Erratum: On the Pressure-Volume and Pressure-Compressibility Relation of Metals

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IN the Letter to the Editor of the above title the formula for the compressibility κ is incorrect and should be replaced by the following one:

$$\frac{1}{\kappa}=\frac{1}{12\pi R}\frac{d^2U}{dR^2}-\frac{1}{6\pi R^2}\frac{dU}{dR},$$

where U denotes the lattice energy of the metal and R the radius of the elementary sphere. Accordingly, in the pressure-compressibility diagrams for the alkali and alkaline earth metals (Figs. 2 and 3) the ordinates have to be divided by $1+\frac{2}{3}\kappa P$, where P denotes the pressure and κ the compressibility at the pressure P represented in the pressure-compressibility diagrams of the quoted paper.

Mirror Levels in Li^7 and Be^7

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THE recently discovered[1] level in Be^7, which appears to be the mirror level of the well-known 478-kev level in Li^7, has an excitation of 429 kev above the ground state. If we assume that $(n-n)$ and $(p-p)$ nuclear forces are exactly equal, we must account for this 49-kev level shift in terms of the different electromagnetic properties of neutron and proton. The customary treatment of energy levels in mirror nuclei has considered only the effect caused by the differing neutron and proton charge. However, it seems that shifts of just the magnitude of that observed in Li^7-Be^7 can occur as a result of the electromagnetic spin-orbit interaction.

The electromagnetic interactions associated with acceleration of spin (Thomas effect) and magnetic interactions (spin-orbit and spin-spin interaction) are well known from atomic systems and must certainly be expected to be present in nuclei. These interactions are very sensitive to the coupling scheme used and so may give information about nuclear structure. In the Li^7-Be^7 mirror system the effect has been calculated assuming that the two levels are $^2P_{3/2,1/2}$ (the ground state has $I=3/2$). The three p-state nucleons are assumed to be coupled together in the spatially symmetric P-state required by the Wigner theory of nuclear structure.[2] Gaussian radial dependence is assumed, and the parameter which determines the range of the wave function is fixed by the experimental coulomb energy difference of the Li^7-Be^7 ground states. For these wave functions only the electrostatic Thomas effect and the magnetic spin-orbit effect contribute, and the calculated amount by which the Li^7 doublet energy should exceed Be^7 is 36.8 kev. A radial dependence which has a greater variation at the origin than the gaussian can be used to get the full 49-kev observed shift. If a wave function is constructed from the $j-j$ coupling model with $[(p_{3/2}p_{3/2})_0p_{3/2}]_{3/2}$ ground state,[3] and $[(p_{3/2}p_{3/2})_0p_{1/2}]_{1/2}$ excited state, the calculated shift is 81.5 kev.

A $j-j$ coupling model for Li^7 has been discussed by Inglis[4] in which the ground state is a mixture of $[(p_{3/2}p_{3/2})_0p_{3/2}]_{3/2}$ and $[(p_{3/2}p_{3/2})_2p_{3/2}]_{3/2}$, while the excited state is $[(p_{3/2}p_{3/2})_2 p_{3/2}]_{1/2,3/2,5/2,7/2}$. In this case the level shift in the excited state is attributed to a change in the coulomb energy. However, the lifetime for the γ-decay of the excited state is uniquely determined by the coupling scheme, since it is a magnetic dipole transition, and the half-life predicted on this model[5] is several times the experimentally observed value.[6] The coupling schemes discussed above both give half-lives in agreement with experiment.

A more complete description of these calculations will appear in the *Communications of the Danish Academy*.

The writer is pleased to express his appreciation to Professor Niels Bohr and the Institute for Theoretical Physics for their hospitality and stimulation and to Harvard University for a Parker Traveling Fellowship.

[1] Brown, Chao, Fowler, and Lauritsen, Phys. Rev. **78**, 88 (1950).
[2] E. Feenberg and E. Wigner, Phys. Rev. **51**, 95 (1937).
[3] We use the following notation: For Li^7 we give first the quantum numbers of the neutrons and their coupling, then the quantum state of the proton, and finally the total angular momentum; for Be^7 interchange neutron and proton.
[4] D. R. Inglis, Phys. Rev. **77**, 724 (1950).
[5] In accord with the angular distribution of the γ-rays, we assume that only $[(p_{3/2}p_{3/2})_2p_{3/2}]_{1/2}$ is found in appreciable amount. The amount of admixture of $[(p_{3/2}p_{3/2})_2p_{3/2}]_{3/2}$ to the ground state is fixed by the magnetic moment of Li^7.
[6] L. G. Elliott and R. E. Bell, Phys. Rev. **76**, 168 (1950).

Angular Distributions of Shower Particles as a Function of Depth*

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WE were nearing the completion of some calculations on the angular distribution of shower particles when we found that, in a recent journal, Borsellino[1] had duplicated independently some of the first stages of our work. We therefore present our results here, insofar as they go beyond Borsellino's.

We have calculated the angular distributions of high energy electrons and photons in air showers as a function of depth in the shower. To do this, we first calculated the moments of these functions and then reconstructed the distribution functions from the moments. Let $\pi(E_0, E, \theta, t)2\pi\theta d\theta dE$ be the number of electrons of energy E to $E+dE$ in the solid angle $2\pi \sin\theta d\theta \cong 2\pi\theta d\theta$ at thickness t (radiation lengths) due to an electron[2] of energy E_0 at $t=0$. The nth moment, $\langle\theta^n\rangle$, of this distribution function is defined by

$$\langle\theta^n\rangle=\frac{\int_0^\infty \pi(E_0, E, \theta, t)\theta^n\theta d\theta}{\int_0^\infty \pi(E_0, E, \theta, t)\theta d\theta}.$$

These moments can be calculated by an extension of the method used to calculate the moments at the maximum of the shower.[3] It turns out that the moments depend on E_0, and t only through the characteristic parameter s of shower theory.[4] For example, one gets, for electrons,

$$\langle\theta^n\rangle_{\text{electrons}}=(E_s/E)^n\delta_n(s),$$

where $\delta_n(s)$ is a certain function of s. The expressions we have obtained for $\delta_n(s)$ agree with those of Borsellino.[1] E_s in these equations is the "characteristic scattering energy," equal to about 21 Mev.

In addition, we have calculated the angular distribution of electrons, which we call $P(E\theta/E_s, s)$, from these moments. If we call $\pi_{\text{long.}}(E_0, E, t)$ the usual longitudinal one-dimensional distribution function and let $x=E\theta/E_s$, then $P(x, s)$ is defined by

$$\pi(E_0, E, \theta, t)2\pi\theta d\theta dE=\pi_{\text{long.}}(E_0, E, t)dE\, P(x, s)2\pi\theta d\theta. \quad (1)$$

We have calculated $P(x, s)$ for $s=0.6$ and $s=1.5$; we have previously calculated it for $s=1.0$. If $s=1.0$ corresponds to the shower maximum, $t_{\max}$, then $s=0.6$ corresponds to about half $t_{\max}$, and $s=1.5$ to twice $t_{\max}$. The shape of $P(x, s)$ is a slowly varying function of s, as one would expect from physical considerations. Our values are given in Table I.

Any method such as ours for reconstructing a function from a finite number of moments does not give the behavior near the origin very accurately; our values for x less than about 0.3 may not be reliable. For larger x, they are probably accurate to within several percent.

We call the angular distribution function for photons $Q(y, s)$,

TABLE I. The electron angular distribution function $P(x, s)$ defined by Eq. (1). Normalization such that $\int_0^\infty P(x, s)xdx = 1$.

x	$P(x, 0.6)$	$P(x, 1.0)$	$P(x, 1.6)$
0	13	9.3	6.1
0.1	9.1	7.1	5.0
0.2	6.3	5.3	4.1
0.4	3.10	2.78	2.60
0.6	1.48	1.52	1.58
0.8	7.10×10^{-1}	8.20×10^{-1}	9.00×10^{-1}
1.0	3.45	4.46	5.10
1.2	1.66	2.32	2.75
1.4	8.00×10^{-2}	1.18	1.50
1.6	3.80	6.00×10^{-2}	8.00×10^{-2}
1.8	1.80	2.90	4.18
2.0	8.20×10^{-3}	1.46	2.18
2.2	3.70	7.13×10^{-3}	1.12
2.4	1.58	3.56	5.60×10^{-3}
2.6	6.7×10^{-4}	1.74	2.75
2.8	2.7	8.29×10^{-4}	1.38
3.0	1.12	3.92	6.50×10^{-4}

where $y = W\theta/E_s$, and W is the photon energy. $Q(y, s)$ is defined analogously to $P(x, s)$.

$Q(y, s)$ is singular at the origin. For $s=1$, $Q(y, 1)$ appears to go as $1/y$ for small y.[3] Our method of moments is particularly unreliable in determining the behavior of the function at the origin if the function is singular. For $s=0.6$ and $s=1.5$ a $1/y$ singularity for $Q(y, s)$ is consistent with our results, although it seems clear that for $s=0.6$ the singularity should be stronger than for $s=1.5$. Mainly for ease in expressing the normalization, we have assumed that the singularity is exactly $1/y$ for $s=0.6$ and $s=1.5$. Our results are given in Table II, where we tabulate

TABLE II. The photon angular distribution function $Q(y, s)$. Tabulated values are $yQ(y, s)$. Normalization such that $\int_0^\infty Q(y, s)ydy = 1$.

y	$yQ(y, 0.6)$	$yQ(y, 1.0)$	$yQ(y, 1.5)$
0	4.7	3.4	2.1
0.1	2.9	2.4	1.8
0.2	1.8	1.7	1.5
0.4	6.50×10^{-1}	8.76×10^{-1}	1.00
0.6	2.61	4.61	6.42×10^{-1}
0.8	1.18	2.21	3.83
1.0	5.06×10^{-2}	1.18	2.22
1.2	2.28	5.47×10^{-2}	1.19
1.4	1.03	2.74	6.28×10^{-2}
1.6	4.80×10^{-3}	1.38	3.30
1.8	2.25	6.87×10^{-3}	1.66
2.0	1.06	3.44	8.44×10^{-3}
2.2	4.83×10^{-4}	1.72	4.02
2.4	2.25	8.59×10^{-4}	1.89
2.6	1.05	4.32	8.90×10^{-4}
2.8	5.15×10^{-5}	2.18	4.12
3.0	2.43	1.07	1.91

$yP(y, s)$. We would like to emphasize that the accuracy of our relative values for y greater than about 0.3 does *not* depend on our assumption as to the singularity at the origin, and that these values are probably good to within several percent.

* This work was supported by the AEC.
[1] A. Borsellino, Nuovo cimento **7**, 700 (1950).
[2] We discuss an incident electron for the sake of definiteness, although our results hold equally well for an incident photon, provided the energy E one is interested in is much less than the energy of the incident particle.
[3] L. Eyges and S. Fernbach, Phys. Rev. **82**, 23 (1951).
[4] See the second of Eqs. (2.55) to (2.60) in B. Rossi and K. Greisen, Revs. Modern Phys. **13**, 284 (1941).

Radial Distribution of Shower Electrons as a Function of Depth*

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THE radial distribution of shower electrons can be determined in the same way as the angular distribution.[1] The procedure is somewhat more involved in that one must calculate the mixed radial and angular moments before obtaining the radial moments alone. Let the number of electrons of energy E to $E+dE$ in the annular ring between r and $r+dr$ at depth t (r and t measured in radiation lengths) be $(E_0, E, r, t)2\pi rdrdE$ when the initiating particle is an electron of energy E_0 at $t=0$. We define the mth moment as

$$\langle r^m \rangle \equiv \int_0^\infty \pi(E_0, E, r, t) r^m r dr \Big/ \int_0^\infty \pi(E_0, E, r, t) r dr.$$

These moments are calculated by an extension of the method previously described.[2] In this case one obtains $\langle r^m \rangle$ as a function of the parameter s, where s is determined from E_0 and t by use of the relation

$$\log(E_0/E) + \lambda_1'(s)t = 0.$$

$\lambda_1'(s)$ is tabulated in Rossi and Greisen's[3] article.

The moments are most simply written as

$$\langle r^m(s) \rangle = (E_s/E)^m \rho_m(s).$$

Figure 1 shows $\rho_m(s)$ plotted for $m=2$, 4, and 6.

From these moments we calculate the distribution function $P_r(Er/E_s, s)$, where $P_r(Er/E_s, s)rdr$ is proportional to the number of electrons of energy E at a depth corresponding to s in an annular ring between r and $r+dr$. The normalization is taken as $\int_0^\infty P_r(x, s)xdx = 1$, where $x = Er/E_s$, E_s being as usual approximately 21 Mev.

In a previous paper,[2] we discussed this calculation for the shower maximum. This corresponds to an $s=1$. In addition, we have now calculated the two cases $s=0.6$ and $s=1.5$ which correspond approximately to half and twice the depth at the shower maximum, respectively.

The radial distribution is singular at the origin, this singularity being of order $r^{-1/3}$ at the shower maximum, if we assume Moliere's[4] calculations to give an accurate picture of the shower spread for small r. In calculating the present distribution function we again assume the functions singular, but have no way of specifying the order of the singularity. This means that we have to guess the behavior of the function for small r. Since the major contribution to the area under the distribution function, namely $\langle r^0 \rangle$, comes from small r, the amplitude of the function is not

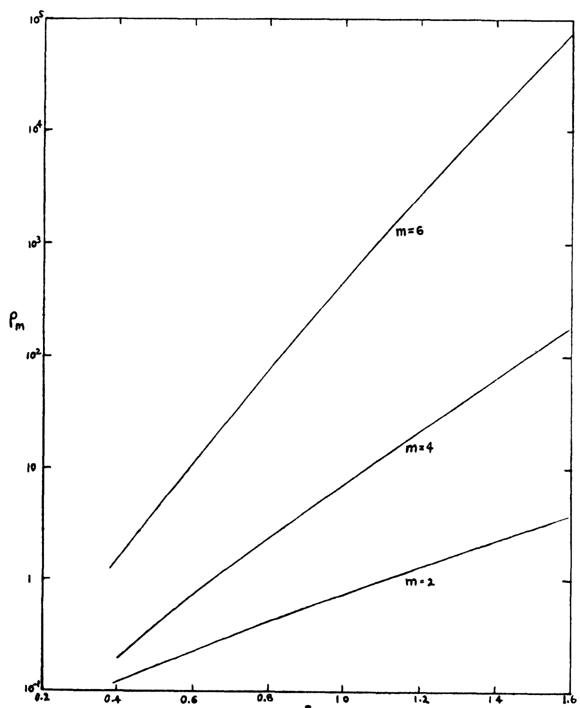

FIG. 1. Radial moments of electron distribution as function of shower depth.