On the Ionization Yields of Heavy Particles

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Since the energy loss of a slow heavy particle is due predominantly to recoiling atoms, ionization by secondary heavy particles contributes a large fraction of the total ionization resulting from a slow heavy particle that is stopped in a gas. If the secondary heavy particle ionization efficiency is low, the over-all efficiency for the production of ion pairs is greatly reduced for low energies of the primary particle. The secondary heavy particle ionization efficiency satisfies an integro-differential equation in which the ratio of electronic to atomic stopping cross sections plays a critical role. Data by Madsen on the ionization by recoiling particles in alpha-decay are used to set limits on the ionization by slow argon particles in argon, from which it is possible to make crude estimates of the energy of fission fragments which fails to give rise to ionization at the usual rate. In argon, ionization defects of the order of several Mev are to be expected.

I. INTRODUCTION

A PARTICLE in its passage through a gas produces direct excitation and ionization of the atoms of the gas and also gives rise to recoil atoms. A fast heavy particle ($v \gg v_0 = e^2/\hbar$) loses energy primarily to electrons. Ion pairs are produced at a rate per unit energy loss ($-dI/dE$) which is very close to that for alpha-particles, for the energy loss per ion pair ω is largely independent of mass, charge, and velocity of fast particles of fixed charge and the process of capture and loss of electrons contributes relatively little to the energy loss and ionization processes when the average particle charge is large. For high velocities, ω tends to approach a value ω^e which depends on the nature of the gas and is practically independent of the nature of the particle. When the velocity of the particle falls below a velocity of the order of v_0, energy loss to recoiling atoms predominates because of the neutralization of the particle through the capture of electrons. Ionization then proceeds through the quasi-adiabatic interpenetration of the electron clouds of the particle and the gas atoms and by means of secondary ionization arising from the recoil atoms. If the secondary ionization efficiency is low, the over-all efficiency for the production of ion pairs is greatly reduced for low energies of the primary particle. A heavy particle of velocity v_0 has an energy of the order of millions of electron volts (a proton of this velocity has an energy of 25 kev). Hence, the energy which fails to give rise to ionization at the high velocity rate $1/\omega^e$ can be rather large unless the secondary ionization is highly efficient, as might be expected for light gases, or there is a marked increase in the efficiency of primary ionization, which seems unlikely for ordinary gases.

II. IONIZATION EFFICIENCY

We write for the energy loss of the particle per unit length of path

$$-dE/dx = N[b^e + b^\nu], \tag{1}$$

where N is the number of atoms per unit volume of the gas, and b^e and b^ν are the energy stopping cross sections per atom for the loss of energy to electrons (inelastic collisions) and to recoiling atoms (elastic collisions), respectively, such a distinction being possible to a high degree of approximation. Similarly, for the ionization per unit length of path

$$-dI/dx = N\left[\sigma^e + \int_0^{E_M'} dE'\sigma(E, E')I'(E')\right], \tag{2}$$

where σ^e is the cross section for the production of ion pairs (including ionization produced by ejected electrons) in the collision of the particle with a gas atom, and $\sigma(E, E')$ is the cross section per unit energy range for the production of a recoil atom of energy E' in such a collision. The quantity $I(E)$ is the total ionization, primary and secondary, resulting from the stopping in the gas of the particle of energy E; and $I'(E')$ is the total ionization resulting from the stopping of a gas particle of energy E' in the same gas.[1] All secondary ionization is associated with that part of the path of the primary particle from which the secondary particles, whether electrons or atoms, producing the ionization originate, although some of the ion pairs may be widely removed therefrom because of the motion of the secondary particles. The maximum energy transferred in elastic collisions is

$$E_M' = 4MM'E/(M+M')^2, \tag{3}$$

where M and M' are the masses of the particle and gas atom, respectively. On combining the above relations, we obtain

$$dI/dE = \left[\sigma^e + \int_0^{E_M'} dE'\sigma(E, E')I'(E')\right] \Big/ [b^e + b^\nu] \tag{4}$$

for the ionization, arising from both primary and secondary processes, per unit energy loss of the primary particle.*

[1] In this discussion all the gas atoms are to be assumed identical.

* *Note added in proof.* A treatment based on the statistics of ionization gives integral equations for I and I' from which Eq. (4) follows at high energies. The expressions derived from the statistical treatment have minor differences with those obtained from the simpler arguments of this paper.

The ionization efficiency is conveniently defined as $\eta=\omega^e I/E$. At very low energies all collisions are practically adiabatic and the ionization efficiency is zero. It is seen from the definition of the constant ω^e that the ionization efficiency approaches unity at high energies.

We introduce the two functions of the energy

$$\mu=\omega^e\sigma^e/(b^e+b^\nu), \quad \text{and} \quad \lambda=b^\nu/(b^e+b^\nu), \tag{5}$$

the second of which is less than unity by definition. It follows from Eq. (4) that the ionization efficiency satisfies the equation

$$d(E\eta)/dE=\mu+\lambda\int_0^{E_M'} dE'k(E,E')\eta'(E'), \tag{6}$$

to which is added the boundary condition $\eta(0)=0$. The kernel is

$$k(E,E')=\sigma(E,E')E' \bigg/ \left[\int_0^{E_M'} dE'\sigma(E,E')E'\right]; \tag{7}$$

and $\eta'=\omega^e I'/E'$ is the ionization efficiency of a gas particle of energy E' in its own gas.[2]

The ionization efficiency η' of a gas particle satisfies the integro-differential equation

$$d(E'\eta')/dE'=\mu'+\lambda'\int_0^{E'} dE''k'(E',E'')\eta'(E''), \quad \eta'(0)=0, \tag{8}$$

obtained from Eq. (6) by regarding the initial particle as identical in nature with the recoil atom. The quantities μ' and λ' are functions of the energy E' of the gas particle and are defined in a manner corresponding to the definitions of μ and λ given above. Likewise, the kernel $k'(E',E'')$ for a gas particle is defined in a manner similar to that given for the kernel for the primary particle.

It is thus seen that the problem of the determination of the ionization efficiency of a heavy particle requires the solution of Eq. (8) for the ionization efficiency of a gas particle. Once this solution is obtained, the ionization efficiency of the primary particle is obtained from Eq. (6) by simple integration.

III. IONIZATION DEFECT

The quantity $\Delta=E-\omega^e I$, which we term the ionization defect, is of particular interest, since it is a measure of the degree to which the total ionization, when converted to energy units by the use of the factor ω^e, fails to give a true measure of the particle energy. An ionization efficiency less than unity corresponds to a positive defect; if $\eta>1$, the defect is negative.

If we write $\Delta=E\chi$ with $\chi=1-\eta$, Eq. (6) becomes

$$d(E\chi)/dE=(1-\mu-\lambda)+\lambda\int_0^{E_M'} dE'k(E,E')\chi'(E'), \tag{9}$$

with $\chi(0)=1$; $\chi'=1-\eta'$.

[2] Any difference in ω^e for the primary particle and for the recoil atoms is neglected.

The quantity λ is nearly unity for the low velocities at which energy loss to recoiling atoms predominates, and approaches zero for very high velocities. The quantity $1-\mu-\lambda=(b^e-\omega^e\sigma^e)/(b^e+b^\nu)$ is zero everywhere if the rate of energy loss to electrons and the rate of primary ionization (including ionization by ejected electrons) are strictly proportional with the proportionality constant ω^e. In any case, $1-\mu-\lambda$ is small for velocities at which energy loss to recoiling atoms is highly predominant, and approaches zero rapidly for high velocities. In the intermediate range it may conceivably have values positive or negative which are appreciably different from zero. In the discussion that follows, $1-\mu-\lambda$ in Eq. (9) is assumed to be negligible and is set equal to zero for all energies.

Under these conditions the ionization defect is simply

$$\Delta=\int_0^E dE\lambda\int_0^{E_M'} dE'k(E,E')\chi'(E'). \tag{10}$$

If $1-\mu'-\lambda'$, which refers to the gas particles, is also set equal to zero, the quantity χ' is determined by

$$d(E'\chi')/dE'=\lambda'\int_0^{E'} dE''k'(E',E'')\chi'(E''), \quad \chi'(0)=1. \tag{11}$$

It is then readily seen that both Δ and $\Delta'=E'\chi'$ are not negative, monotonically increasing, bounded functions of E and E', respectively, provided λ and λ' go to zero sufficiently rapidly for high energies. Moreover, if λ' is a monotonically decreasing function of E', and if $dk'(E',E'')/dE'\leq 0$, χ' is a positive, monotonically decreasing function of E'. Also under similar conditions, χ is a positive monotonically decreasing function of E.

IV. THE KERNEL $k(E, E')$ AND THE STOPPING CROSS SECTION b^ν

An elastic collision can be described with considerable accuracy as the direct action of the screened nuclear coulomb field of the particle on the screened nuclear charge of the atom. The process is conveniently discussed with the aid of Bohr's screening parameter[3] $\zeta=b/a$, in which b is the collision diameter

$$[(M+M')/M'](ZZ'e^2/E),$$

and a is the screening length $a_0\bar{Z}^{-\frac{1}{3}}$. In terms of $E_S=ZZ'e^2/a$, the screening parameter is simply

$$\zeta=(M+M')E_S/M'E=E_2/E, \tag{12}$$

where $E_2=(M+M')E_S/M'$. In these expressions Z and Z' are the atomic numbers of the particle and gas atom, respectively, $\bar{Z}=(Z^{\frac{2}{3}}+Z'^{\frac{2}{3}})^{\frac{3}{2}}$, and $a_0=\hbar^2/me^2$.

For $\zeta\gg 1$, which is the condition of almost complete shielding, the collision cross section is essentially the

[3] N. Bohr, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **18**, nr. 8 (1948).

kinetic theory area πd^2, with $d \approx a_0 Z_k^{1/6}$, where $Z_k = (Z^{2/3} + Z'^{2/3})^{3/2}$; the mean energy transferred in a collision is $E_M'/2$. For $\zeta > \sim 1$, which Bohr describes as a condition of excessive screening, the principal scattering is at a separation distance of about the screening length a; the cross section is estimated by Bohr to be of the order of $\pi a^2 \zeta / 2.72$, and the scattering is still approximately uniform in the center-of-gravity system. For smaller values of ζ, coulomb scattering takes place with a limitation of the minimum energy transferred due to the effect of shielding on distant collisions (minor screening).

Correspondingly, the order of magnitude of the differential scattering coefficient is given by

$$\sigma(E, E') \sim \begin{cases} \pi d^2 / E_M', \\ \pi a^2 (M+M') E_S / M' E E_M', \\ \pi a^2 M E_S^2 / M' E E'^2, \end{cases} \tag{13}$$

for the three conditions of complete, excessive, and minor screening, respectively, except that for minor screening, scattering with recoil energies less than $ME_S^2/M'E$ is negligible. For $E < E_2$, the scattering coefficient is practically independent of E' in the entire range 0 to E_M'. However, for $E_2 < E$, it is practically zero from 0 to $ME_S^2/M'E$ and proportional to $1/E'^2$ from the latter value to E_M'. It might seem reasonable to compensate somewhat for the discontinuous behavior at $E = E_2$ in these estimates by requiring that the collision cross section, which is the integral of the scattering coefficient over E', be continuous. This requirement could be satisfied by multiplying the value in (13) for excessive screening, which is the most uncertain of the three expressions, by the number $\frac{3}{4}$. However, it is more satisfactory in the present discussion to require that the stopping cross section b^ν be continuous. For this purpose the appropriate factor is $\log 2 = 0.69$.

Since the kernel $k(E, E')$ is independent of factors of proportionality, only the dependence of $\sigma(E, E')$ on E' is important in its determination. Thus, in the approximation here described,

$$k(E, E') \approx \begin{cases} 2E'/E_M'^2, & 0 \leq E' \leq E_M', \quad 0 \leq E < E_2, \\ 0, & 0 \leq E' \leq E_m' \\ \dfrac{1}{2E' \log(2E/E_2)}, & E_m' \leq E' \leq E_M' \end{cases} \Bigg\} E_2 < E, \tag{14}$$

where

$$E_m' = ME_S^2/M'E = MM'E_2^2/(M+M')^2 E \tag{15}$$

is the effective value of the minimum energy transferred in an elastic collision in a screened coulomb field.

The stopping cross section, as calculated from Eq. (13) with the additional factor log2 for the intermediate condition, is

$$b^\nu \approx \frac{4MM'}{(M+M')^2} \begin{cases} \frac{1}{2}\pi d^2 E, & 0 \leq E \leq E_1, \\ \frac{1}{2}\pi a^2 E_2 \log 2, & E_1 \leq E \leq E_2, \\ \frac{1}{2}\pi a^2 (E_2^2/E) \log(2E/E_2), & E_2 \leq E, \end{cases} \tag{16}$$

in which $E_1 = (a/d)^2 E_2 \log 2$ has been so chosen to make the first two approximate expressions pass continuously from one to the other. For an argon particle in argon $E_1 = 113$ ev, and the corresponding velocity is $v_0/95$. It is of interest to note that for $E = E_1$, the screening parameter, which is $\zeta = E_2/E$, has the value

$$d^2/a^2 \log 2 \approx 1.45 Z_k^{2/3} \bar{Z}^{2/3}, \tag{17}$$

which is much greater than unity for a heavy particle ($Z \gg 1$) in any gas. The stopping cross section as given by Eq. (16) increases linearly with E to E_1, is constant to E_2, rises to a maximum at $1.36E_2$ (the increase being only 6 percent), and then decreases. It should be emphasized that the particular features of Eq. (16) are highly arbitrary and that this entire description is at the best only semiquantitative.

V. THE ELECTRONIC STOPPING CROSS SECTION

Inelastic collisions are possible for particle energies above $(M+M')/M'$ times the energy ϵ of the first excited level of the particle or atom, whichever has the lower energy. The corresponding threshold velocity is

$$v_t = [2m\epsilon/\mu\epsilon_0]^{1/2} v_0,$$

where $\mu = MM'/(M+M')$, and $\epsilon_0 = me^4/\hbar^2$. The threshold velocity for argon particles in argon is $v_0/210$.

An energetic heavy particle loses energy primarily through the action of its average charge. The average charge decreases as the particle velocity decreases through the process of capture and loss of electrons.[4] The electronic-stopping cross section in a gas of high atomic number is approximately a linear function of the velocity in the velocity range corresponding to electron capture, giving rise to the familiar linear portion of the range-velocity curves for fission fragments.

As the particle velocity falls below about $v_a = v_0 Z^{-1/3}$, which is a measure of average velocity of the slowest electrons in the neutral particle, the mechanism for inelastic collisions is no longer through an average charge, which is practically zero, but rather through the relative kinetic energy which induces transitions during the interpenetration of the electron clouds of the particle and atom. Statistical arguments indicate that here again the electronic stopping cross section might be expected to be proportional to the particle velocity.

These considerations suggest that the electronic stopping cross section be expressed in ascending and

[4] N. Bohr, Phys. Rev. **58**, 654 (1940); **59**, 270 (1941). W. E. Lamb, Phys. Rev. **58**, 696 (1940); **59**, 687 (1941). J. K. Knipp and E. Teller, Phys. Rev. **59**, 659 (1941). Brunings, Knipp, and Teller, Phys. Rev. **60**, 657 (1941).

descending powers of the velocity in the following manner:

$$b^e = \begin{cases} 0, & 0 \leq v \leq v_t, \\ b_1(v - v_t/v_a) + b_2(v - v_t/v_a)^2 + \cdots, & v_t \leq v \leq v_a, \\ B_1(v/v_a) + B_0 + B_{-1}(v_a/v) + \cdots, & v_a \leq v < \sim Zv_0,\ Z'v_0. \end{cases} \quad (18)$$

Except for B_1, we have very little theoretical basis for the calculation of the coefficients in these series because of the intrinsic difficulty of the theory of quasi-adiabatic processes.

For B_1 we have the expression

$$B_1 \approx (4\pi e^4/mv_a v_a')\langle\log\rangle_{Av}, \quad (19)$$

which is obtained from the familiar formula for the energy loss of a charged particle by replacing the particle charge by $(v/v_a)e$ and estimating the number of atomic electrons effective in an encounter as $(2v/v_a')$; $\langle\log\rangle_{Av}$ is an average value of the usual logarithmic factor for these electrons. It has been shown by Bohr[3] that such a factor can be calculated from the stopping of alpha-particles by means of

$$(2v/v_a')\langle\log\rangle_{Av} = [(3/4\kappa^{\frac{1}{2}}) + (1/4\kappa)]L_\alpha, \quad (20)$$

where L_α is the total logarithmic factor for alpha-particles. In this expression, $\kappa \approx 2Z^{\frac{1}{3}}$. Measurements in argon give[5] $L_\alpha \approx 13.9(v/v_0)$. From Eq. (20) we find $\langle\log\rangle_{Av} \approx 1.21$ for argon particles in argon. For the average light fission fragment in argon $\langle\log\rangle_{Av}$ is found to be 1.15 and for the average heavy fragment 1.10.

VI. IONIZATION BY SLOW HEAVY PARTICLES

The atomic scattering of a heavy particle is nearly spherically symmetrical in the center-of-gravity system if $v < v_2$, where $v_2 = [ZZ'\bar{Z}^{\frac{1}{3}} 2m/\mu]^{\frac{1}{2}} v_0$. The equation for χ, when written in terms of velocities, is

$$\chi = \frac{8}{v^2}\int_0^v dv \frac{1}{(1+b^e/b^\nu)v^3\gamma'^4}\int_0^{\gamma' v} dv' v'^3 \chi'(v'), \quad v < v_2, \quad (21)$$

in which $\gamma' = 2M/(M+M')$. Moreover, the secondary atomic scattering is also spherically symmetrical if $\gamma' v < v_2'$, where $v_2' = [(2)^{\frac{1}{2}} Z^{7/3} 4m/M]^{\frac{1}{2}} v_0$. The equation for χ' is

$$(1 + b^{e'}/b^{\nu'})(d/dv')(v'^2\chi') = (8/v'^3)\int_0^{v'} dv'' v''^3 \chi'(v''),$$
$$\chi'(0) = 1, \quad v' < v_2'. \quad (22)$$

For simplicity, we neglect v_t in Eq. (18) for b^e and suppose that b^ν is practically constant; we treat $b^{e'}$ and $b^{\nu'}$ similarly. For $v' < v_2'$ and $v < v_2$, we write

$$b^{e'}/b^{\nu'} = \beta_1'(v'/v_2') + \beta_2'(v'/v_2')^2 + \cdots,$$
$$b^e/b^\nu = \beta_1(v/v_2) + \beta_2(v/v_2)^2 + \cdots, \quad (23)$$

and seek expressions in series form:

$$\chi' = 1 - \alpha_1'(v'/v_2') - \alpha_2(v'/v_2')^2 - \cdots,$$
$$\chi = 1 - \alpha_1(v/v_2) - \alpha_2(v/v_2)^2 - \cdots. \quad (24)$$

From Eq. (22) we find for the coefficients in χ':

$$\alpha_1' = \frac{10}{7}\beta_1', \quad \alpha_2' = -\frac{45}{28}\beta_1'^2 + \frac{3}{4}\beta_2', \text{ etc.} \quad (25)$$

Using these, we obtain from Eq. (21) for the coefficients in the expansion of χ:

$$\alpha_1 = \frac{2}{3}\beta_1 + \frac{16}{21}\beta_1'\gamma,$$
$$\alpha_2 = -\frac{1}{2}\beta_1^2 - \frac{4}{7}\beta_1\beta_1'\gamma - \frac{15}{28}\beta_1'^2\gamma^2 + \frac{1}{2}\beta_2 + \frac{1}{4}\beta_2'\gamma^2, \text{ etc.,} \quad (26)$$

in which $\gamma = \gamma' v_2/v_2'$.

In argon with a five percent admixture of air, the ionization by single recoil particles in the alpha-decay of Po, ThC, and ThC′ has been measured by Madsen.[6] The recoil energies are 104 kev, 118 kev, and 170 kev, respectively, and correspond to a velocity range of $0.14v_0$ to $0.18v_0$. We take[7] $Z=82$, $Z'=18$, $M=208$ proton masses, and $M'=40$ proton masses and find $v_2 = 0.493v_0$, $v_2' = 0.255v_0$, and $v_2'/\gamma' = 0.151v_0$. Hence, it is seen that the primary atomic scattering is spherically symmetrical but that some nonspherically symmetrical secondary scattering is beginning to set in. We fit the Madsen data with $\alpha_1 = 0.91$ and $\alpha_2 = -0.3$. It does not seem unreasonable to suppose that the values so determined are accurate enough to be used in Eq. (26), which applies to lower velocities. The first expression in Eq. (26) places upper limits on β_1 and β_1', since it gives

$$(\beta_1/1.36) + (\beta_1'/0.37) = 1. \quad (27)$$

The most that can be obtained from the second is that

$$0.6 \leq (\beta_2/1.3) + (\beta_2'/0.3) \leq 1, \quad (28)$$

indicating that quadratic terms in Eq. (23) are not negligible even at these low velocities.

VII. IONIZATION BY ARGON IN ARGON

We discuss first the behavior of χ' for velocities below v_2' on the assumption that terms in the velocity higher than quadratic are negligible in both $b^{e'}/b^{\nu'}$ and χ'. We assume also that β_2 and β_2' in Eq. (28) have the same sign; they then are positive and $\beta_2' \leq 0.3$. We seek a lower bound for χ' in the range $0 \leq v' \leq v_2'$. Using the

[5] G. Mano, Ann. phys. **11**, 407 (1934). N. O. Lassen, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **25**, nr. 11 (1949).

[6] B. S. Madsen, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **23**, nr. 8 (1945).

[7] R. C. Ling and J. K. Knipp, Phys. Rev. **80**, 106 (1950).

optimum value of β_1' and the maximum value of β_2', we find 0.53 for α_1' and approximately zero for α_2', indicating that $\chi' > \sim 0.5$ on this basis. In any case

$$\chi' > \sim [1-0.53(v'/v_2')] \text{ if } v' \leq v_2'.$$

Above the velocity v_2' the equation for χ' is

$$\frac{1}{v'}\log\left(\frac{\sqrt{2}v'}{v_2'}\right)\left(1+\frac{b^{e'}}{b^{\nu'}}\right)\frac{d(v'^2\chi')}{dv'} = \int_{v_2'^2/2v'}^{v'} \chi'(v'')dv''/v'', \quad (29)$$

the solution of which is to have the value $\chi'(v_2')$ at v_2', as determined by the solution of (22). For an argon particle in argon, $v_2'=0.255v_0$ and $v_a'=0.382v_0$. We write

$$\frac{b^{e'}}{b^{\nu'}} = \frac{1}{\log[(2)^{\frac{1}{2}}v'/v_2']}\left(\frac{v'}{v_2'}\right)\left[\gamma_1'\left(\frac{v'}{v_2'}\right)+\gamma_0'+\cdots\right], \quad v_2'<v, \quad (30)$$

where, if the descending series in Eq. (18) extends down to v_2',

$$\gamma_1' \approx (v_2'/v_a')B_1'/\pi a'^2 E_2', \quad \gamma_0' \approx B_0'/\pi a'^2 E_2', \quad \text{etc.} \quad (31)$$

From Eq. (19) the value of γ_1' for an argon particle in argon is found to be 0.127.

It is readily seen that

$$v_2'^2\chi'(v_2') + \chi'(v_2'/\sqrt{2})\int_{v_2'}^{v'} v'\lambda' dv' \leq v'^2\chi' \leq 2\int_0^{v'} v'\lambda' dv'. \quad (32)$$

The integral in the lower bound has the asymptotic value $7.8v_2'^2$ for large values of v' if Eq. (30) is used in λ', with $\gamma_1'=0.13$, $\gamma_0'=0$, etc.

VIII. IONIZATION BY FISSION FRAGMENTS

Extensive studies have been made of the ionization produced by fission fragments when they are stopped in various gases. Ionization yields are usually converted to energies by comparison with the ionization of an alpha-particle of known energy. The ionization of an alpha-particle is directly proportional to its energy over a very wide energy range because the energy lost to recoiling atoms is but a small fraction of the energy of the alpha-particle and hence, even if the recoiling atoms have a reduced ionization efficiency, the total ionization yield of the primary particle is largely unaffected. The situation is quite different with fission fragments in a heavy gas such as argon. A reduced efficiency of the recoiling gas atoms brings about an appreciable decrease in the ionization yield of the fragments.

There is some indication, from experiments on fission fragments from U^{235} produced by thermal neutrons, that these particles have positive ionization defects. Estimates of the energy based on ionization yields[8] are appreciably smaller than the calorimetric measurement of Henderson.[9] Recently Leachman[10] has shown that certain differences between fission fragment mass and ionization distributions can be attributed to a variation in ω, the energy loss per ion pair, with fragment mass.

For $v<v_2$, the quantity χ is given by Eq. (21). For $v_2<v$, we have

$$\chi = \frac{8}{v^2}\int_0^{v_2}\frac{dv}{(1+b^e/b^\nu)v^3\gamma'^4}\int_0^{\gamma' v} v'^3\chi'(v')dv' + \frac{1}{v^2}\int_{v_2}^{v}\frac{dv}{(1+b^e/b^\nu)v^{-1}\log(\sqrt{2}v/v_2)} \times \int_{\gamma' v_2^2/2v}^{\gamma' v}(v')^{-1}\chi'(v')dv', \quad v_2<v. \quad (33)$$

Lower and upper bounds for the ionization defect are given by

$$E_2[\chi(v_2)+\chi'(\gamma' v_2/(2)^{\frac{1}{2}})v_2^{-2}\int_{v_2}^{v} v\lambda dv] \leq \Delta \leq 2E_2 v_2^{-2}\int_0^v v\lambda dv. \quad (34)$$

The integral in the lower bound has the asymptotic value $9.6v_2^2$ for large values of v if an expression similar to Eq. (30) is used in λ, with $\gamma_1=0.10$, $\gamma_0=0$, etc., where

$$\gamma_1 \approx (v_2/v_a)B'/[4MM'\pi a^2 E_2/(M+M')^2]. \quad (35)$$

The value 0.10 for γ_1 is the result obtained from Eq. (35) for both the average light and the average heavy fission fragments in argon.[11]

IX. NUMERICAL ESTIMATES

Owing to our lack of knowledge concerning the ratio of the stopping cross sections for velocities below v_a, which is the velocity at which the neutralization of the particle by electron capture is practically complete, it is not possible to do better than make very rough estimates of the actual magnitudes of the ionization efficiencies and ionization defects for various particles. We describe briefly a sample calculation which has been made on the basis of what would seem to be reasonable, although quite arbitrary, assumptions.

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[8] Flammersfeld, Jensen, and Gentner, Z. Physik **120**, 450 (1943); W. Jentsche, Z. Physik **120**, 165 (1943); M. Deutsch and M. Ramsey, MDDC 945 (1946); D. C. Brunton and G. C. Hanna, Can. J. Research **A28**, 190 (1950).

[9] M. C. Henderson, Phys. Rev. **58**, 744 (1940).

[10] R. B. Leachman, Phys. Rev. **79**, 197 (A) (1950).

[11] We use $Z=37$ and $M=95$ proton masses for the average light fission fragment, which give $v_a=0.300v_0$ and $v_2=0.330v_0$; and $Z=55$ and $M=139$ proton masses for the average heavy fragment, which give $v_a=0.363v_0$ and $v_2=0.400v_0$.

Equations (22) and (29) are solved numerically for an argon particle in argon using, for $b^{e'}/b^{\nu'}$, only the first terms in the series (23) and (30), with $\beta_1'=0.24$ and $\gamma_1'=0.13$. The value for β_1' is that obtained from Eq. (27) under the assumption that $\beta_1'/v_2'=\beta_2/v_2$. One sees that there is introduced a discontinuity in $b^{e'}/b^{\nu'}$ at v_2', which might be expected to be smoothed out by higher terms in the series, were they included. It is found that $\chi'=0.78$ at $v'=v_2'/\sqrt{2}$, 0.72 at v_2', 0.53 at $2v_2'$, 0.40 at $3v_2'$, etc., and that $\chi'\rightarrow 12(v_2'/v')^2$ as $v'\rightarrow\infty$, corresponding to an ionization defect of 780 kev for an argon particle of very high velocity. The asymptotic value $v'^2\chi'$ is very closely the mean of the values obtained from the upper and lower bounds in Eq. (32).

For the average light fission fragment, we take $\beta_1=0.31$ and $\gamma_1=0.10$, and for the average heavy fission fragment, we take $\beta_1=0.38$ and $\gamma_1=0.10$, other coefficients in the series expressions being put equal to zero. We then obtain, in argon, an ionization defect of 2.5 Mev for a light fragment of energy 98 Mev and an ionization defect of 4.2 Mev for a heavy fragment of energy[12] 67 Mev. These quantities are rather insensitive to the the behavior of b^e/b^ν for velocities below v_2, since the contribution of the second integral of Eq. (33) is much larger than that of the first. However, a behavior of $b^{e'}/b^{\nu'}$ which is radically different than that assumed would lead to quite different values for the ionization defects of both argon and the fission fragments.

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A Rapid Method of Calculating log(ft) Values for β-Transitions

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This paper contains several graphs and nomographs which make it possible to obtain, very quickly, log(ft) values for most β-decays. The use of these figures is discussed.

I. INTRODUCTION

BETA-DECAYS can be divided into several classes of allowedness and forbiddenness according to their $\log_{10}(ft)$ values.[1,2] In conjunction with Gamow-Teller selection rules, such a classification agrees, in nearly all cases, with predictions from the nuclear shell model.[2-4] This paper contains several graphs and nomographs which make it possible to obtain, very quickly, log(ft) values for most β^+ emissions, β^- emissions, and K-captures.

Log(ft) can be written as the sum of three additive terms. The first term, $\log(f_0t)$, is the value for a β-decay if the effect of the coulomb field is ignored and if there is no branching. The second term, $\log(C)$, is the coulomb correction term. The third term, $\Delta\log(ft)$, appears if there is branching. In this paper all logarithms are taken to the base 10.

II. CONSTRUCTION OF THE FIGURES

(a) Construction of Nomograph for $\log(f_0t)$

f_0 for β^+ emission is given by the formulas:

$$f_0=\int_1^{W_0} W(W^2-1)^{\frac{1}{2}}(W_0-W)^2dW,$$

and

$$f_0=[(W_0^4/30)-(3/20)W_0^2-(2/15)][(W_0^2-1)]^{\frac{1}{2}} +(2.302/4)W_0\log[W_0+(W_0^2-1)^{\frac{1}{2}}],$$

where W_0 is the maximum energy of the β-particles, including rest mass, in units of mc^2.

For K-capture, f_0 is given by $f_0=(W_0+E_K)^2$. Here E_K is the binding energy of a K-electron, which, of course, depends on the atomic number of the decaying nucleus. f_0 depends only on W_0 for $\beta^\pm$ emission. For K-capture, one can write for f_0, instead of the above expression:

$$f_0=W_0^2.$$

The error introduced by ignoring E_K in the expression for f_0 is negligible except for large Z and small W_0, as discussed in Sec. III. Here t is the half-life in seconds for all modes of decay.

The nomograph of Fig. 1, for getting $\log f_0t$ values, consists of three columns; a, b, and c. Column a contains two sets of entries of energies E_0. For $\beta^\pm$ emission, E_0 is the maximum energy of the β-particles in Mev, not including the rest mass, and the entries on the right side apply. For K-capture, E_0 is the decay energy in Mev, and the entries on the left side apply. The spacing of energies on Column a is not uniform but is proportional to $\log(f_0)$. Column b gives t in seconds, days, etc. The spacing is uniform in $\log(t)$. Column b is constructed exactly halfway between Columns a and c. Column c

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