nucleus by observing the angular correlation of the emitted particles as a function of time.[2] One would expect a correlation to persist to times of the order of the total spin relaxation time T_2 of the intermediate nuclei j in the sample, perhaps milliseconds or better.[3] We show here how the theoretical delayed coincidence correlation function describing the effect of the magnetic field is to be obtained from the usual correlation function.

The equations of the Weisskopf-Wigner treatment of the decay[4] can be made to yield a function $W(\Omega_1;\Omega_2;t-t')$ which can be interpreted as the rate (steradian^{-2} sec^{-1}) at which particles of the second transition are emitted at time t with direction $\Omega_2\equiv\theta_2,\varphi_2$ when the preceding particle in the cascade was emitted at time t' with direction $\Omega_1\equiv\theta_1,\varphi_1$, the initial nuclei j_1 being randomly oriented at time zero. As indicated, this rate comes out to be a function of $t-t'$ only, and we refer to it as the delayed coincidence angular correlation function.[5] If ω_0 is the circular Larmor frequency of the intermediate nucleus j in the external magnetic field (along the z-axis), then the delayed coincidence correlation is given by

$$W(\theta_1,\varphi_1;\theta_2,\varphi_2;t)=4\pi\tau_1\exp(-t/\tau)\times\Sigma_{mm'}X_{mm'}(\theta_1,\varphi_1;\theta_2,\varphi_2)\exp(i\omega_0(m-m')t), \quad (1)$$

where

$$X_{mm'}(\theta_1,\varphi_1;\theta_2,\varphi_2)=(2j_1+1)^{-1}(2\pi/\hbar)^2\rho_1\rho_2\mathcal{S}\Sigma_{m_1m_2}\times[(j_2m_2|\Omega_2,\sigma_2|jm)(jm|\Omega_1,\sigma_1|j_1m_1)\times(j_2m_2|\Omega_2,\sigma_2|jm')^*(jm'|\Omega_1,\sigma_1|j_1m_1)^*]. \quad (2)$$

In Eq. (1), τ_1 and τ are the lifetimes of the initial and intermediate nuclei, respectively, and in Eq. (2), $\mathcal{S}$ is the invariant summation over the spin coordinates σ_1 and σ_2 of the emitted particles, and ρ_1 and ρ_2 are densities-in-energy of the states of the emitted particles. The usual instantaneous angular correlation[6] is

$$W(\theta_1,\varphi_1;\theta_2,\varphi_2)=\tau W(\theta_1,\varphi_1;\theta_2,\varphi_2;0)=4\pi\tau\tau_1\Sigma_{mm'}X_{mm'}(\theta_1,\varphi_1;\theta_2,\varphi_2), \quad (3)$$

normalized to $\int W(\theta_1,\varphi_1;\theta_2,\varphi_2)(d\Omega_1/4\pi)d\Omega_2=1$. It is actually a function only of the angle θ between Ω_1 and Ω_2, of course.

The transformation properties of the nuclear matrix elements[7] can be used to obtain

$$X_{mm'}(\theta_1,\varphi_1;\theta_2,\varphi_2)=X_{mm'}(\theta_1,\varphi_1;\theta_2,\varphi_2+\alpha)\exp(i\alpha(m'-m))$$

for rotations of Ω_2 by angle α around the z-axis. If this is substituted into Eq. (1) with $\alpha=\omega_0t$, one discovers that

$$W(\theta_1,\varphi_1;\theta_2,\varphi_2;t)=(1/\tau)\exp(-t/\tau)W(\theta_1,\varphi_1;\theta_2,\varphi_2+\omega_0t). \quad (4)$$

The delayed coincidence angular correlation pattern is just the instantaneous pattern precessing with the intermediate nucleus around the external field.[8] If the instantaneous correlation is given as a power series in $\cos\theta$, the corresponding delayed coincidence correlation is obtained as a fourier series in ω_0t by transforming the instantaneous correlation to a Legendre series:[9] $W(\theta)=\Sigma\gamma_lP_l(\cos\theta)$, and then applying the spherical harmonic addition theorem[9] to

$$P_l(\cos\theta)=P_l(\cos\theta_1\cos\theta_2+\sin\theta_1\sin\theta_2\cos(\varphi_1-\varphi_2)).$$

For example, the instantaneous correlation $W(\theta)=1+r\cos^2\theta=1+(r/3)+(2r/3)P_2(\cos\theta)$ gives the delayed coincidence correlation (omitting the decay and normalization factors):

$$W(\theta_1,\varphi_1;\theta_2,\varphi_2;t)=1+(r/3)+(r/6)(3\cos^2\theta_1-1)(3\cos^2\theta_2-1)+(r/2)\sin2\theta_1\sin2\theta_2\cos(\varphi_1-\varphi_2-\omega_0t)+(r/2)\sin^2\theta_1\sin^2\theta_2\cos2(\varphi_1-\varphi_2-\omega_0t). \quad (5)$$

In general, the odd harmonics of ω_0t drop out if either counter is in the equatorial plane, θ_1 or $\theta_2=90°$; and there are no oscillating terms at all if either counter is on the axis, θ_1 or $\theta_2=0°$ or 180°. The relative amplitudes of the various harmonics are complicated functions of θ_1 and θ_2, in general, and the best position for the counters has to be worked out for each case separately. For the correlation (5), one maximum of the first harmonic amplitude relative to the non-oscillating terms occurs when both counters are on the cone:

$$\cos^2\theta_1=\cos^2\theta_2=(1/2)-(r/(8+6r))+O(r^3).$$

It is to be noted that both the instantaneous correlation and the gyromagnetic ratio of nucleus j can be determined by measuring the distribution of pulses in time in one counter; e.g., if the counter is in the equatorial plane, $W(90°,\varphi;90°,\varphi;t)=(1/\tau)\exp(-t/\tau)W(\omega_0t)$, where $W(\theta)$ is the instantaneous correlation.

If the highest power of $\cos\theta$ in the instantaneous correlation is $\cos^L\theta$, the delayed correlation contains harmonics up to $\cos(L\omega_0t)$, and the total gate width Δt should be narrow enough so that these harmonics are not washed out by averaging over the gates: if $L\omega_0\Delta t\geq1$, gate width correction factors must be introduced. For the same reason, the variation $(T_2^*)^{-1}$ of the Larmor frequency due to inhomogeneities in the applied magnetic field should be kept small compared to $(L\tau)^{-1}$.

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Electron Density and Light Intensity Decay in Cesium Afterglows*

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THE decay of the electrodeless discharge through purified cesium has been studied by the previously described techniques[1] of microwave electron density, optical spectrographic, and optical light intensity measurements as a function of time after termination of the exciting fields. In the pressure range 0.02 to 0.1 mm, we found that the electron density, n, decays exponentially with a time constant, τ_1, increasing with pressure from 0.4 millisecond at 0.02 mm to about 1 millisecond at 0.1 mm. A minimum in the light intensity is found at 40 microseconds after cut-off, followed at about 80 microseconds by a broad maximum. The ratio I_{max}/I_{min} is about 8 at the lowest pressure and decreases with pressure. Higher input power during the discharge moves the maximum to shorter times. After the maximum, the light intensity decay is exponential with an associated time constant τ_2 consistently closely equal to $\tau_1/2$, which shows that the light intensity is proportional to n^2. Spectrograms show an intense line spectrum during the discharge, the usual CsI spectrum coexisting with a great number of other lines belonging to the excited cesium ion. In the afterglow, however, the spectrum is practically a pure band spectrum, which was not analyzed in detail, but which shows up in the same wavelength range as the bands found previously in absorption in cesium vapor.[2]

The low pressure electron density data follows the behavior expected for ambipolar diffusion quite well. In order to compare the ion mobility data for cesium ions through cesium vapor with previous measurements[3] of the mobility of cesium ions through rare gases, the electron temperature at moderate times in the afterglow at a pressure of 0.036 mm was estimated by a standard kinetic theory calculation as 1400°K. On this basis, our data gives a value of the mobility, k_0, of about 50 cm^2/volt-sec calculated for 1-mm pressure and 0°C. This value is rather surprising, since the measured values of the mobility of cesium ions through rare

gases are on the order of 10^3. The formation of molecular ions and the possibility of some mechanism such as charge exchange may be used to explain the low observed value of the effective mobility of cesium ions in cesium.

Above about 0.3-mm pressure, electron density decay follows the recombination law, $dn/dt=-\alpha n^2$, very well. The recombination coefficient, α, has a value of the order of 10^{-7} cm^3/ion-sec. At constant temperature this coefficient is found to be pressure dependent. At 270°C, for instance, α varies from 3.5×10^{-7} at 0.32-mm pressure to 1.2×10^{-6} at 0.68 mm. At 285°C α varies from 3.5×10^{-7} at 1 mm to 1.45×10^{-6} at 2.6 mm. At times in the afterglow greater than 400 microseconds the light intensity is again found to be proportional to the square of electron density. At shorter times, however, this relationship does not hold, indicating that recombination is initially somewhat slower. The use of simple optical filters shows that the red portion of the spectrum decays initially somewhat faster than the violet. Spectrograms show much less of the cesium ion line spectrum during the discharge than was found at low pressures. The afterglow spectrum is again almost purely molecular.

Our data are in agreement with the following mechanism for recombination-type electron removal in the afterglow. Cesium molecular ions are formed in triple collisions between an atomic ion and two neutral molecules. This is followed by recombination between molecular ions and electrons. This process results in the formation of excited molecules which radiate a band spectrum in the afterglow, thus making dissociative recombination an unlikely explanation for the high values of recombination coefficient observed. The initial maximum of the light intensity as observed at low pressure would then correspond to the initial increase of molecular ion concentration (since the concentration of such ions during the discharge is probably small, as evidenced by the line spectrum observed) followed by a decrease due to diffusion to the walls and recombination. It is quite possible that some equilibrium ratio between the density of atomic and molecular ions develops in the afterglow. This ratio would depend on pressure, a larger proportion of the ions being molecular at higher pressures. This would explain the observed pressure variation of α if we assume also that the recombination coefficient for molecular ions is the limiting value observed at high pressure and that the recombination coefficient for atomic ions is much smaller.

No dependable values for the absolute light intensity were obtained in this work (as have been obtained in this laboratory for several other gases), because the spectrograms showed that most of the energy was concentrated in the infrared, beyond the range of our photomultiplier equipment.

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Charge Renormalization in the Hartree Approximation*

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THE field equations and canonical commutation relations of the Dirac-Maxwell fields are

$$\{\gamma_\mu[(\partial/\partial x_\mu)-ie_0A_\mu{}^0/\hbar]+\kappa\}\psi(x)=0, \tag{1}$$

$$\Box A_\mu{}^0=J_\mu{}^{0\,\mathrm{ext}}(x)+\tfrac{1}{2}ie_0[\psi^+(x),\gamma_\mu\psi(x)], \tag{2}$$

$$[\psi^+(\mathbf{r},t),\psi(\mathbf{r}',t)]_+=\gamma^4\delta(\mathbf{r}-\mathbf{r}'), \tag{3}$$

$$[A_\mu{}^0(\mathbf{r},t),\partial A_\mu{}^0(\mathbf{r}',t)/\partial t]=i\hbar\delta(\mathbf{r}-\mathbf{r}'), \tag{4}$$

in which $J_\mu{}^{0\,\mathrm{ext}}(x)$ is the external charge and current distribution. If one uses the Hartree approximation of replacing the electron-positron current operator $J_\mu{}^0(x)=\frac{1}{2}ie_0[\psi^+(x),\gamma_\mu\psi(x)]$ by its expectation value $\langle J_\mu{}^0(x)\rangle$ in Eq. (2), one obtains

$$\Box A_\mu{}^0=J_\mu{}^{0\,\mathrm{ext}}(x)+\langle J_\mu{}^0(x)\rangle. \tag{5}$$

Any covariant gauge invariant evaluation of $\langle J_\mu{}^0(x)\rangle$ for the "vacuum" state[1] gives the following form:

$$\langle J_\mu{}^0(x)\rangle_{\mathrm{vac}}=-a\Box A_\mu{}^0(x)-\int K^0(x-x')\Box' A_\mu{}^0(x')dx' \tag{6}$$

for the parts of $\langle J_\mu{}^0(x)\rangle$ which are linear in the field strength, and we will neglect all nonlinear effects. In Eq. (6) $K^0(x-x')$ is an invariant function of $(x-x')$ which describes the Uehling effect and which is not as singular as a δ-function. Combining Eq. (5) and Eq. (6), one obtains

$$(1+a)\Box A_\mu{}^0+\int K^0(x-x')\Box A_\mu{}^0(x')dx'=J_\mu{}^{0\,\mathrm{ext}}(x). \tag{7}$$

Carrying out a charge and field renormalization, one obtains

$$\begin{aligned} &A_\mu=(1+a)^{\frac{1}{2}}A_\mu{}^0, && e=(1+a)^{-\frac{1}{2}}e_0,\\ &J_\mu{}^{\mathrm{ext}}(x)=(1+a)^{-\frac{1}{2}}J_\mu{}^{0\,\mathrm{ext}}, && K(x-x')=(1+a)^{-1}K^0(x-x'),\\ &J_\mu(x)=(1+a)^{-\frac{1}{2}}J_\mu{}^0(x). \end{aligned} \tag{8}$$

Under this renormalization, Eqs. (1) and (5) remain unaltered in form apart from the Uehling terms, but e_0 is replaced by e, and $e_0A_\mu{}^0$ is replaced by eA_μ. Equation (3) remains unaltered, while (4) becomes

$$[A_\mu(\mathbf{r},t),\partial A_\mu(\mathbf{r}',t)/\partial t]=i\hbar(1+a)\delta(\mathbf{r}-\mathbf{r}'). \tag{9}$$

The fact that these equations maintain the same form shows that a charge renormalization will not change the energy levels of atomic systems except for the Uehling terms implied by $K(x-x')$, if the experimental value of $e^2/\hbar$ is used. However, by Eq. (9) we see that the value of Planck's constant for the electromagnetic field will be larger than the Planck's constant of the matter field by the factor $1+a=(e_0/e)^2$. We thus see that charge renormalization is not consistent with the canonical formalism, at least in the Hartree approximation. We note, however, that if one insists on evaluating a so that it is nonvanishing, the error introduced by this can be corrected by reducing the electromagnetic field commutators by the factor $1/(1+a)$.

If the commutator relation Eq. (9) is to remain consistent in time with the equations of motion, Eq. (7), one may infer that $\Box A_\mu(x)=J_\mu{}^*(x)$, in which $J_\mu{}^*(x)$ is the experimental external current distribution. Thus, one obtains

$$J_\mu{}^*(x)+\int K(x-x')J_\mu{}^*(x')dx'=J_\mu{}^{\mathrm{ext}}(x). \tag{10}$$

We now note that the conventional evaluation[2] of $K(x-x')$ is such that $J_\mu{}^*(x)$ is not uniquely determined by the fictitious external charge and current distribution $J_\mu{}^{\mathrm{ext}}(x)$, because Eq. (10) has nonvanishing solutions when $J_\mu{}^{\mathrm{ext}}(x)$ is put equal to zero. Since it is $J_\mu{}^*(x)$ which determines the energy levels of atomic systems, it is convenient to arrange the calculation of the ambiguous quantity $\langle J_\mu(x)\rangle_{\mathrm{vac}}$ so that its parts, which are linear in the field strengths, vanish identically. This can be done.[3]

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Diffuse X-Ray Scattering by Disordered Binary Alloys

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THE x-ray scattering from binary alloys of the substitutional type shows two effects due to the degree of order (or disorder): (1) Sharp superlattice reflections when long range order is present; and (2) diffuse scattering which depends on the degree of short range order. Theoretical intensity formulas for these two types of scattering have been given by MacGillavry and Strijk[1]