Equations (22) and (29) are solved numerically for an argon particle in argon using, for $b^{e\prime}/b^{\nu\prime}$, only the first terms in the series (23) and (30), with $\beta_1'=0.24$ and $\gamma_1'=0.13$. The value for β_1' is that obtained from Eq. (27) under the assumption that $\beta_1'/v_2'=\beta_2/v_2$. One sees that there is introduced a discontinuity in $b^{e\prime}/b^{\nu\prime}$ at v_2', which might be expected to be smoothed out by higher terms in the series, were they included. It is found that $\chi'=0.78$ at $v'=v_2'/\sqrt{2}$, 0.72 at v_2', 0.53 at $2v_2'$, 0.40 at $3v_2'$, etc., and that $\chi'\rightarrow 12(v_2'/v')^2$ as $v'\rightarrow\infty$, corresponding to an ionization defect of 780 kev for an argon particle of very high velocity. The asymptotic value $v'^2\chi'$ is very closely the mean of the values obtained from the upper and lower bounds in Eq. (32).

For the average light fission fragment, we take $\beta_1=0.31$ and $\gamma_1=0.10$, and for the average heavy fission fragment, we take $\beta_1=0.38$ and $\gamma_1=0.10$, other coefficients in the series expressions being put equal to zero. We then obtain, in argon, an ionization defect of 2.5 Mev for a light fragment of energy 98 Mev and an ionization defect of 4.2 Mev for a heavy fragment of energy[12] 67 Mev. These quantities are rather insensitive to the the behavior of b^e/b^ν for velocities below v_2, since the contribution of the second integral of Eq. (33) is much larger than that of the first. However, a behavior of $b^{e\prime}/b^{\nu\prime}$ which is radically different than that assumed would lead to quite different values for the ionization defects of both argon and the fission fragments.

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A Rapid Method of Calculating log(ft) Values for β-Transitions

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This paper contains several graphs and nomographs which make it possible to obtain, very quickly, log(ft) values for most β-decays. The use of these figures is discussed.

I. INTRODUCTION

BETA-DECAYS can be divided into several classes of allowedness and forbiddenness according to their $\log_{10}(ft)$ values.[1,2] In conjunction with Gamow-Teller selection rules, such a classification agrees, in nearly all cases, with predictions from the nuclear shell model.[2-4] This paper contains several graphs and nomographs which make it possible to obtain, very quickly, log(ft) values for most β^+ emissions, β^- emissions, and K-captures.

Log(ft) can be written as the sum of three additive terms. The first term, $\log(f_0t)$, is the value for a β-decay if the effect of the coulomb field is ignored and if there is no branching. The second term, $\log(C)$, is the coulomb correction term. The third term, $\Delta\log(ft)$, appears if there is branching. In this paper all logarithms are taken to the base 10.

II. CONSTRUCTION OF THE FIGURES

(a) Construction of Nomograph for log(f_0t)

f_0 for β^+ emission is given by the formulas:

$$f_0=\int_1^{W_0} W(W^2-1)^{\frac{1}{2}}(W_0-W)^2dW,$$

and

$$f_0=[(W_0^4/30)-(3/20)W_0^2-(2/15)][(W_0^2-1)]^{\frac{1}{2}}+(2.302/4)W_0\log[W_0+(W_0^2-1)^{\frac{1}{2}}],$$

where W_0 is the maximum energy of the β-particles, including rest mass, in units of mc^2.

For K-capture, f_0 is given by $f_0=(W_0+E_K)^2$. Here E_K is the binding energy of a K-electron, which, of course, depends on the atomic number of the decaying nucleus. f_0 depends only on W_0 for $\beta^\pm$ emission. For K-capture, one can write for f_0, instead of the above expression:

$$f_0=W_0^2.$$

The error introduced by ignoring E_K in the expression for f_0 is negligible except for large Z and small W_0, as discussed in Sec. III. Here t is the half-life in seconds for all modes of decay.

The nomograph of Fig. 1, for getting $\log f_0t$ values, consists of three columns; a, b, and c. Column a contains two sets of entries of energies E_0. For $\beta^\pm$ emission, E_0 is the maximum energy of the β-particles in Mev, not including the rest mass, and the entries on the right side apply. For K-capture, E_0 is the decay energy in Mev, and the entries on the left side apply. The spacing of energies on Column a is not uniform but is proportional to $\log(f_0)$. Column b gives t in seconds, days, etc. The spacing is uniform in $\log(t)$. Column b is constructed exactly halfway between Columns a and c. Column c

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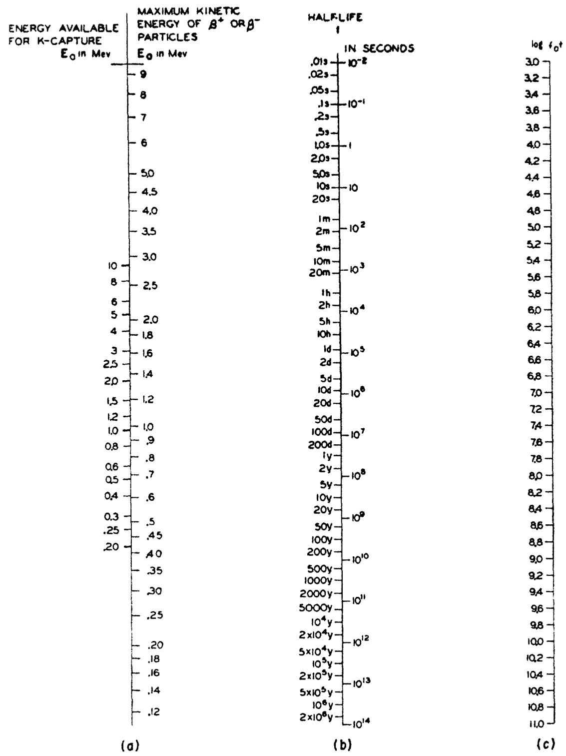

FIG. 1. Log(f_0t) as a function of E_0 and t.

contains log(f_0t), spaced uniformly. These spacings are such that for equal geometrical intervals,

$$d(\log f_0t)=\tfrac{1}{2}d(\log t)=-d(\log f_0),$$

where $d(X)$ denotes the change in X. It is then possible to get log(f_0t) by drawing a straight line through the proper values of E_0 (Column a) and t (Column b), noting its crossing point with Column c.

(b) Construction of Graphs for log(C)

Log(C) is the coulomb correction term, dependent upon W_0, and Z, the the atomic number of the initial nucleus, and is different for β^- emission, β^+ emission, or K-capture. For β^- or β^+ emission, it is a relatively slowly varying function of W_0, and Z given by

$$C=\int_1^{W_0}F(Z',W)W(W^2-1)^{\frac{1}{2}}(W_0-W)^2dW/f_0.$$

$F(Z', W)$, the density of electrons at the edge of the nucleus, is given by

$$F(Z',W)=\frac{1+S}{2}\frac{4}{|\Gamma(2S+1)|^2}[2(W^2-1)^{\frac{1}{2}}R]^{2S-2}\times\exp\left(\frac{\pi\alpha Z'W}{(W^2-1)^{\frac{1}{2}}}\right)\left|\Gamma\left(S+\frac{i\alpha Z'W}{(W^2-1)^{\frac{1}{2}}}\right)\right|^2,$$

where $Z'=\mp Z+1$ for $\beta^{\pm}$ emission, $S=(1-\alpha^2Z'^2)^{\frac{1}{2}}$, α = fine structure constant = 1/137, R = nuclear radius in units of $\hbar/mc=0.0039A^{\frac{1}{3}}$, and Γ = the gamma-function. Figures 2(a) and 2(b) show log(C) as a function of maximum energy for various values of Z, for β^- and β^+ emission.

The calculation of log(C) was done as follows. In the expression for C given above, the factor $F(Z', W)$ can be replaced by $a(W-1)^b$, where a and b depend only slightly on W, and more strongly on Z'. This is apparent from graphs for $F(Z', W)$, which are not shown here. To get C for a given value of Z and W_0, it is necessary in principle, to integrate $F(Z', W)$, multiplied by the weighting factor $W(W^2-1)^{\frac{1}{2}}(W_0-W)^2$ over the energy. C is equal to the resulting integral divided by

$$f_0=\int_1^{W_0}W(W^2-1)^{\frac{1}{2}}(W_0-W)^2dW.$$

The weighting factor has a maximum for $W-1\sim0.3(W_0-1)$ and, in fact, most of the contribution to the integral comes from values of $W-1$ between

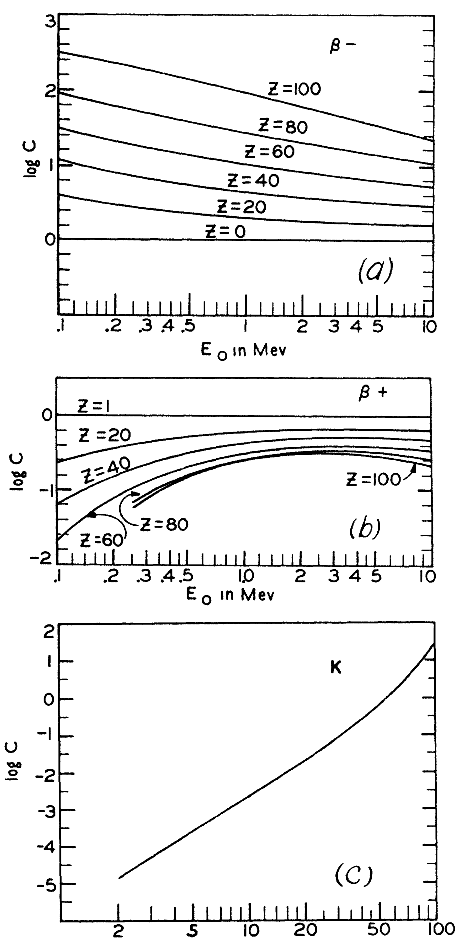

FIG. 2. (a) Log(C) as a function of E_0 and Z for β^- emission. (b) Log(C) as a function of E_0 and Z for β^+ emission. (c) Log(C) as a function of Z for K-capture.

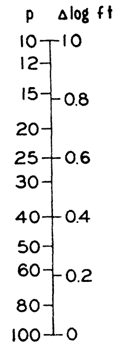

FIG. 3. $\Delta\log(ft)$ as a function of p.

$0.2(W_0-1)$ and $0.5(W_0-1)$, which is a relatively small range of values. It is therefore permissible to approximate $F(Z', W)$ by $a(W-1)^b$, where a and b are constants read off the graph for F. The constants should be read off at $Z'=\mp Z+1$ for $\beta^{\pm}$ emission and in the neighborhood of $W=1+0.3(W_0-1)$.

It is not desirable to carry out the integration explicitly, since the integrals depend so strongly on W_0, and the integration can be performed analytically only for $W_0-1\ll 1$ and for $W_0\gg 1$. Instead, $C(Z, W_0)$ may be written as $F(Z', W_{Av})$, where W_{Av} is some average value of the energy. If W_{Av} is written as $1+r(W_0-1)$, using the definition for C given above with $F(Z', W)=a(W-1)^b$, it is seen that r is given as a function of b and W_0:

$$r=\left[\frac{\int_1^{W_0}(W-1)^bW(W^2-1)^{\frac{1}{2}}(W_0-W)^2dW}{(W_0-1)^bf_0}\right]^{1/b}.$$

r can be calculated analytically in the limit $W_0-1\ll 1$ as well as for $W_0\gg 1$. r depends only slightly on W_0 in contrast to the integrals separately. Therefore, in the intermediate case of $W_0-1\sim 1$, r can be obtained quite accurately by interpolation. Then $C(Z, W_0)$ can be read off from the graph for F using the value of r obtained here.

For K-capture, with 2 electrons in the K-shell initially, C is given by

$$C=2\pi(\alpha Z_K)^3(2\alpha Z_KR)^{2S_0-2}\frac{2+E_K}{\Gamma(2S_0+1)}e^{-2\alpha Z_KR},$$

where $Z_K=Z-0.35$, $S_0=(1-\alpha^2Z_K^2)^{\frac{1}{2}}$, $E_K=-\alpha^2Z_K^2/2$. α, R and Γ are defined as for $\beta^{\pm}$ emission. Log(C) as a function of Z for K-captures is shown in Fig. 2(c).

(c) Construction of Table for the Branching Correction Term, $\Delta\log(ft)$

If there are several modes of decay occurring simultaneously with a half-life t, and the percentage p of total decay occurring in the mode under consideration is known, the term $\Delta\log(ft)$ appears and is given by

$$\Delta\log(ft)=2-\log p.$$

TABLE I. The function $E_0'(Z)$.

Z	E_0' (Mev)
60	0.18
70	0.24
80	0.32
90	0.40
100	0.50

This is shown in Fig. 3. If only one mode of decay occurs, $p=100$ and $\Delta\log ft=0$.

III. RANGE OF USEFULNESS OF THE FIGURES

The figures can be used for $\beta^{\pm}$ maximum energy of 120 kev to 9 Mev, and give results accurate to within 0.1 of the true value of $\log(ft)$. For K-captures, results to within 0.2 are obtained for energies of 200 kev to 10 Mev, with certain qualifications which are due to the approximations for f_0 made in Sec. II (a). Let $E_0'(Z)$ be the value of E_0 below which $\log(ft)$ for K-capture, as obtained from the figures, deviates by more than 0.2 from the correct value. Table I shows E_0' for various values of Z.

The branching correction obtainable from Fig. 3 is not restricted to values of $p>10$, since if p is replaced by $10^{-n}p$, $\Delta\log(ft)$ is replaced by $n+\Delta\log(ft)$.

IV. USE OF THE FIGURES

For a given type decay, given energy, half-life, etc., the figures of this article permit calculation of $\log(ft)$. Here the following notation is used: E_0 for $\beta^{\pm}$ emission is the maximum kinetic energy of the particles in Mev; E_0 for K-capture is the decay energy in Mev.

When a β^+ emission and K-capture go from and to the same level; E_0 for K capture $=E_0$ for β^+ emission $+1.02$ Mev; t is the total half-life in seconds (s), minutes (m), hours (h), days (d), or years (y); Z is the atomic number of the initial nucleus; and, p is the percentage of decay occurring in the mode under consideration. When no branching occurs, $p=100$.

V. PROCEDURE FOR OBTAINING LOG(FT)

(1) To obtain $\log(f_0t)$, use Fig. 1. E_0 is read off the left-hand side of the E_0 column for K-capture, and off the right-hand side for $\beta^{\pm}$ emission. Put a straight edge over the given values of E_0 and t and note where it crosses the column of $\log(f_0t)$ values.

(2) Read off $\log(C)$ from Figs. 2(a), 2(b), and 2(c), for β^- emission, β^+ emission, and K-capture, respectively.

(3) Get $\Delta\log(ft)$ from Fig. 3 if $p<100$. When $p=100$, $\Delta\log(ft)=0$.

(4) $\text{Log}(ft)=\log(f_0t)+\log(C)+\Delta\log(ft)$.

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