculate the dependence of each on L. The aperiodic solution can then be found by equating the constant term in (6) to the product of the roots:

$$\lambda_3=-L\omega_i^2/(\lambda_1\lambda_2)=-L\omega_i^2/(a_2^2+b_2^2).$$

In the periodic regions the general solution of (2″) and (4″) for the meniscus level x can then be written:

$$x=D'e^{-bt}\cos(at-\phi)+D''\exp[-L\omega_i^2t/(a_2^2+b_2^2)], \quad (8)$$

where D', D'', ϕ, are arbitrary constants. Hence, we obtain an aperiodic decrease of the average temperature and pressure differences between bath and container, plus the damped vibrations.

In Fig. 2 we show the solutions calculated for $T=2°K$ and $h=1$ cm, as functions of L/ω_a. The ordinates are in units of ω_a in and

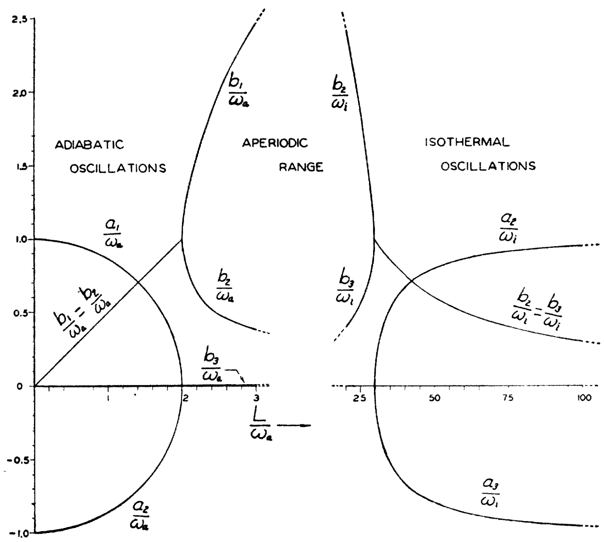

FIG. 2. Frequencies (a) and damping (b) as functions of the heat exchange L, calculated for $h=1$ cm, $T=2°K$. The aperiodic solution in the adiabatic range is $\lambda_3=-b_3$, and in the isothermal range is $\lambda_1=-b_1$.

adjacent to the adiabatic region, and in units of ω_i in and adjacent to the isothermal region. (For the scales employed, the aperiodic root, λ_3, is almost too small to be visible in the adiabatic range, and is too large in the isothermal range to be shown in the plot.) Figure 2 shows that if the adiabatic oscillations are to persist over several periods with only slight diminution in amplitude, we must have at most $L/\omega_a<0.1$. In this case we have

$$a=\omega_a\{1-\tfrac{1}{8}(L/\omega_a)^2\};\quad b=\tfrac{1}{2}L;\quad \lambda_3=-(\omega_i/\omega_a)^2L.$$

The vibrations die out more rapidly than the average meniscus level sinks.

To see whether the adiabatic oscillations are observable we refer to Keesom and Saris,[4] who found that for one of their experimental arrangements the total heat leakage from an insulated container (volume 1 cm³) to the bath was about 40 erg deg^{-1} sec^{-1} for $T<T_\lambda$. This would give roughly $L\approx 2\times10^{-4}T^{5.6}$, where we have taken $\rho C_p\approx 4\times10^{-3}T^{5.6}$ cal deg^{-1} sec^{-1} cm^{-3}. Accordingly, it should be possible to provide insulation which will reduce the damping sufficiently to permit observation of the adiabatic oscillations. This might provide a method of measuring the temperature dependence of the entropy, or rather of $(S-S_s)$, at lowest temperatures.

The author is indebted to Professor F. London for suggesting this question and for numerous discussions.

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[1] J. F. Allen and A. D. Misener, Proc. Roy. Soc. (London) **A172**, 467 (1939).
[2] K. R. Atkins, Proc. Roy. Soc. (London) **A203**, 119, 239 (1950).
[3] See, for instance, R. B. Dingle, Proc. Phys. Soc. (London) **A62**, 648 (1949).
[4] W. H. Keesom and B. Saris, Physica **7**, 241 (1940). See also P. H. Keesom, Physica **11**, 339 (1945).

ft-Values of β-Decay and the Shell Models

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STARTING from the classification of *ft*-values of β-decay, we[1] have recently analyzed the β-decay schemes and selection rules of β-rays by Mayer's shell model.[2] Our general conclusion agrees with the result of recent researches made by Wu[3] and also by Feenberg and Trigg.[4] As regards some of the β-decays, Mayer's theory gives a more simple explanation than those of Feenberg and Hammack[5] and of Nordheim.[6] We would like to lay special stress on the following three points:

(1) Fermi's formula for the allowed transition, which has been tentatively employed in calculating the *ft*-values of β-decay, is subject, in some cases of forbidden transitions, to errors ranging up to about one hundred. For instance, Feenberg and Hammack[5] held that in a group obeying the first-forbidden selection rule $\Delta J=\pm2$, parity change yes, respective *ft*-values are too great to be included in the first-forbidden category. However, our reexamination[7] by means of the correct forbidden formula of Nakamura, Shima, and Kobayashi[8] resulted in improvement by a factor of as much as sixty. It is interesting to note that the reexamination reveals that all of the above-mentioned group with "*a*"-type spectrum have almost the same *ft*-values (see Table I), and that their magnitudes are not unreasonably great for the first-forbidden transition if allowance is made for the reduction of

TABLE I. Improvement of *ft*-values by the first-forbidden formula:*

$$f(W_0)=\left(\frac{W_0^6}{630}-\frac{59W_0^4}{5040}-\frac{407W_0^2}{10080}-\frac{1}{630}\right)(W_0^2-1)^{\frac{1}{2}}+\left(\frac{W_0^3}{24}+\frac{W_0}{96}\right)\log[W_0+(W_0^2-1)^{\frac{1}{2}}]. \quad (1)$$

Elements of "*a*"-type	W_0 (mc^2)	t (sec)	*ft* uncorrected	*ft* corrected by Eq. (1)
Y^{90}	5.29	2.2×10^5	1.1×10^8	3×10^7
A^{39}	2.108	4.5×10^8	2.1×10^8	4×10^7
Kr^{85}	2.17	2.96×10^8	3.0×10^8	4×10^7
Cl^{38}	10.43	2.3×10^3	4.0×10^7	5×10^7
Sr^{89}	3.88	4.7×10^6	4.9×10^8	6×10^7
Sr^{90}	2.05	8×10^8	1.4×10^9	7×10^7
Y^{91}	4.01	5.3×10^6	5.4×10^8	8×10^7
Rb^{86}	4.6	1.36×10^6	3.8×10^8	8×10^7
K^{42}	8	6.0×10^4	1.1×10^8	1.1×10^8

* F. B. Shull and E. Feenberg, Phys. Rev. **75**, 1768 (1949), replaced it by $(W_0^2-1)f$ and found, as a mean value, $(W_0^2-1)ft\sim10^{10}$ for the "*a*"-type group.