

TABLE I. Radial distribution of electrons for $s=0.6, 1.0,$ and 1.5 .

x	$P_r(x, s)$		
	$s=0.6$	$s=1.0$	$s=1.5$
0.2	11.	7.6	5.0
0.4	2.85	2.74	2.00
0.6	8.00×10^{-1}	1.01	1.08
0.8	2.30	4.84×10^{-1}	6.10×10^{-1}
1.0	8.50	2.52	3.75
1.2	4.00	1.47	2.40
1.4	2.18	8.72×10^{-2}	1.60
1.6	1.22	5.36	1.14
1.8	7.40×10^{-2}	3.49	8.20×10^{-2}
2.0	4.50	2.26	6.00
2.2	2.90	1.53	4.50
2.4	2.00	1.02	3.50
2.6	1.37	7.20×10^{-3}	2.75
2.8	9.50×10^{-3}	5.16	2.15
3.0	6.70	3.73	1.70
3.5	3.00	1.74	9.80×10^{-3}
4.0	1.36	8.72×10^{-4}	6.00
4.5	6.20×10^{-4}	4.52	3.65
5.0	2.99	2.51	2.35
5.5	1.39	1.46	1.47
6.0	6.90×10^{-5}	8.80×10^{-5}	9.80×10^{-4}
7.0	1.58		4.70

greatly changed at large r so long as we continue the curve back towards the origin smoothly and make $\langle r^0 \rangle$ unity. The shape of the function for large r is correct, since it depends only on the higher moments and not at all on the type of singularity. The amplitude, however, does depend to a slight extent on how well we have guessed the behavior at the origin. The normalization, therefore, may be off by several percent. In Table I we present our results for $x > 0.2$, but we do not believe the value for $x = 0.2$ to be very reliable.

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¹ L. Eyges and S. Fernbach, Phys. Rev. **82**, 287 (1951).

² L. Eyges and S. Fernbach, Phys. Rev. **82**, 23 (1951).

³ Rossi and Greisen, Revs. Modern Phys. **13**, 285 (1941).

⁴ G. Moliere, *Cosmic Radiation*, ed. Heisenberg (Dover Publications, New York, 1946).

The Quadrupole Moment Ratio of Cl^{35} and Cl^{37} from Pure Quadrupole Spectra*

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RESONANCE absorption lines due to nuclear quadrupole splittings were first observed by Dehmelt and Krüger¹ for chlorine in *trans*-dichloroethylene at 90°K. The splitting is caused by the interaction of the chlorine nuclear electric quadrupole moment with the gradient of the electric field at the nucleus. The electric field gradient can be considered to arise from charge distribution within the molecule and from other charge distributions in the crystal lattice.

Resonance absorptions similar to the above have been observed at this laboratory in several compounds. These are listed in Table I. A frequency modulated regenerative oscillator was used, and the absorption lines were viewed on an oscilloscope. In making frequency measurements, markers formed by harmonics from a Signal Corps type BC-221 frequency meter were superposed on the lines. The meter was calibrated with harmonics from a 1-Mc crystal oscillator which in turn was calibrated against radio station WWV at 10 Mc. The measured frequencies in Table I should be accurate to 0.01 percent; however, the frequency ratios may be somewhat better, because the harmonics selected for the Cl^{37} and Cl^{35} measurements were such that only a small part of the frequency meter dial travel was used. All frequencies listed are the average of eight measurements. Approximately 25-cc samples were used, and all observations were made at liquid nitrogen temperature (80°K). The relative intensities of the Cl^{35} and Cl^{37} lines agreed satisfactorily with the isotopic abundance ratio. In some cases two absorption lines were seen for each isotope. A similar effect has been observed by Dehmelt² for the iodine

TABLE I. Measured frequencies and ratios for solid chlorine compounds.

Compound	$\nu(\text{Cl}^{35})$ Mc	$\nu(\text{Cl}^{37})$ Mc	$\nu(\text{Cl}^{35})/\nu(\text{Cl}^{37})$
SOCl_2	32.0908	25.2935	1.26874
	31.8874	25.1331	1.26874
POCl_3	28.9835	22.8432	1.26880
	28.9378	22.8067	1.26883
CH_2Cl_2	35.9912	28.3673	1.26876
CHCl_3	38.3081	30.1921	1.26881
	38.2537	30.1500	1.26878
$\text{C}_6\text{H}_5\text{Cl}$	34.6216	27.2872	1.26879

resonance in SnI_4 . The effect was ascribed by him to crystallographically different sets of iodines in the lattice. Each set of iodine nuclei thus experiences its own particular electric field gradient. The splittings observed here could be caused by the same effect. In forming the frequency ratios of Table I, the higher Cl^{35} frequency (higher electric field gradient) line has been associated with the higher Cl^{37} frequency.

The frequency ratios in Table I should be the nuclear quadrupole moment ratio of Cl^{35} and Cl^{37} . The average gives

$$Q(\text{Cl}^{35})/Q(\text{Cl}^{37}) = 1.26878 \pm 0.00015.$$

All of the eight ratios agree to within ± 0.00005 .

Dehmelt and Krüger's value for the moment ratio using *trans*-dichloroethylene was 1.2661 ± 0.0002 .

Geschwind, Gunther-Mohr, and Townes³ have recently summarized published values of quadrupole coupling ratios and shown that in many cases the values for the ratio determined by different investigators varied outside of estimated experimental errors. The variation, at least in part, depended on the molecule used. This was also true for the three accurately remeasured values by the above authors for CH_3Cl , ClCN , and GeH_3Cl , which gave ratios of 1.2691 ± 0.0003 , 1.2682 ± 0.0006 , and 1.2670 ± 0.0005 , respectively. These measurements were made by observing quadrupole splittings in rotational transitions in the microwave region. Gunther-Mohr, Geschwind, and Townes⁴ considered that a reasonable explanation of the variation in experimental values on different molecules could be made by assuming a nuclear polarization by the external electric field. This effect is absent in the measurements reported here.

* This work was performed for the AEC.

¹ H. G. Dehmelt and H. Krüger, Naturwiss. **37**, 111 (1950).

² H. G. Dehmelt, Naturwiss. **37**, 398 (1950).

³ Geschwind, Gunther-Mohr, and Townes, Phys. Rev. **81**, 288 (1951).

⁴ Gunther-Mohr, Geschwind, and Townes, Phys. Rev. **81**, 289 (1951).

On Einstein's Unified Field Theory

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SOME of the most general implications of Einstein's new theory¹ are considered in a paper to appear elsewhere. The results obtained are described in the following shortened version.

By neglecting the cubes and higher orders of antisymmetric field variables a simple expression for the affine field is found in terms of the generalized nonsymmetric tensor $g_{\alpha\beta} = g_{\alpha\beta} + g_{\alpha\beta}^{\sim} = \mathbf{A}_{\alpha\beta} + \Phi_{\alpha\beta}$, and its first-order partial derivatives $g_{\alpha\beta, \gamma}$, where $\mathbf{A}_{\alpha\beta} = g_{\alpha\beta} = g_{\beta\alpha}$ and $\Phi_{\alpha\beta} = g_{\alpha\beta}^{\sim} = -g_{\beta\alpha}^{\sim}$. I am indebted to the referee of *The Physical Review* for directing my attention to a solution of the Γ -field obtained by Ingraham² in a slightly different way from ours.

The antisymmetric part of the Γ -field is given by

$$\Gamma_{\beta\gamma}^{\alpha} = -\frac{1}{2}I_{\beta\gamma}^{\alpha} + \mathbf{A}^{\alpha\mu}\Phi_{\beta\gamma|\mu} \quad (1)$$

subject to the four conditions:

$$\Gamma_{\alpha\gamma}^{\alpha} = \Gamma_{\gamma} = \mathbf{A}^{\alpha\mu}\Phi_{\alpha\gamma|\mu} = 0, \quad (2)$$

where the stroke ($\dot{}$) implies covariant differentiation with respect to the Christoffel symbols of the $\mathbf{A}_{\alpha\beta}$ -tensor, and

$$I_{\alpha\beta\gamma} = \epsilon_{\alpha\beta\gamma\delta} J^{\delta} \quad (J^{\delta} = 4 \text{ "current density" vector}),$$

and

$$\Phi_{\alpha,\beta,\gamma} + \Phi_{\beta\gamma,\alpha} + \Phi_{\gamma\alpha,\beta} = I_{\alpha\beta\gamma}. \quad (3)$$

The complete solution for the Γ -field is (approximately)

$$\Gamma_{\beta\gamma}^{\alpha} = \{\beta\gamma\}^{\alpha} \mathbf{A} - \frac{1}{2} \mathbf{A}^{\alpha\mu} (\Phi_{\beta\nu} I_{\gamma\mu}^{\nu} + \Phi_{\nu\gamma} I_{\mu\beta}^{\nu}) + \mathbf{A}^{\nu\rho} \mathbf{A}^{\alpha\mu} (\Phi_{\beta\nu} \Phi_{\gamma\mu|\rho} + \Phi_{\nu\gamma} \Phi_{\mu\beta|\rho}) + \mathbf{A}^{\alpha\rho} \Phi_{\beta\gamma|\rho} - \frac{1}{2} I_{\beta\gamma}^{\alpha}.$$

The contravariant tensor densities $\mathbf{G}^{\alpha\beta} = (-g)^{\frac{1}{2}} g^{\alpha\beta}$, and $\mathbf{G}^{\alpha\beta} = (-g)^{\frac{1}{2}} g^{\alpha\beta}$ were obtained by Schrödinger³ in the form

$$\mathbf{G}^{\alpha\beta} = \frac{(-\mathbf{A})^{\dagger} [\mathbf{A}^{\alpha\beta} (1 + \Omega) - \Phi^{\alpha\mu} \Phi_{\mu}^{\beta}]}{(1 + \Omega - \Lambda^2)^{\frac{1}{2}}}, \quad (4)$$

$$\mathbf{G}^{\alpha\beta} = \frac{(-\mathbf{A})^{\dagger} (\Phi^{\alpha\beta} - \Lambda \Phi^{\alpha\beta})}{(1 + \Omega - \Lambda^2)^{\frac{1}{2}}}, \quad (5)$$

where

$$* \Phi^{\alpha\beta} = \frac{1}{2(-\mathbf{A})^{\dagger}} \epsilon^{\alpha\beta\mu\nu} \Phi_{\mu\nu}$$

$$\Lambda = \frac{1}{2} \Phi^{\alpha\beta} \Phi_{\alpha\beta}$$

and

$$\Omega = \frac{1}{2} \Phi^{\alpha\beta} \Phi_{\alpha\beta}.$$

From the field equations $\mathbf{G}^{\alpha\beta},_{\beta} = 0$, and from Eq. (3) we have

$$\begin{aligned} ((-\mathbf{A})^{\dagger} \Phi^{\alpha\beta})_{,\beta} &= -\mathbf{J}^{\alpha}, \quad \mathbf{G}^{\alpha\beta},_{\beta} = 0, \\ (\mathbf{A} = \text{Det} \mathbf{A}_{\alpha\beta}, \quad g = \text{Det} g_{\alpha\beta}). \end{aligned} \quad (6)$$

In this form Eqs. (6) may be compared with Born's⁴ electro-dynamics.

With the assumed approximation for the Φ 's and identifying the $\mathbf{A}_{\alpha\beta}$ -field as in a "Minkowski world," in the absence of gravitation we find that the field equations

$$R_{\alpha\beta} = 0, \quad \mathbf{G}^{\alpha\beta},_{\gamma} = 0, \quad \Gamma_{\alpha} = 0, \quad \mathbf{G}^{\alpha\beta},_{\beta} = 0$$

reduce to

$$\Gamma_{\beta\gamma,\alpha} = 0, \quad \mathbf{G}^{\alpha\beta},_{\beta} = -\mathbf{G}^{\mu\beta} \Gamma_{\mu\beta}^{\alpha}, \quad \Phi^{\alpha\beta},_{\beta} = 0. \quad (7)$$

(The equations $\Gamma_{\alpha} = 0$, $\mathbf{G}^{\alpha\beta},_{\beta} = 0 \doteq \Phi^{\alpha\beta},_{\beta}$ are equivalent, and they represent the vanishing of the magnetic current density in Maxwell's electrodynamics.)

In a particular Lorentz frame (x, y, z, ct), the first and the third sets of equations of (7) can be put into a vector form as

$$\left. \begin{aligned} \nabla^2 \mathbf{H} - \frac{1}{c^2} \frac{\partial^2 \mathbf{H}}{\partial t^2} &= -\frac{1}{2} \nabla \times \mathbf{J} \\ \nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} &= \frac{1}{2} \left(\frac{\partial \mathbf{J}}{\partial t} + \nabla \rho \right) \end{aligned} \right\}, \quad (8)$$

$$\left. \begin{aligned} \nabla \times \mathbf{E} + \frac{1}{c} \frac{\partial \mathbf{H}}{\partial t} &= 0 \\ \nabla \cdot \mathbf{H} &= 0 \end{aligned} \right\}. \quad (9)$$

The second set of equations of (7) lead to

$$\begin{aligned} [\eta^{\alpha\beta} \frac{1}{2} \Phi_{\mu\nu} \Phi^{\mu\nu} - \Phi^{\alpha\mu} \Phi_{\mu}^{\beta}]_{,\beta} &= [\eta^{\alpha\beta} \frac{1}{2} \Phi_{\mu\nu} \Phi^{\mu\nu} - \Phi^{\alpha\mu} \Phi_{\mu}^{\beta}]_{,\beta} \\ &= -\Phi^{\alpha\delta} J^{\delta} \end{aligned} \quad (10)$$

$$(\eta_{ij} = -\delta_{ij}, \quad \eta_{i4} = \eta_{4i} = 0, \quad \eta_{44} = 1, \quad i, j = 1, 2, 3).$$

Equations (10) could be regarded as a proof for the compatibility of the field equations only if the R.H.S. were proportional to the four vector $d^2 x^{\alpha}/ds^2$; this, on the other hand, is to be looked for in the field equations $R_{\alpha\beta} = 0$.

In the above we took

$$(\Phi_{23}, \Phi_{31}, \Phi_{12}; \Phi_{41}, \Phi_{42}, \Phi_{43}) \equiv (E_1, E_2, E_3; H_1, H_2, H_3).$$

The factor ($\frac{1}{2}$) in (8) is due to the fact that gravito-electromagnetic units are not introduced.

¹ A. Einstein, *The Meaning of Relativity* (1950).

² Ingraham, *Ann. Math.* **52**, 743 (1950).

³ E. Schrödinger, *Proc. Roy. Irish Acad.* **A LI**, 213 (1948).

⁴ M. Born, *Nature* **132**, 282 (1933). M. Born and L. Infeld, *Proc. Roy. Soc. (London)* **A144**, 425 (1934).

Intensity Distribution of the First Negative ($B^2\Sigma \rightarrow X^2\Sigma$) Band System of N_2^+

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THE theoretical work associated with a laboratory investigation of the mechanisms of excitation of $\text{N}_2^+(B^2\Sigma)$ in discharge tubes has required the calculation of the fractional transition probabilities of the vibrational bands of the first negative system of this molecule. The method of calculation employed is in all respects similar to that used in a set of previous calculations for the second positive system of nitrogen.¹ Because the first negative system forms an important part of the auroral spectrum, and from its measured intensity distribution, conclusions may be drawn regarding the state of the upper atmosphere,² it has been considered worthwhile to present here the results of these calculations.

The fractional transition probability $f(v', v'')$, shown for each band in Table I, is defined in the expression for the integrated intensity $I(v', v'')$ of a molecular band:

$$I(v', v'') = CN(v')E(V', v'')f(v', v''),$$

TABLE I. Fractional transition probabilities, $f(v', v'')$. Transitions where bands are not reported to be observed are indicated in italics.

$v'' \setminus v'$	0	1	2	3	4	5	6	7	8	9	10	11
0	0.701	0.223	0.047	0.024	0.006							
1	0.334	0.266	0.266	0.067	0.037	0.028	0.002					
2	0.058	0.435	0.078	0.242	0.089	0.048	0.043	0.006				
3	0.001	0.147	0.438	0.011	0.175	0.083	0.064	0.070	0.011			
4		0.001	0.220	0.395	0.000	0.115	0.075	0.069	0.085	0.038	0.001	
5			0.005	0.306	0.334	0.007	0.07	0.058	0.065	0.094	0.06	0.000

where v' and v'' are the vibrational quantum numbers of the two levels involved in the transition. $N(v')$ is the population of molecules in the upper level, and $E(v', v'')$ is the energy separation between the two levels. C , the constant of proportionality is so adjusted that $\sum_{v''} f(v', v'') = 1$ for each v' sequence.

In order to express the results as fractional transition probabilities, it was necessary to perform calculations for some transitions where bands are not reported to be observed.³ These are indicated in italics. With the exception of $f(5, 3)$ each of the $f(v', v'')$ values in italics is small enough to suggest that the respective band would, in fact, be difficult to observe. The fact that $f(5, 3)$ is not reported in spite of its high $f(v', v'')$ value perhaps may be explained by its calculated wavelength ($\lambda 3274.24$) lying between those of the $(3, 3)$ ($\lambda 3285.3$) and $(4, 4)$ ($\lambda 3268.1$) bands of the second positive system. Possibly, development of the rotational structure of these bands may tend to overlap and obscure $f(5, 3)$. Experimental work on this point is in progress.

Comparison between the aforementioned calculations and a less extensive previous set⁴ shows fair agreement.

¹ R. W. Nicholls, *Phys. Rev.* **77**, 421 (1950).

² D. R. Bates, *Proc. Roy. Soc. (London)* **A196**, 562 (1949).

³ T. R. Merton and J. G. Pilley, *Phil. Mag.* **50**, 195 (1925).

⁴ D. R. Bates, *Proc. Roy. Soc. (London)* **A196**, 223 (1949).