

TABLE I. The electron angular distribution function $P(x, s)$ defined by Eq. (1). Normalization such that $\int_0^\infty P(x, s) dx = 1$.

x	$P(x, 0.6)$	$P(x, 1.0)$	$P(x, 1.6)$
0	13	9.3	6.1
0.1	9.1	7.1	5.0
0.2	6.3	5.3	4.1
0.4	3.10	2.78	2.60
0.6	1.48	1.52	1.58
0.8	7.10×10^{-1}	8.20×10^{-1}	9.00×10^{-1}
1.0	3.45	4.46	5.10
1.2	1.66	2.32	2.75
1.4	8.00×10^{-2}	1.18	1.50
1.6	3.80	6.00×10^{-2}	8.00×10^{-2}
1.8	1.80	2.90	4.18
2.0	8.20×10^{-3}	1.46	2.18
2.2	3.70	7.13×10^{-3}	1.12
2.4	1.58	3.56	5.60×10^{-3}
2.6	6.7×10^{-4}	1.74	2.75
2.8		8.29×10^{-4}	1.38
3.0	1.12	3.92	6.50×10^{-4}

where $y = W\theta/E_s$, and W is the photon energy. $Q(y, s)$ is defined analogously to $P(x, s)$.

$Q(y, s)$ is singular at the origin. For $s=1$, $Q(y, 1)$ appears to go as $1/y$ for small y .³ Our method of moments is particularly unreliable in determining the behavior of the function at the origin if the function is singular. For $s=0.6$ and $s=1.5$ a $1/y$ singularity for $Q(y, s)$ is consistent with our results, although it seems clear that for $s=0.6$ the singularity should be stronger than for $s=1.5$. Mainly for ease in expressing the normalization, we have assumed that the singularity is exactly $1/y$ for $s=0.6$ and $s=1.5$. Our results are given in Table II, where we tabulate

TABLE II. The photon angular distribution function $Q(y, s)$. Tabulated values are $yQ(y, s)$. Normalization such that $\int_0^\infty Q(y, s) y dy = 1$.

y	$yQ(y, 0.6)$	$yQ(y, 1.0)$	$yQ(y, 1.5)$
0	4.7	3.4	2.1
0.1	2.9	2.4	1.8
0.2	1.8	1.7	1.5
0.4	6.50×10^{-1}	8.76×10^{-1}	1.00
0.6	2.61	4.61	6.42×10^{-1}
0.8	1.18	2.21	3.83
1.0	5.06×10^{-2}	1.18	2.22
1.2	2.28	5.47×10^{-2}	1.19
1.4	1.03	2.74	6.28×10^{-2}
1.6	4.80×10^{-3}	1.38	3.30
1.8	2.25	6.87×10^{-3}	1.66
2.0	1.06	3.44	8.44×10^{-3}
2.2	4.83×10^{-4}	1.72	4.02
2.4	2.25	8.59×10^{-4}	1.89
2.6	1.05	4.32	8.90×10^{-4}
2.8	5.15×10^{-5}	2.18	4.12
3.0	2.43	1.07	1.91

$yP(y, s)$. We would like to emphasize that the accuracy of our relative values for y greater than about 0.3 does not depend on our assumption as to the singularity at the origin, and that these values are probably good to within several percent.

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¹ A. Borsellino, Nuovo cimento **7**, 700 (1950).

² We discuss an incident electron for the sake of definiteness, although our results hold equally well for an incident photon, provided the energy E one is interested in is much less than the energy of the incident particle.

³ L. Eyges and S. Fernbach, Phys. Rev. **82**, 23 (1951).

⁴ See the second of Eqs. (2.55) to (2.60) in B. Rossi and K. Greisen, Revs. Modern Phys. **13**, 284 (1941).

Radial Distribution of Shower Electrons as a Function of Depth*

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THE radial distribution of shower electrons can be determined in the same way as the angular distribution.¹ The procedure is somewhat more involved in that one must calculate the mixed radial and angular moments before obtaining the radial moments alone. Let the number of electrons of energy E to $E+dE$ in the

annular ring between r and $r+dr$ at depth t (r and t measured in radiation lengths) be $(E_0, E, r, t)2\pi r dr dE$ when the initiating particle is an electron of energy E_0 at $t=0$. We define the m th moment as

$$\langle r^m \rangle \equiv \int_0^\infty \pi(E_0, E, r, t) r^m r dr / \int_0^\infty \pi(E_0, E, r, t) r dr.$$

These moments are calculated by an extension of the method previously described.² In this case one obtains $\langle r^m \rangle$ as a function of the parameter s , where s is determined from E_0 and t by use of the relation

$$\log(E_0/E) + \lambda_1'(s)t = 0.$$

$\lambda_1'(s)$ is tabulated in Rossi and Greisen's³ article.

The moments are most simply written as

$$\langle r^m(s) \rangle = (E_0/E)^m \rho_m(s).$$

Figure 1 shows $\rho_m(s)$ plotted for $m=2, 4$, and 6 .

From these moments we calculate the distribution function $P_r(Er/E_s, s)$, where $P_r(Er/E_s, s)dr$ is proportional to the number of electrons of energy E at a depth corresponding to s in an annular ring between r and $r+dr$. The normalization is taken as $\int_0^\infty P_r(x, s) dx = 1$, where $x = Er/E_s$, E_s being as usual approximately 21 Mev.

In a previous paper,² we discussed this calculation for the shower maximum. This corresponds to an $s=1$. In addition, we have now calculated the two cases $s=0.6$ and $s=1.5$ which correspond approximately to half and twice the depth at the shower maximum, respectively.

The radial distribution is singular at the origin, this singularity being of order $r^{-1/3}$ at the shower maximum, if we assume Moliere's⁴ calculations to give an accurate picture of the shower spread for small r . In calculating the present distribution function we again assume the functions singular, but have no way of specifying the order of the singularity. This means that we have to guess the behavior of the function for small r . Since the major contribution to the area under the distribution function, namely $\langle r^0 \rangle$, comes from small r , the amplitude of the function is not

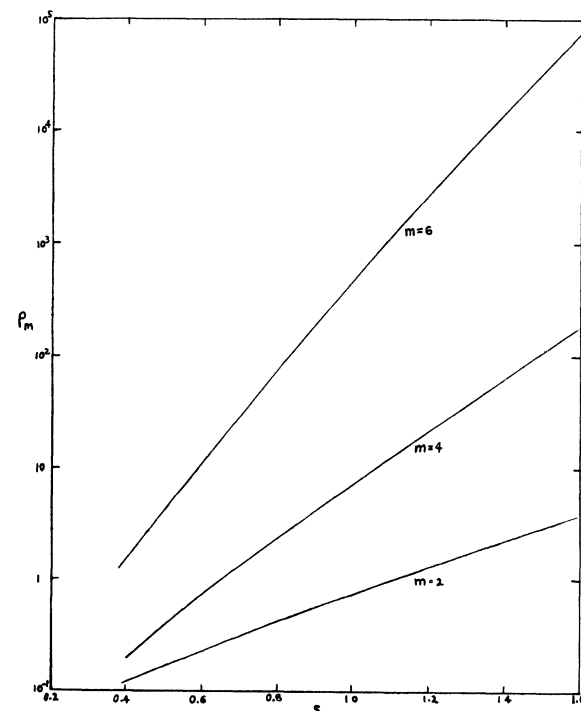


FIG. 1. Radial moments of electron distribution as function of shower depth.

TABLE I. Radial distribution of electrons for $s=0.6$, 1.0, and 1.5.

x	$s=0.6$	$P_r(x, s)$ $s=1.0$	$s=1.5$
0.2	11.	7.6	5.0
0.4	2.85	2.74	2.00
0.6	8.00×10^{-1}	1.01	1.08
0.8	2.30	4.84×10^{-1}	6.10×10^{-1}
1.0	8.50	2.52	3.75
1.2	4.00	1.47	2.40
1.4	2.18	8.72×10^{-2}	1.60
1.6	1.22	5.36	1.14
1.8	7.40×10^{-2}	3.49	8.20×10^{-2}
2.0	4.50	2.26	6.00
2.2	2.90	1.53	4.50
2.4	2.00	1.02	3.50
2.6	1.37	7.20×10^{-3}	2.75
2.8	9.50×10^{-3}	5.16	2.15
3.0	6.70	3.73	1.70
3.5	3.00	1.74	9.80×10^{-3}
4.0	1.36	8.72×10^{-4}	6.00
4.5	6.20×10^{-4}	4.52	3.65
5.0	2.99	2.51	2.35
5.5	1.39	1.46	1.47
6.0	6.90×10^{-5}	8.80×10^{-5}	9.80×10^{-4}
7.0	1.58		4.70

greatly changed at large r so long as we continue the curve back towards the origin smoothly and make $\langle r^0 \rangle$ unity. The shape of the function for large r is correct, since it depends only on the higher moments and not at all on the type of singularity. The amplitude, however, does depend to a slight extent on how well we have guessed the behavior at the origin. The normalization, therefore, may be off by several percent. In Table I we present our results for $x > 0.2$, but we do not believe the value for $x = 0.2$ to be very reliable.

* This work was sponsored by the AEC.

¹ L. Eyges and S. Fernbach, Phys. Rev. **82**, 287 (1951).

² L. Eyges and S. Fernbach, Phys. Rev. **82**, 23 (1951).

³ Rossi and Greisen, Revs. Modern Phys. **13**, 285 (1941).

⁴ G. Moliere, *Cosmic Radiation*, ed. Heisenberg (Dover Publications, New York, 1946).

The Quadrupole Moment Ratio of Cl^{35} and Cl^{37} from Pure Quadrupole Spectra*

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RESONANCE absorption lines due to nuclear quadrupole splittings were first observed by Dehmelt and Krüger¹ for chlorine in *trans*-dichloroethylene at 90°K. The splitting is caused by the interaction of the chlorine nuclear electric quadrupole moment with the gradient of the electric field at the nucleus. The electric field gradient can be considered to arise from charge distribution within the molecule and from other charge distributions in the crystal lattice.

Resonance absorptions similar to the above have been observed at this laboratory in several compounds. These are listed in Table I. A frequency modulated regenerative oscillator was used, and the absorption lines were viewed on an oscilloscope. In making frequency measurements, markers formed by harmonics from a Signal Corps type BC-221 frequency meter were superposed on the lines. The meter was calibrated with harmonics from a 1-Mc crystal oscillator which in turn was calibrated against radio station WWV at 10 Mc. The measured frequencies in Table I should be accurate to 0.01 percent; however, the frequency ratios may be somewhat better, because the harmonics selected for the Cl^{37} and Cl^{35} measurements were such that only a small part of the frequency meter dial travel was used. All frequencies listed are the average of eight measurements. Approximately 25-cc samples were used, and all observations were made at liquid nitrogen temperature (80°K). The relative intensities of the Cl^{35} and Cl^{37} lines agreed satisfactorily with the isotopic abundance ratio. In some cases two absorption lines were seen for each isotope. A similar effect has been observed by Dehmelt² for the iodine

TABLE I. Measured frequencies and ratios for solid chlorine compounds.

Compound	$\nu(\text{Cl}^{35})$ Mc	$\nu(\text{Cl}^{37})$ Mc	$\nu(\text{Cl}^{35})/\nu(\text{Cl}^{37})$
SOCl_2	32.0908	25.2935	1.26874
	31.8874	25.1331	1.26874
POCl_3	28.9835	22.8432	1.26880
	28.9378	22.8067	1.26883
CH_2Cl_2	35.9912	28.3673	1.26876
CHCl_3	38.3081	30.1921	1.26881
	38.2537	30.1500	1.26878
$\text{C}_6\text{H}_5\text{Cl}$	34.6216	27.2872	1.26879

resonance in SnI_4 . The effect was ascribed by him to crystallographically different sets of iodines in the lattice. Each set of iodine nuclei thus experiences its own particular electric field gradient. The splittings observed here could be caused by the same effect. In forming the frequency ratios of Table I, the higher Cl^{35} frequency (higher electric field gradient) line has been associated with the higher Cl^{37} frequency.

The frequency ratios in Table I should be the nuclear quadrupole moment ratio of Cl^{35} and Cl^{37} . The average gives

$$Q(\text{Cl}^{35})/Q(\text{Cl}^{37}) = 1.26878 \pm 0.00015.$$

All of the eight ratios agree to within ± 0.00005 .

Dehmelt and Krüger's value for the moment ratio using *trans*-dichloroethylene was 1.2661 ± 0.0002 .

Geschwind, Gunther-Mohr, and Townes³ have recently summarized published values of quadrupole coupling ratios and shown that in many cases the values for the ratio determined by different investigators varied outside of estimated experimental errors. The variation, at least in part, depended on the molecule used. This was also true for the three accurately remeasured values by the above authors for CH_3Cl , ClCN , and GeH_3Cl , which gave ratios of 1.2691 ± 0.0003 , 1.2682 ± 0.0006 , and 1.2670 ± 0.0005 , respectively. These measurements were made by observing quadrupole splittings in rotational transitions in the microwave region. Gunther-Mohr, Geschwind, and Townes⁴ considered that a reasonable explanation of the variation in experimental values on different molecules could be made by assuming a nuclear polarization by the external electric field. This effect is absent in the measurements reported here.

* This work was performed for the AEC.

¹ H. G. Dehmelt and H. Krüger, Naturwiss. **37**, 111 (1950).

² H. G. Dehmelt, Naturwiss. **37**, 398 (1950).

³ Geschwind, Gunther-Mohr, and Townes, Phys. Rev. **81**, 288 (1951).

⁴ Gunther-Mohr, Geschwind, and Townes, Phys. Rev. **81**, 289 (1951).

On Einstein's Unified Field Theory

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SOME of the most general implications of Einstein's new theory¹ are considered in a paper to appear elsewhere. The results obtained are described in the following shortened version.

By neglecting the cubes and higher orders of antisymmetric field variables a simple expression for the affine field is found in terms of the generalized nonsymmetric tensor $g_{\alpha\beta} = g_{\alpha\beta} + g_{\alpha\beta} = \mathbf{A}_{\alpha\beta} + \Phi_{\alpha\beta}$, and its first-order partial derivatives $g_{\alpha\beta,\gamma}$, where $\mathbf{A}_{\alpha\beta} = g_{\alpha\beta} = g_{\beta\alpha}$ and $\Phi_{\alpha\beta} = g_{\alpha\beta} = -g_{\beta\alpha}$. I am indebted to the referee of *The Physical Review* for directing my attention to a solution of the Γ -field obtained by Ingraham² in a slightly different way from ours.

The antisymmetric part of the Γ -field is given by

$$\Gamma_{\beta\gamma}^\alpha = -\frac{1}{2}I_{\beta\gamma}^\alpha + \mathbf{A}^{\alpha\mu}\Phi_{\beta\gamma|\mu} \quad (1)$$

subject to the four conditions:

$$\Gamma_{\alpha\gamma}^\alpha = \Gamma_\gamma = \mathbf{A}^{\alpha\mu}\Phi_{\alpha\gamma|\mu} = 0, \quad (2)$$