where the stroke (|) implies covariant differentiation with respect to the Christoffel symbols of the $\mathbf{A}_{\alpha\beta}$-tensor, and

$$I_{\alpha\beta\gamma}=\epsilon_{\alpha\beta\gamma\delta}\mathbf{J}^{\delta}\quad (\mathbf{J}^{\delta}=4 \text{ “current density” vector}),$$

and

$$\Phi_{\alpha\beta,\gamma}+\Phi_{\beta\gamma,\alpha}+\Phi_{\gamma\alpha,\beta}=I_{\alpha\beta\gamma}. \qquad (3)$$

The complete solution for the Γ-field is (approximately)

$$\begin{aligned}\Gamma^{\alpha}_{\beta\gamma}=\left\{{\alpha \atop \beta\gamma}\right\}_{\mathbf{A}}-\tfrac{1}{2}\mathbf{A}^{\alpha\mu}(\Phi_{\beta\nu}I^{\nu}_{\gamma\mu}+\Phi_{\nu\gamma}I^{\nu}_{\mu\beta})\\ +\mathbf{A}^{\nu\rho}\mathbf{A}^{\alpha\mu}(\Phi_{\beta\nu}\Phi_{\gamma\mu|\rho}+\Phi_{\nu\gamma}\Phi_{\mu\beta|\rho})+\mathbf{A}^{\alpha\rho}\Phi_{\beta\gamma|\rho}-\tfrac{1}{2}I^{\alpha}_{\beta\gamma}.\end{aligned}$$

The contravariant tensor densities $\mathbf{G}^{\underline{\alpha\beta}}=(-g)^{\frac{1}{2}}g^{\underline{\alpha\beta}}$, and $\mathbf{G}^{\underset{\vee}{\alpha\beta}}=(-g)^{\frac{1}{2}}g^{\underset{\vee}{\alpha\beta}}$ were obtained by Schrödinger[3] in the form

$$\mathbf{G}^{\underline{\alpha\beta}}=\frac{(-\mathbf{A})^{\frac{1}{2}}[\mathbf{A}^{\alpha\beta}(1+\Omega)-\Phi^{\alpha\mu}\Phi^{\beta}_{.\mu}]}{(1+\Omega-\Lambda^2)^{\frac{1}{2}}}, \qquad (4)$$

$$\mathbf{G}^{\underset{\vee}{\alpha\beta}}=\frac{(-\mathbf{A})^{\frac{1}{2}}(\Phi^{\alpha\beta}-\Lambda\overset{*}{\Phi}{}^{\alpha\beta})}{(1+\Omega-\Lambda^2)^{\frac{1}{2}}}, \qquad (5)$$

where

$$\overset{*}{\Phi}{}^{\alpha\beta}=\frac{1}{2(-\mathbf{A})^{\frac{1}{2}}}\epsilon^{\alpha\beta\mu\nu}\Phi_{\mu\nu}$$

$$\Lambda=\tfrac{1}{4}\overset{*}{\Phi}{}^{\alpha\beta}\Phi_{\alpha\beta}$$

and

$$\Omega=\tfrac{1}{2}\Phi^{\alpha\beta}\Phi_{\alpha\beta}.$$

From the field equations $\mathbf{G}^{\underset{\vee}{\alpha\beta}}{}_{,\beta}=0$, and from Eq. (3) we have

$$\begin{gathered}((-\mathbf{A})^{\frac{1}{2}}\overset{*}{\Phi}{}^{\alpha\beta})_{,\beta}=-\mathbf{J}^{\alpha},\quad \mathbf{G}^{\underset{\vee}{\alpha\beta}}{}_{,\beta}=0,\\ (\mathbf{A}=\mathrm{Det}\mathbf{A}_{\alpha\beta},\quad g=\mathrm{Det}g_{\alpha\beta}).\end{gathered} \qquad (6)$$

In this form Eqs. (6) may be compared with Born's[4] electrodynamics.

With the assumed approximation for the Φ's and identifying the $\mathbf{A}_{\alpha\beta}$-field as in a "Minkowski world," in the absence of gravitation we find that the field equations

$$R_{\underset{\vee}{\alpha\beta}}=0,\quad \mathbf{G}^{\alpha\beta}{}_{;\gamma}=0,\quad \Gamma_{\alpha}=0,\quad \mathbf{G}^{\underset{\vee}{\alpha\beta}}{}_{,\beta}=0$$

reduce to

$$\Gamma^{\alpha}_{\underset{\vee}{\beta\gamma},\alpha}=0,\quad \mathbf{G}^{\underline{\alpha\beta}}{}_{,\beta}=-\mathbf{G}^{\mu\beta}\Gamma^{\alpha}_{\mu\beta},\quad \overset{*}{\Phi}{}^{\alpha\beta}{}_{,\beta}=0. \qquad (7)$$

(The equations $\Gamma_{\alpha}=0$, $\mathbf{G}^{\underset{\vee}{\alpha\beta}}{}_{,\beta}=0\div\overset{*}{\Phi}{}^{\alpha\beta}{}_{,\beta}$ are equivalent, and they represent the vanishing of the magnetic current density in Maxwell's electrodynamics.)

In a particular Lorentz frame (x, y, z, ct), the first and the third sets of equations of (7) can be put into a vector form as

$$\left.\begin{aligned}\nabla^2\mathbf{H}-\frac{1}{c^2}\frac{\partial^2\mathbf{H}}{\partial t^2}=-\tfrac{1}{2}\nabla\times\mathbf{J}\\ \nabla^2\mathbf{E}-\frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}=\tfrac{1}{2}\left(\frac{\partial\mathbf{J}}{\partial t}+\nabla\rho\right)\end{aligned}\right\}, \qquad (8)$$

$$\left.\begin{aligned}\nabla\times\mathbf{E}+\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t}=0\\ \nabla\cdot\mathbf{H}=0\end{aligned}\right\}. \qquad (9)$$

The second set of equations of (7) lead to

$$\begin{aligned}[\eta^{\alpha\beta}\tfrac{1}{4}\Phi_{\mu\nu}\Phi^{\mu\nu}-\Phi^{\alpha\mu}\Phi^{\beta}_{.\mu}]_{,\beta}&=[\eta^{\alpha\beta}\tfrac{1}{4}\overset{*}{\Phi}_{\mu\nu}\overset{*}{\Phi}{}^{\mu\nu}-\overset{*}{\Phi}{}^{\alpha\mu}\overset{*}{\Phi}{}^{\beta}_{.\mu}]_{,\beta}\\ &=-\overset{*}{\Phi}_{\alpha\delta}J^{\delta}\end{aligned} \qquad (10)$$

$$(\eta_{ij}=-\delta_{ij},\quad \eta_{i4}=\eta_{4i}=0,\quad \eta_{44}=1,\quad i,j=1,2,3).$$

Equations (10) could be regarded as a proof for the compatibility of the field equations only if the R.H.S. were proportional to the four vector d^2x^{α}/ds^2; this, on the other hand, is to be looked for in the field equations $R_{\underline{\alpha\beta}}=0$.

In the above we took

$$(\Phi_{23},\Phi_{31},\Phi_{12};\Phi_{41},\Phi_{42},\Phi_{43})\equiv(E_1,E_2,E_3;H_1,H_2,H_3).$$

The factor $(\frac{1}{2})$ in (8) is due to the fact that gravito-electromagnetic units are not introduced.

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Intensity Distribution of the First Negative ($B^2\Sigma\rightarrow X^2\Sigma$) Band System of N_2^+

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THE theoretical work associated with a laboratory investigation of the mechanisms of excitation of $N_2^+(B^2\Sigma)$ in discharge tubes has required the calculation of the fractional transition probabilities of the vibrational bands of the first negative system of this molecule. The method of calculation employed is in all respects similar to that used in a set of previous calculations for the second positive system of nitrogen.[1] Because the first negative system forms an important part of the auroral spectrum, and from its measured intensity distribution, conclusions may be drawn regarding the state of the upper atmosphere,[2] it has been considered worthwhile to present here the results of these calculations.

The fractional transition probability $f(v', v'')$, shown for each band in Table I, is defined in the expression for the integrated intensity $I(v', v'')$ of a molecular band:

$$I(v', v'')=CN(v')E(V', v'')f(v', v''),$$

where v' and v'' are the vibrational quantum numbers of the two levels involved in the transition. $N(v')$ is the population of molecules in the upper level, and $E(v', v'')$ is the energy separation between the two levels. C, the constant of proportionality is so adjusted that $\sum_{v''} f(v', v'')=1$ for each v' sequence.

In order to express the results as fractional transition probabilities, it was necessary to perform calculations for some transitions where bands are not reported to be observed.[3] These are indicated in italics. With the exception of $f(5, 3)$ each of the $f(v', v'')$ values in italics is small enough to suggest that the respective band would, in fact, be difficult to observe. The fact that (5, 3) is not reported in spite of its high $f(v', v'')$ value perhaps may be explained by its calculated wavelength (λ3274.24) lying between those of the (3, 3) (λ3285.3) and (4, 4) (λ3268.1) bands of the second positive system. Possibly, development of the rotational structure of these bands may tend to overlap and obscure (5, 3). Experimental work on this point is in progress.

Comparison between the aforementioned calculations and a less extensive previous set[4] shows fair agreement.

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TABLE I. Fractional transition probabilities, $f(v', v'')$. Transitions where bands are not reported to be observed are indicated in italics.

v' \ v''	0	1	2	3	4	5	6	7	8	9	10	11
0	0.701	0.223	0.047	0.024	0.006							
1	0.334	0.266	0.266	0.067	0.037	*0.028*	*0.002*					
2	0.058	0.435	0.078	0.242	0.089	0.048	0.043	*0.006*				
3	*0.001*	0.147	0.438	0.011	0.175	0.083	0.064	0.070	*0.011*			
4		*0.001*	0.220	0.395	0.000_2	0.115	0.075	0.069	0.085	*0.038*	*0.001*	
5			*0.005*	*0.306*	0.334	*0.007*	*0.07*	0.058	*0.065*	*0.094*	*0.06*	*0.000_2*