The Absorption of Slow π^- Mesons in Deuterium

S. TAMOR,*†
University of Rochester, Rochester, New York
(Received October 5, 1950)

The possible mechanisms for absorption of π^- mesons in deuterium are considered. A phenomenological treatment of the simple "neutron absorption" enables one to calculate the competition between the processes independently of the value of the π^- nucleon coupling constant. It is found that various meson theories for the charged meson lead to markedly different predictions for the fraction of events leading to gamma-ray emission. The results are compared with experiment and lead to the conclusion that the charged meson is either pseudoscalar or pseudovector. Comparison with the $(\pi^0:\gamma)$ ratio from the absorption in hydrogen indicates that if the charged meson is pseudoscalar the neutral meson is also pseudoscalar.

I. INTRODUCTION

THE recent experiments on the absorption of π^- mesons in hydrogen[1] provide a very sensitive method of determining the π^0 rest mass. Furthermore, the $(\pi^0:\gamma)$ ratio enables one to calculate the strength of the π^0 nucleon coupling for any assumed theories of the charged and neutral meson.[2] The experiment does not, however, provide any clearcut information concerning the properties of the charged meson. It has been suggested[3] that an investigation of the absorption process in deuterium would lead to a method of determining the spin and parity of the charged meson.

There are three ways in which this process can go: "neutron absorption" (simple disintegration of the deuteron into two neutrons), radiative absorption (absorption accompanied by gamma-emission), and mesic absorption (absorption accompanied by emission of a neutral meson). In a completely field-theoretic calculation the lowest order in which the neutron absorption can take place is the third. The radiative and mesic processes can be treated as second-order processes, as in hydrogen. In this calculation the nucleon-nucleon interaction is treated phenomenologically, by means of an empirical potential. If this potential is included in the unperturbed nucleon hamiltonian, the neutron absorption can be treated as a first-order process in complete analogy with the photo-disintegration of the deuteron.[4] This method has the advantage that all three modes of absorption are of first-order in the π^- coupling constant, so that the ratios of the transition probabilities are independent of the strength of this coupling.

The observed decay of the neutral meson into two gamma-rays shows that the π^0 cannot have spin one. However, no such conclusion can be drawn for the charged meson. Calculations are, therefore, performed for neutral mesons of the type $S(S)$, $PS(PS)$, and $PS(PV)$, and for charged mesons of the type $S(S)$, $PS(PS)$, $PS(PV)$, $V(V)$, and $PV(PV)$. [$PS(PV)$ denotes pseudoscalar meson with pseudovector coupling, and similarly is for the others.] Gradient couplings for the S, V, and PV fields do not lead to any essentially new results.

II. THE METHOD OF CALCULATION

It has been shown by Wightman[5] that a meson passing through liquid hydrogen or deuterium will be slowed down, captured into a Bohr orbit, and cascade down to the K shell in much less time than the natural $\pi-\mu$-decay time. We, therefore, assume that all mesons are absorbed from the $1S$ state of the mesic deuterium atom. It is true that the $2S$ state is metastable so that an appreciable fraction of the mesons may be absorbed from this level. However, everything said about the $1S$ level also applies to all other S states except for a normalization factor.

Since the deuteron wave function is a ${}^3S_1+{}^3D_1$ state, the spatial wave function of the initial state has even parity. However, the exclusion principle applied to the final neutrons requires that the wave function be completely antisymmetric in their coordinates. This, combined with parity conservation, introduces selection rules which strongly affect the probability for neutron absorption.

The entire method of treatment is inherently nonrelativistic. The problem, therefore, resolves itself into two parts: first, to find the nonrelativistic approximation to the interaction operator for the process under consideration, and second, to evaluate the integral representing the overlap of the initial and final wave functions. If the interaction operator is odd, its nonrelativistic form is most easily found by application of the canonical transformation discussed by Foldy and Wouthuysen.[6] The exclusion principle is taken into account by explicitly antisymmetrizing the final neutron wave function. Since the deuteron wave function is completely symmetric with respect to interchange of

* AEC predoctoral fellow.
† Now at Oak Ridge National Laboratory, Oak Ridge, Tennessee.
[1] Panofsky, Aamodt, and York, Phys. Rev. **78**, 825 (1950).
[2] R. Marshak and A. Wightman, Phys. Rev. **76**, 114 (1949) and Marshak, Tamor, and Wightman, Phys. Rev. **80**, 765 (1950).
[3] B. Ferretti, "Report on the International Conference on Low Temperatures and Fundamental Particles," Cambridge (1946), p. 75.
[4] C. Marty and J. Prentki, J. phys. et rad. **10**, 156 (1949).
[5] A. Wightman, Phys. Rev. **77**, 521 (1950); and private communication.
[6] L. L. Foldy and S. A. Wouthuysen, Phys. Rev. **78**, 29 (1950).

the nucleon coordinates, only that part of the interaction which is antisymmetric in the nucleon coordinates leads to nonvanishing matrix elements. For the two-step processes of radiative and mesic absorption, it is important to use the relativistic interactions and only to pass to the Pauli approximation after the sum over intermediate states has been performed. Otherwise, transitions through states of negative energy, which are frequently important, will not be included. All matrix elements are calculated to the lowest nonvanishing order in the nucleon velocities and μ/M, where μ and M are the meson and nucleon masses respectively. Since the radius of the mesic Bohr orbit is much larger than the radius of the deuteron, the meson wave function may be replaced by its value at the origin, and terms involving the gradient of the meson wave function may be neglected.

(A) Neutron Absorption

The nonrelativistic forms of the $S(S)$ and $PV(PV)$ interactions may be written down immediately as ϕ and $\boldsymbol{\sigma}\cdot\boldsymbol{\phi}$ respectively; the Foldy-Wouthuysen transformation gives for the $PS(PV)$ and $V(V)$ cases

$$PS(PV)\ (\hbar/Mc)\boldsymbol{\sigma}\cdot\mathbf{p}\phi,\quad V(V)\ (\hbar/Mc)\boldsymbol{\phi}\cdot\mathbf{p},$$

where the vectors $\boldsymbol{\sigma}$ and $\mathbf{p}$ are respectively the nucleon spin and momentum operators, and

$$\phi=\frac{\hbar c}{(2E_l)^{\frac{1}{2}}}(a_l+b_l^*)\varphi_l\tau^-,\quad \boldsymbol{\phi}=\frac{\hbar c}{(2E_l)^{\frac{1}{2}}}\mathbf{e}(a_l+b_l^*)\varphi_l\tau^-.$$

Here, φ_l is the wave function of a meson in the l'th Bohr orbit, and $\mathbf{e}$ is a unit vector in the direction of polarization of the meson; for a meson at rest, the fourth component of the vector meson field is zero.

The matrix element for the absorption of a $PS(PS)$ meson is found by use of the equivalence theorem.[7] Dyson has pointed out, however, that the equivalence theorem must be modified in the presence of a phenomenological nucleon-nucleon interaction which has a charge exchange character. Application of his transformation to a system with a potential interaction V, having an arbitrary spin and charge dependence, leads to the modified equivalence relation (keeping only terms to first order in g)

$$\boldsymbol{\sigma}\cdot\nabla\phi-(1/c)\gamma_5(\partial\phi/\partial t)=2iM\beta\gamma_5\phi+\gamma_5[V,\phi].$$

The commutator vanishes when the potential V does not contain isotopic spin operators.

After antisymmetrizing the interactions, the matrix elements become (except for factors of i)

$$(4\pi)^{\frac{1}{2}}g[\hbar c/(2E)^{\frac{1}{2}}](\chi_f^*|Q|\chi_0)\varphi(0)\int\psi_0\psi_f d\tau,$$

where χ_0 and χ_f are the initial and final nucleon spin functions, ψ_0 and ψ_f are the spatial part of the initial and final wave functions, and the operator Q is given by

$S(S)$	$0,$
$PS(PV)$	$(\hbar/Mc)\frac{1}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{K},$
$V(V)$	$(\hbar/Mc)\mathbf{e}\cdot\mathbf{K},$
$PS(PV)$	$\frac{1}{2}(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\mathbf{e}.$

Here $\mathbf{K}$ denotes the final nucleon momentum in the center-of-mass system of the final neutrons (which in this case coincides with the laboratory system). If one assumes a nucleon-nucleon interaction of the form[8] $V(r)(1+P_M)/2$ (P_M is the Majorana operator), the $PS(PS)$ matrix element becomes

$$(4\pi)^{\frac{1}{2}}g\frac{\mu}{2M}\frac{\hbar}{Mc}\left(\chi_f^*\left|\frac{\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2}{2}\cdot\mathbf{K}\right|\chi_0\right)\varphi(0)\times\int\psi_0\left(1-\frac{V(r)}{\mu c^2}\right)\psi_f d\tau.$$

After squaring the matrix element, we find that the sums over initial and final spins proceed in exactly the same manner as in the photo-disintegration of the deuteron.[9] On multiplication by the density of final states, the transition probabilities per unit time are in units of $\frac{1}{3}(g^2/hc)N^2(\hbar/\mu c)^3\cdot c\cdot|\varphi(0)|^2$;

$S(S)$	$0,$
$PS(PS)$	$2\pi(\mu/M)^3(\hbar K/\mu c)^3|I'|^2$
$PS(PV)$	$8\pi(\mu/M)(\hbar K/\mu c)^3|I|^2,$
$V(V)$	$4\pi(\mu/M)(\hbar K/\mu c)^3|I|^2,$
$PV(PV)$	$12\pi(M/\mu)(\hbar K/\mu c)|I|^2;\quad (J=0),$

where

$$I=\int(u_0(r)/r)\psi_f(\mathbf{r})d\tau,$$

$$I'=\int(u_0(r)/r)(1-V(r)/\mu c^2)\psi_f(\mathbf{r})d\tau.$$

u_0 denotes the radial part of the deuteron wave function, and N is its normalization factor.

It should be noted that for mesons of spin one there are three possible values for the total angular momentum of the meson-deuteron system. For the case of pseudovector mesons the above matrix elements correspond to absorption from the state $J=0$ of the meson-deuteron system. Since the deuteron is partially in a ${}^3D_1$ state, neutron absorption from the state $J=2$ is also possible. This is calculated by replacing χ_0 by $2^{-\frac{1}{2}}S_{12}\chi_0$ (S_{12} is the tensor operator) and using the appropriate radial wave function for the ${}^3D_1$ state. Neutron absorption from the state $J=1$ is forbidden.[10]

[7] F. J. Dyson, Phys. Rev. **73**, 929 (1948); K. M. Case, Phys. Rev. **76**, 14 (1949).

[8] R. S. Christian and E. W. Hart, Phys. Rev. **77**, 441 (1950).

[9] W. Rarita and J. Schwinger, Phys. Rev. **59**, 436 (1941).

[10] The author is indebted to Dr. K. A. Brueckner for calling his attention to this point.

(B) Radiative Absorption

Both the radiative and mesic absorptions may be treated as second-order processes. Here, only the proton takes part in the interaction, since interactions of the neutron with the meson or electromagnetic fields can only occur in higher order. It is easy to see that if the depth of the nucleon-nucleon potential is small compared to Mc^2, the deuteron may be treated as a superposition of free Dirac wave functions with a momentum distribution corresponding to the fourier transform of the nonrelativistic deuteron wave function. For every value of the final relative momentum of the nucleons, $\mathbf{K}$, and of the γ-ray (or meson) momentum, $\mathbf{k}$, there corresponds a matrix element for the interaction which can be expressed as a function of $\mathbf{K}$ and $\mathbf{k}$. If this is a slowly varying function of $\mathbf{K}$, it is possible to write the matrix element for the absorption of the meson and emission of a quantum of momentum $\mathbf{k}$ as

$$\int d\mathbf{K} M(\mathbf{k}, \mathbf{K}) \int \psi_0(\mathbf{r}) e^{-i\frac{1}{2}(\mathbf{k}\cdot\mathbf{r})} \psi_f(\mathbf{r}) d\tau,$$

where M is the corresponding matrix element for a free proton, and ψ_f is the final neutron wave function which is asymptotically a plane wave representing a relative momentum $\mathbf{K}$. The magnitudes of $\mathbf{K}$ and $\mathbf{k}$ are related to one another through the requirement of energy conservation.

Thus, the problem is again separated into two parts; the determination of the nonrelativistic matrix element and the evaluation of an overlap integral. The matrix element $M(\mathbf{k}, \mathbf{K})$ is calculated in the same way as in the hydrogen case except that the proton can no longer be considered to be initially at rest. The calculation is easily performed using either the standard methods of perturbation theory or the new techniques of Feynman.

In this calculation the anomalous magnetic moment of the nucleon is treated phenomenologically by adding an additional interaction between the nucleon and the radiation field.[11] A completely field-theoretic calculation would require consideration of higher order processes involving the virtual emission and absorption of mesons. In view of the fact that, at present, meson theory does not give quantitatively the correct nucleon moments, and since higher order terms would defeat the purpose of the present calculation by introducing higher powers of g, we simply add this anomalous interaction to the ordinary spin moment of the nucleon.

After summing over intermediate states and passing to the Pauli approximation, we find for the matrix elements for the absorption of a meson with the emission of a gamma-ray of wave number $\mathbf{k}$,

$$2\pi e g \frac{\hbar c}{(2E)^{\frac{1}{2}}(2\hbar\omega)^{\frac{1}{2}}} \frac{\hbar}{Mc} (\chi_f^* | Q | \chi_0) \int \psi_f e^{-i\frac{1}{2}(\mathbf{k}\cdot\mathbf{r})} \psi_0 d\tau,$$

where Q is given by

$S(S)$	$\Gamma(\hbar/\mu c)\boldsymbol{\sigma}_1\cdot\mathbf{k}\times\boldsymbol{\varepsilon}$,
$PS(PS)$	$\boldsymbol{\sigma}_1\cdot\boldsymbol{\varepsilon}$,
$PS(PV)$	$(2M/\mu)\boldsymbol{\sigma}_1\cdot\boldsymbol{\varepsilon}$,
$V(V)$	$(M/\mu)\mathbf{e}\cdot\boldsymbol{\varepsilon}+i\boldsymbol{\sigma}_1\cdot\mathbf{e}\times\boldsymbol{\varepsilon}$,
$PV(PV)$	$(M/\mu)\boldsymbol{\sigma}_1\cdot\boldsymbol{\varepsilon}(\mathbf{k}\cdot\mathbf{e})/\lvert k\rvert$.

Here, Γ is the difference of the neutron and proton magnetic moments in units of the ordinary Dirac moment ($\Gamma=4.7$), and $\boldsymbol{\varepsilon}$ is a unit vector in the direction of polarization of the gamma-ray.

The probabilities for transitions to the singlet state are, in units of $\frac{1}{3}(g^2/\hbar c)N^2(\hbar/\mu c)^3 c|\varphi(0)|^2|I_s(k)|^2$,

$S(S)$	$2\Gamma^2(\mu/M)(\hbar/\mu c)^5 Kk^3dk$,
$PS(PS)$	$2(\mu/M)(\hbar/\mu c)^3 Kkdk$,
$PS(PV)$	$8(M/\mu)(\hbar/\mu c)^3 Kkdk$,
$V(V)$	$(4\mu/3M)(\hbar/\mu c)^3 Kkdk$,
$PV(PV)$	$(2M/\mu)(\hbar/\mu c)^5 Kk^3dk$, $(J=1)$

where

$$I_s = \int (u_0(r)/r) e^{-i\frac{1}{2}(\mathbf{k}\cdot\mathbf{r})} \psi_s(\mathbf{r}) d\tau,$$

and ψ_s is the symmetric part of the final neutron wave function.

Similarly, if we replace I_s by I_t, where the neutron wave function is now antisymmetric in space, the transition probabilities to triplet states are, in the same units,

$S(S)$	$4\Gamma^2(\mu/M)(\hbar/\mu c)^5 Kk^3dk$,
$PS(PS)$	$4(\mu/M)(\hbar/\mu c)^3 Kkdk$,
$PS(PV)$	$16(M/\mu)(\hbar/\mu c)^3 Kkdk$,
$V(V)$	$2(M/\mu)(\hbar/\mu c)^3 Kkdk$,
$PV(PV)$	$2(M/\mu)(\hbar/\mu c)^5 Kk^3dk$. $(J=0, 2)$

Since the deuteron is a loosely bound system, the most probable transitions are those in which the gamma-ray carries off most of the available energy. Therefore, the final state is predominantly S, and the most important transitions (except in the vector case) are those to the singlet state.

(C) Mesic Absorption

The calculation of the probability for mesic absorption proceeds in exactly the same way as for the radiative case. Since there are so many possible combinations of π^0 and π^- meson theories, we shall not list all the matrix elements but simply discuss the important features of the interactions.

As a first approximation, the matrix elements for mesic absorption in deuterium should be the same as those for hydrogen, except for the operation of the exclusion principle. The antisymmetrization in most cases simply introduces a factor of $\frac{1}{3}$ arising from the averaging over initial spins. The only exceptions occur when both mesons are scalar or both pseudoscalar. In these cases, transitions to singlet states are impossible so that the P parts of the final nucleon and meson wave

[11] W. Pauli, *Handbuch der Physik*, Vol. 24/1, p. 233.

functions contribute to the integral. This reduces the probabilities for these processes by a large factor. In the other cases, one would expect the $(\pi^0:\gamma)$ ratios to be about the same as in the hydrogen case. However, the binding energy of the deuteron uses up a considerable fraction of the $(\pi^- - \pi^0)$ mass difference so that the volume of phase space available to the meson is diminished. This has the effect of decreasing all the mesic absorption probabilities by factors of 10 to 100.

III. RESULTS

The various integrals were evaluated using the deuteron wave function $u_0(r)=N(e^{-\gamma r}-e^{-\beta r})$ corresponding to the Hulthén potential $(\gamma^2-\beta^2)/(e^{(\beta-\gamma)r}-1)$. This potential closely approximates the Yukawa potential but has the advantage of permitting an analytic solution of the wave equation. The parameters γ and β are determined from the effective range and scattering length of the $n-p$ interaction.[12] In the case of neutron absorption, the energy available in the center of mass system is 138 Mev so that the final neutron wave function is very nearly a plane wave. However, in the radiative and mesic processes the neutrons carry off very little energy, and the distortion of the wave function due to mutual interaction may be large. The S part of the wave function may be approximated at low energies by

$$(1-e^{-\eta r})\sin(Kr+\delta)/Kr,$$

where the parameter η is determined from the effective singlet range of the $n-n$ interaction. The parameters for the $n-n$ interaction are obtained by assuming that this interaction is the same as the nuclear part of the $p-p$ interaction. The phase shift, δ, is obtained from

$$K\cot\delta=-\alpha+\tfrac{1}{2}Kr_0^2,$$

where r_0 is the effective range of the singlet interaction, and α is the reciprocal scattering length. These approximations are valid throughout the range of energies available for the mesic absorption process. For radiative absorption the approximation breaks down for nucleon energies much over 10 Mev, but this is already well beyond the peak in the gamma-ray spectrum. At these energies the distortion of the P state wave function may be neglected.

The integrals have been evaluated using a meson mass of $277m_e$, a deuteron binding energy of 2.24 Mev, and a neutron-proton mass difference [13] of 1.31 Mev. For the triplet interaction an effective range of $1.60\cdot 10^{-13}$ cm was used, while for the singlet interaction $r_0=2.43\cdot 10^{-13}$ cm, $\alpha=-0.421\cdot 10^{12}$ cm^{-1}, and $\eta=12.8\cdot 10^{12}$ cm^{-1}. In Table I are listed the transition probabilities for neutron absorption, and the integrated radiative absorption probabilities. The energy spectrum of the gamma-rays from the absorption of pseudoscalar mesons is shown in Fig. 1. The shapes of the spectra arising from scalar and pseudovector mesons are almost indistinguishable from this, because k is essentially constant over the width of the maximum. The singlet probabilities were also calculated using undistorted wave functions for the final state. The effect of ignoring the neutron-neutron interaction is to increase the half-width of the maximum from something less than 2 Mev to about 12 Mev, but the integrated transition probabilities are relatively unaffected.

TABLE I. Transition probabilities for neutron and radiative absorption in units of $\frac{1}{3}(g^2/\hbar c)N^2(\hbar/\mu c)^3 c|\varphi(0)|^2=0.91\cdot 10^{16}$ sec^{-1}.

Meson theory	$S(S)$	$PS(PS)$	$PS(PV)$	$V(V)$	$PV(PV)$
Neutron absorption	0	$2.7\cdot 10^{-3}$	0.25	0.12	$2.5(J=0)$ $0\ (J=1)$ $0.15(J=2)$
Radiative	0.012	$6.6\cdot 10^{-4}$	0.12	$2.2\cdot 10^{-3}$	$2.10\cdot 10^{-2}(J=1)$ $2.4\cdot 10^{-3}(J=0, 2)$
Ratio	0	4.1	2.1	55	2

The probabilities for mesic absorption are given in Table II. The integrals were evaluated for $(\pi^- - \pi^0)$ mass differences of 4.25 and 4.75 Mev.[1,14] These numbers were all obtained using the distorted wave function for the S wave. The effect of ignoring the $n-n$ interaction is to decrease the transition probability by a factor of 5 to 20.

These results indicate that the relative probabilities for the three absorption mechanisms depend strongly upon the assumed spin and parity of the charged meson.

(A) Scalar Mesons

Here, all absorption leads to either gamma-rays or neutral mesons. If the π^0 is also scalar, no neutral mesons should be found, but the assumption of a pseudoscalar neutral meson leads to a probability for

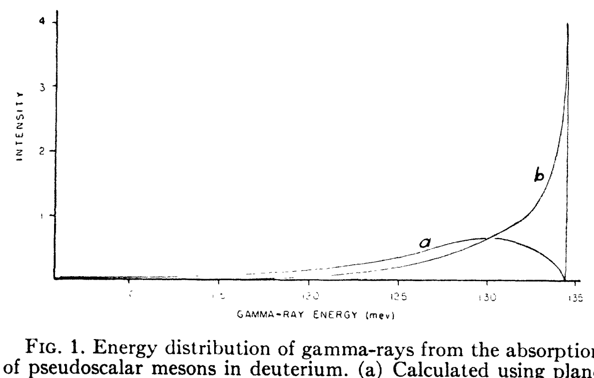

FIG. 1. Energy distribution of gamma-rays from the absorption of pseudoscalar mesons in deuterium. (a) Calculated using plane waves for final neutrons. (b) Calculated using distorted wave for S part of neutron wave function.

[12] J. M. Blatt and J. D. Jackson, Phys. Rev. 76, 18 (1949); H. A. Bethe and C. Longmire, Phys. Rev. 77, 647 (1950). Where possible, the notation of these papers is used here.

[13] R. Bell and L. Elliot, Phys. Rev. 74, 1552 (1948).

[14] W. Panofsky, private communication. According to the newest data, the $(\pi^- - \pi^0)$ mass difference is closer to 6 Mev. This will have the effect of increasing the probabilities for mesic absorption by about a factor of 2 compared to those given for a mass difference of 4.75 Mev. None of the qualitative features of the results are changed however.

TABLE II. Transition probabilities for mesic absorption for various meson theories in units of $g^2/hc \cdot 0.91 \cdot 10^{16}$ sec^{-1}. $\Delta\mu$ is the $(\pi^- - \pi^0)$ mass difference. In the last column are given the appropriate π^0 coupling constants and their values as determined from the absorption of mesons in hydrogen (reference 2). The notation here is the same as in ref. 2; $\Delta g = g_n - g_p$, $\bar{g} = g_n + g_p$. The actual probability is the product of column 3 or 4 with column 5.

π^-	π^0	$W(\Delta\mu=4.25$ Mev)	$W(\Delta\mu=4.75$ Mev)	Coupling constant
$S(S)$	$S(S)$	$1.0 \cdot 10^{-5}$	$6.8 \cdot 10^{-5}$	$(\Delta g)^2 = 0.008$
$PS(PS)$	$PS(PS)$	$0.59 \cdot 10^{-7}$	$3.9 \cdot 10^{-7}$	$(\bar{g})^2 = 0.06$
$PS(PV)$	$PS(PV)$	$0.59 \cdot 10^{-7}$	$3.9 \cdot 10^{-7}$	$(\bar{g})^2 = 11$
$PS(PS)$	$PS(PV)$	$0.59 \cdot 10^{-7}$	$3.9 \cdot 10^{-7}$	$(\Delta g)^2 = 0.06$
$PS(PV)$	$PS(PS)$	$0.59 \cdot 10^{-7}$	$3.9 \cdot 10^{-7}$	$(\Delta g)^2 = 11$
$S(S)$	$PS(PS)$	$1.1 \cdot 10^{-5}$	$3.9 \cdot 10^{-5}$	$(\Delta g)^2 = 27$
$S(S)$	$PS(PV)$	$1.9 \cdot 10^{-3}$	$6.8 \cdot 10^{-3}$	$(\Delta g)^2 = 0.15$
$PS(PS)$	$S(S)$	$0.62 \cdot 10^{-7}$	$2.2 \cdot 10^{-7}$	$(\Delta g)^2 = 210$
$PS(PV)$	$S(S)$	$1.1 \cdot 10^{-5}$	$3.9 \cdot 10^{-5}$	$(2g_p)^2 = 210$
$V(V)$	$S(S)$	$1.8 \cdot 10^{-5}$	$6.9 \cdot 10^{-5}$	$(\Delta g)^2 = 52$[a]
$V(V)$	$PS(PS)$	$2.0 \cdot 10^{-3}$	$3.6 \cdot 10^{-3}$	$(\Delta g)^2 = 0.93$
$V(V)$	$PS(PV)$	$2.0 \cdot 10^{-3}$	$3.6 \cdot 10^{-3}$	$(\bar{g})^2 = 0.93$
$PV(PV)$	$S(S)$	0.35	0.64	$(\Delta g)^2 = 0.005$
$PV(PV)$	$PS(PS)$	$0.73 \cdot 10^{-5}$	$2.6 \cdot 10^{-5}$	$(\bar{g})^2 = 26$[b]
$PV(PV)$	$PS(PV)$	$1.3 \cdot 10^{-3}$	$4.5 \cdot 10^{-3}$	$(\bar{g})^2 = 0.15$[b]

[a] This assumes a symmetric theory. A neutral theory applied to the hydrogen experiment gives the same π^0 coupling constant but in deuterium gives a zero transition probability in this approximation.

[b] Here, a neutral theory is assumed. A symmetric theory for the hydrogen experiment gives coupling constants half as large but gives zero for the deuteron absorption.

mesic absorption of about 10 percent. There are two effects which tend to invalidate the selection rule against neutron absorption of scalar mesons. The first is the possibility of absorption from states of higher angular momentum, especially from the $2P$ state. The absorption probability from the $2P$ state has been calculated and is $(g^2/hc)1.71 \cdot 10^{10}$ sec^{-1}. This is to be compared with the probability of radiative transitions to the $1S$ state. The probability for the $2P-1S$ transition is given by [15]

$$W_{2P-1S} = c(\mu c/h)(e^2/hc)^5(\tfrac{2}{3})^8$$

and is $1.75 \cdot 10^{11}$ sec^{-1}. Therefore, in order for an appreciable fraction of the mesons to be absorbed before reaching the ground state, the π^- coupling constant must be about 10. For higher P states it is easy to show that the ratio of the absorption probability to that for radiative transitions to lower states is essentially independent of n. For states of higher angular momentum the absorption probability falls off approximately as $1/(2l)!\,(r/a_0)^2$, where r is the radius of the deuteron, and a_0 is the mesic Bohr radius. Assuming a value of $\frac{1}{3}$ for the scalar meson coupling constant, not more than 4 percent of the mesons are absorbed before reaching either the $1S$ or the $2S$ state.

The other effect tending to invalidate the selection rule is the presence of relativistic terms in a correct field theoretic calculation. The relativistic matrix element for the third-order process is easily calculated, and again turns out to be identically zero. (This result was first obtained by A. S. Wightman.) We can conclude, therefore, that if the charged meson is scalar, the vast majority of absorptions would lead to gamma-ray emission, with about 10 percent neutral mesons if the π^0 is pseudoscalar.

[15] H. A. Bethe, *Handbuch der Physik*, Vol. 24/1, p. 440.

(B) Pseudoscalar Mesons

In this case the neutron and radiative processes compete. The $PS(PS)$ case gives a ratio of neutron to radiative absorption of 4.1 to 1, or 20 percent gamma-emission, while the $PS(PV)$ theory gives a ratio 2.1 to 1, or 32 percent gamma-emission. If the equivalence theorem were valid, both would give a ratio of 2.1 to 1. The assumption of a pseudoscalar neutral meson gives no observable number of mesic absorptions. However, the probability of emission of a scalar π^0 is about 10 percent of the radiative.

The neutron absorption probability in the $PS(PV)$ case was also calculated using a square well for the $n-p$ interaction. This leads to a neutron radiative ratio of 1.4 to 1 or a probability for gamma-emission of 42 percent. (The probability for radiative absorption is quite insensitive to the shape of the potential.) If the charged meson is pseudoscalar, we would, therefore, expect to observe a high energy gamma-ray in 20 to 40 percent of the events. A pseudoscalar neutral meson would not be observed, but a scalar meson should be detectable

(C) Vector Mesons

The absorption of a vector meson should lead to no observable number of either gamma-rays or neutral mesons.

(D) Pseudovector Mesons

Here, the crucial effect is not the relative rates of the neutron and radiative absorptions but the relative populations of the states $J=0, 1, 2$. If one assumes them to be populated according to their statistical weights, the ratio of neutron to radiative absorption is 2 to 1. Strictly speaking, however, the populations of these states depend on the previous history of the meson during its cascade to the ground state. To see how much difference this makes, one can assume that in some higher energy level the degenerate states are populated according to their statistical weights (this should be true for very high levels) and then trace the meson down to the ground state (or $2S$ state). This was done for several initial energy levels, and in no case was a significant deviation from the statistical distribution found.

Radiative absorption can, indeed, take place from the states $J=0$ and 2, but this matrix element brings down additional powers of the nucleon momentum and, therefore, leads to a smaller transition probability than for $J=1$. The absorption probability for $J=2$ was calculated by numerical integration using the D state wave function given by Rarita and Schwinger.[9] While the result is admittedly very approximate, it does show that from the state $J=2$ the predominant process is neutron absorption.

The number given for the mesic absorption probability also represents absorption from the $J=0$ state and is much smaller than the neutron probability. For states of higher J, additional powers of the meson and nucleon momenta appear which make the probability for mesic absorption extremely small. We may, therefore, say that no neutral mesons should be observed, regardless of their parity.

IV. CONCLUSIONS

Preliminary results of the experiment of Panofsky, Aamodt, and Hadley on the absorption of π^- mesons in deuterium[14] indicate a gamma-ray spectrum with a large peak in the neighborhood of 130 Mev, while there is no detectable peak in the spectrum at 70 Mev where the decay quanta from the neutral meson would appear. They estimate that of the total number of absorption events, about $\frac{1}{3}$ actually give rise to gamma-rays. These results are inconsistent with the assumption of either a scalar or vector character for the charged meson but do fit with both the pseudoscalar and pseudovector theories. If the charged meson is assumed to be pseudoscalar, the absence of neutral mesons indicates that they too are pseudoscalar. If, on the other hand, the charged meson is assumed to be pseudovector, nothing can be said about the parity of the neutral meson field.

The author wishes to express his appreciation to Professor R. E. Marshak for having suggested this problem and for his guidance throughout the course of the work. He also wishes to thank Dr. A. S. Wightman for a number of helpful conversations and Professor W. Panofsky for having communicated his new results on the absorption in hydrogen and deuterium.

Note added in proof:—Owing to failure to correct for the $n-p$ mass difference, all energies in Fig. 1 are too high by 1.3 Mev.

PHYSICAL REVIEW VOLUME 82, NUMBER 1 APRIL 1, 1951

Energy Levels of B^{10}

FAY AJZENBERG
University of Wisconsin, Madison, Wisconsin*
(Received December 14, 1950)

The neutron spectrum at 0° and at 80° from the $Be^9(d,n)B^{10}$ reaction has been investigated by means of nuclear emulsions. Energy levels of B^{10} at 0.77, 1.79, 2.22, 3.59, 4.79, 5.12, 5.91, 6.11, 6.57, and 6.81 Mev have been observed as have possible levels at 5.58, 5.68, and 6.38 Mev. There is evidence for multiplicities at 5.12 and, possibly, at 6.11 Mev. No neutron groups were observed which would correspond to levels at 1.4 and 2.85 Mev. This result is in disagreement with the picture of equally spaced levels in B^{10}. An attempt is made to explain the results that led to this picture.

I. INTRODUCTION

In the past few years, much attention has been devoted to the energy levels of B^{10}. The low-lying levels with excitation energies (E_x) of less than 6 Mev have been investigated by means of the reactions $Be^9(d,n)B^{10}$ and $Li^7(\alpha,n)B^{10}$, while the high energy levels ($E_x \geq 6.5$ Mev) have been studied by bombarding Be^9 with protons. Studies of the neutron spectrum by observation of proton recoils in Wilson cloud chambers[1,2] or in nuclear emulsions[3,4] have indicated the presence of levels at approximately 0.7, 2.2, 3.5, and 5.1 Mev above the ground state of B^{10}. Haxel and Stuhlinger[5] have detected neutrons from the $Li^7(\alpha,n)B^{10}$ reaction by means of a boron counter. This work indicated levels at approximately 0.8, 1.3, and 2.1 Mev. Bonner *et al.*,[6] using the method of neutron thresholds to study the neutrons from the $Be^9(d,n)B^{10}$ reaction, have investigated the range from 5 to 6 Mev. They have located levels[7] at 5.098, 5.156, and 5.920 Mev. Rasmussen, Hornyak, Lauritsen, and Chao[8,9] have studied the gamma-rays from the same reaction by means of a magnetic lens spectrometer. They have attributed to B^{10*} gamma-rays of energies 413.5, 716.6, 1022, 1433, 2151, 2871, 3604, 3970, 4470, and 5200 kev, and have assigned corresponding levels to all but the 3970 and the 4470-kev gamma-rays which they assumed to be due to cascade transitions. Rasmussen *et al.*[8,10] state that the levels corresponding to the 716.6, 1433, 2151, 2871, and 3604-kev gamma-rays appear to constitute an equally spaced set of levels leading to the picture of a harmonic oscillator.

* Work supported by the AEC and the Wisconsin Alumni Research Foundation.

[1] T. W. Bonner and W. M. Brubaker, Phys. Rev. **50**, 308 (1936).
[2] H. Staub and W. E. Stephens, Phys. Rev. **55**, 131 (1939).
[3] C. F. Powell, Proc. Roy. Soc. (London) **A181**, 344 (1942).
[4] W. D. Whitehead and C. E. Mandeville, Phys. Rev. **77**, 732 (1950).
[5] O. Haxel and E. Stuhlinger, Z. Physik **114**, 178 (1939).
[6] Bonner, Butler, and Risser, Phys. Rev. **79**, 240 (1950).
[7] T. W. Bonner, private communication.
[8] Rasmussen, Hornyak, and Lauritsen, Phys. Rev. **76**, 581 (1949).
[9] Chao, Lauritsen, and Rasmussen, Phys. Rev. **76**, 582 (1949).
[10] Hornyak, Lauritsen, Morrison, and Fowler, Revs. Modern Phys. **22**, 291 (1950).