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The Density of States in Light Nuclei

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The number of states of energy less than 25 Mev is calculated for a simple model of a nucleus of 20 nucleons. Kinetic energy and the effect of symmetry on the potential energy were considered to be the only systematic contributions to the excitation. A method of classification of states by supermultiplets, using mainly their projections on the S–T plane, is presented. An empirical formula for level density in light nuclei is deduced.

I. INTRODUCTION

THE neutron scattering measurements of Barschall *et al.* at Wisconsin and of the group at Minnesota[1] have made valuable additions to our knowledge of the energy levels in light nuclei. These results, combined with those on proton capture and the well-known frequency of resonance capture of slow neutrons in heavy elements, provide a considerable body of empirical information on the density of energy levels in nuclei, at least for relatively low angular momentum.

Unfortunately, the empirical evidence cannot be considered to be complete, except in limited instances, because of practical difficulties in the resolution of individual peaks and in the identification of the angular momenta of the resonance levels. Moreover, the statistical theories are not wholly adequate to an interpretation of most of the more precise information since this covers a range of intermediate mass numbers, $20 \leq A \leq 40$, in which such theories barely apply, even for low angular momenta.

In order to assist in the study of nuclear energy levels, we have undertaken a computation of the level distribution in a model nucleus. For this purpose, we have chosen $A=20$ as a compromise between feasibility and a reasonable representation of light nuclei. The work, thus, goes farther than the previous study by Bardeen and Feenberg[2] in that it relates the kinetic and potential energies to an explicit nuclear model, with more recent estimates of the importance of the potential energy, and, therefore, leads to a more complete description of the angular momenta and parities of the levels. The underlying assumption remains the same; namely, that the over-all density of levels in the true nucleus with $A=20$ is essentially the same as that calculated in the model which considers the kinetic energy and the effect of symmetry on the potential energy to be the main systematic contributions to the energy of excitation. It is also assumed that spin-orbit splitting, correlations, coulomb energy, etc., remove the degeneracy left by considering only kinetic and symmetry effects, except, of course, that a state of total angular momentum (nuclear spin) j is $(2j+1)$-fold degenerate. No claim is made concerning the exact location of any individual level.

A secondary purpose of this work is to enlarge upon a calculational scheme which may be adapted to machine computations for more complex nuclei, either for level densities or for ground-state calculations. The $A=20$ results are also used to derive a rough empirical formula for level density which may be applicable to light nuclei. Qualitatively, the results of the work presented here are similar to those of reference 2.

II. MODEL OF THE NUCLEUS $A=20$

The calculations below are similar to those made on lighter nuclei by Feenberg and Phillips[3] and by Motz and Feenberg.[4] It is assumed that the nuclear wave function can be written as a sum of products of individual particle wave functions. The individual nuclear waves are taken to be the stationary states of definite n, l, and m for a particle moving freely in a spherical potential of finite depth[5] inside the radius R:

$$\psi_{n,l,m}(\mathbf{r}) = c(\pi/2k_n r)^{\frac{1}{2}} J_{l+\frac{1}{2}}(k_n r) P_l^m(\theta) e^{im\phi}, \qquad r<R \quad (1)$$

$$= c'(\pi/2k_n r)^{\frac{1}{2}} H_{l+\frac{1}{2}}^{(1)}(\kappa_n r) P_l^m(\theta) e^{im\phi}, \quad r>R$$

$$H_{l+\frac{1}{2}}^{(1)}(\kappa_n R) J_{l+\frac{1}{2}}(k_n R)' - J_{l+\frac{1}{2}}(k_n R) H_{l+\frac{1}{2}}^{(1)}(\kappa_n R)' = 0, \quad (2)$$

and $J_{l+\frac{1}{2}}(k_n r_i)=0$ for $n-l$ values of $|r_i|<R$. If the potential energy of the well is zero for $r>R$ and

$$v_0 = -\hbar^2 k_0^2/2\mu, \quad r \leqq R, \quad (3)$$

then $\kappa_n^2 = k_0^2 - k_n^2$, with μ as the reduced mass of the nucleon.

The determination of the solution for k_n and κ_n of Eqs. (2) and (3) was effected by graphical methods and by using recursion relations to determine the functions. Thus Eq. (2) is readily proved to be equivalent to

$$\xi_l(kR) + \eta_l(\kappa R) = 0, \quad (4)$$

with

$$\xi_l(x) = xJ_{l-\frac{1}{2}}(x)/J_{l+\frac{1}{2}}(x), \quad \eta_l(y) = iyH_{l-\frac{1}{2}}^{(1)}(iy)/H_{l+\frac{1}{2}}^{(1)}(iy),$$

$$\xi_0(x) = x\cot x, \quad \eta_0(y) = y,$$

$$\xi_l(x) = x^2/(2l-1-\xi_{l-1}), \quad \eta_l(y) = y^2/(2l-1+\eta_{l-1}).$$

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[1] Hornyak, Lauritsen, Morrison, and Fowler, Revs. Modern Phys. **22**, 291 (1950); R. K. Adair, Revs. Modern Phys. **22**, 249 (1950).

[2] J. Bardeen and E. Feenberg, Phys. Rev. **54**, 809 (1938).

[3] E. Feenberg and M. Phillips, Phys. Rev. **51**, 597 (1937).

[4] L. Motz and E. Feenberg, Phys. Rev. **54**, 1055 (1938).

[5] H. Margenau, Phys. Rev. **46**, 613 (1934).

The value of the nuclear radius assumed here is that deduced for the ground states by Wigner's analysis,[6]

$$\begin{aligned} R &= 1.45A^{\frac{1}{3}}\times 10^{-13}\text{ cm} \\ &= 3.07\times 10^{-13}\text{ cm for } A=20. \end{aligned} \tag{5}$$

In this model the depth of the potential is assumed to be the same for neutrons and protons. Since the value chosen for the depth is an arbitrary matter, we have chosen such a depth that 16.8 Mev is required to remove a neutron, leaving the residual nucleus in its lowest state. A description in terms of individual nucleons probably is better served by such a choice than by matching the total binding energy. The effective depth for protons may be a little lower, but the exact choice of V_0 is a minor detail which need not be considered too closely.

Wigner's analysis[6] is also used to obtain the magnitude of the effect of symmetry on the potential energy. A state belonging to the supermultiplet (STY) has a positive potential energy V_m (in addition to a constant negative potential):

$$\begin{aligned} V_m &= \tfrac{1}{2}L[S(S+4)+T(T+2)+Y^2], \\ L &= 40/A \text{ Mev}, \quad S\geq T\geq Y. \end{aligned} \tag{6}$$

It is in this description of the potential energy that the method of this paper and that of reference 2 departs from the earlier statistical analyses of Bethe[7] and of van Lier and Uhlenbeck.[8] In those papers the number of states was computed for a given number of neutrons and protons without counting the multiplicity of states that can be obtained by exchanging charge between neutrons and protons. Hence, they approximate the effect of symmetry by implying infinite Heisenberg forces.

III. CALCULATION OF SYMMETRY CHARACTERS

The kinetic energy of a given configuration is determined by the number of particles in each of the individual orbits; 1*s*, 2*p*, 3*d*, 2*s*, 4*f*, etc. The next step is to determine the number of states of given orbital angular momentum that can be obtained for a given configuration. This is done without reference to whether a nucleon is a neutron or proton, or to its spin orientation. A given configuration has a certain maximum value M for the sum over individual quantum numbers m, which is obtained by putting as many nucleons into the highest m orbits as possible but no more than four to an orbit. The number of ways of selecting individual orbits in order to obtain sums over m_i lower than M is then determined by inspection. In principle these sums run from M, $M-1$, $\cdots$ to $-M$ but the determination is stopped at $\Sigma m_i=0$ because of the symmetry in positive and negative values of Σm_i. When $\Sigma m_i<M$, there are, in general, a number of possible arrangements of nucleons each of which is characterized by a number, ν_4 of orbits filled by four nucleons, ν_3 by three nucleons, ν_2 by two, and ν_1 by one. It is convenient to denote each such arrangement by the symbol $\{\nu_4\nu_3\nu_2\nu_1\}$. However, a given set of values ν_4, ν_3, ν_2, ν_1 may occur more than once for a given Σm_i if there are alternate ways of distributing the nucleons among distinct orbits of distinct n, l, m quantum numbers.

After one determines the number of ways a value of Σm_i can be achieved in terms of the $\{\nu_4\nu_3\nu_2\nu_1\}$ distributions, the next point of interest is the number of ways in which a particular $\{\nu_4\nu_3\nu_2\nu_1\}$ can be achieved with a given number of neutrons and protons and a given net spin. Each distinct arrangement of charge and spin represents, of course, a distinct wave function, and functions of definite symmetry type are linear superpositions of them. It is to be noted first that ν_4 orbits, which are occupied by four nucleons, have no freedom as to composition. Furthermore, substitution into any of the ν_3 orbits, occupied by three nucleons, is restricted by the Pauli principle so that the freedom of choice is the same as in a singly occupied orbit so far as spin and charge are concerned. The third zeta-component, $Y_\zeta=\frac{1}{2}\Sigma s_z\tau_\zeta$ in Wigner's notation,[9] differs in sign for threefold and singly occupied orbits, but the necessity for taking this into consideration is so rare among the lower supermultiplets that it is convenient to ignore it except in the special cases in which it is decisive. This means that we are interested primarily in distributing N neutrons and P protons among which occur α upward spins and β downward spins in $\nu_1+\nu_3$ singly occupied spaces and ν_2 doubly occupied spaces. The numbers N and P are not true neutron and proton numbers, nor are α and β true spin numbers, but rather "excess" numbers related to the ν's and to spin component S_z and isotopic spin component T_ζ by

$$\begin{aligned} N+P=\alpha+\beta=\nu_1+\nu_3+2\nu_2, \\ T_\zeta=\tfrac{1}{2}(N-P), \quad S_z=\tfrac{1}{2}(\alpha-\beta). \end{aligned} \tag{7}$$

Let p be the number of orbits (n, l, m) occupied by two protons and n the number occupied by two neutrons; then ν_2-n-p is the number occupied by one neutron and one proton. By combinatorial analysis, one readily computes the number of arrangements in a given $\{\nu_4\nu_3\nu_2\nu_1\}$ and given n and p. The total number of wave functions occurring with particular values of S_z and T_ζ is a sum over all n and p.

$$\begin{aligned} f_{S_zT_\zeta}(\nu_2, \nu_1+\nu_3) = \sum_n \sum_p \binom{\nu_2}{n}\binom{\nu_2-n}{\nu_2-n-p} \\ \times\binom{\nu_1+\nu_3}{P-\nu_2-p+n}\binom{N+P-2n-2p}{\alpha-n-p}, \end{aligned} \tag{8}$$

where $\binom{x}{y}=x!/y!(x-y)!$

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[6] E. Wigner, *University of Pennsylvania Bicentennial Conference* (University of Pennsylvania Press, Philadelphia, 1941).

[7] H. A. Bethe, Phys. Rev. **50**, 332 (1936).

[8] C. van Lier and G. E. Uhlenbeck, Physica **4**, 531 (1937).

[9] E. Wigner, Phys. Rev. **51**, 106 (1937).

TABLE I. Reduction of space arrangements.

$\{\nu_4\nu_3\nu_2\nu_1\}$	$\Sigma a_w(STY)_w$
{2000}	(000)
{1101}	(110)+ (000)
{1020}	(200)+ (110)+ (000)
{1012}	(211)+ (200)+2(110)+ (000)
{0210}	(21–1)+ (200)+2(110)+ (000)
{1004}	(222)+3(211)+2(200)+3(110)+ (000)
{0202}	(220)+ (211)+ (21–1)+ (200)+4(110)+2(000)
{0121}	(310)+ (220)+2(211)+2(21–1)+3(200)+5(110) +2(000)
{0113}	(321)+2(310)+ (222)+3(220)+3(21–1)+6(211) +5(200)+9(110)+3(000)
{0105}	(332)+4(321)+5(310)+5(222)+6(220)+15(211) +5(21–1)+10(200)+15(110)+4(000)

The numbers $f_{S_zT_\zeta}$ form a matrix in the S–T space with elements corresponding (in our case of 20 particles) to positive and negative integral values of S_z and T_ζ. They are the sums of the numbers of arrangements of the nucleons and indicate nothing concerning the symmetry of the wave function. Since the potential energy of the nucleus is assumed to depend strongly on the symmetry character, the next step is to determine the composition of a given arrangement $\{\nu_4\nu_3\nu_2\nu_1\}$ as a sum of arrangements having definite symmetry. The most convenient manner of designating states of definite symmetry is the (STY) formalism of Wigner. The substates for a given (STY) are readily computed, as shown in reference 9, by expanding those for the four-valued spin in terms of the substates of the three-valued spin and expanding the latter in terms of the ordinary two-valued spin. All substates of supermultiplets used in this work were determined by this method. Summing over the Y_ζ, one gets the projection of a given (STY) on the S–T plane

$$(S_zT_\zeta)_w=\sum_{Y_\zeta}(S_zT_\zeta Y_\zeta)_w, \tag{9}$$

where the subscript w refers to a given supermultiplet, $(STY)=w$. The problem is, then, one of reducing the matrix with elements given by Eq. (8) to

$$f_{S_zT_\zeta}=\sum_w a_w(S_zT_\zeta)_w, \tag{10}$$

thereby reducing all arrangements $\{\nu_4\nu_3\nu_2\nu_1\}$ to sums over supermultiplets w with multiplicity a_w. The results for the arrangements used in this work are given in Table I. In most instances the expansion of $f_{S_zT_\zeta}$, Eq. (10), is unique. There are exceptions, however, since the projection on the S–T plane of (211) and (21–1) are identical, and also the projections of (222) and (220) are linearly related through lower supermultiplets, *viz.*[10]

$$\begin{aligned}&\text{Projection}[(222)+(211)+(200)]\\&\qquad=\text{Projection}[(220)+(110)+(000)].\end{aligned} \tag{11}$$

Actually, the ambiguity in (211) and (21–1) is unimportant so far as the energy is concerned, Eq. (6). The method used for resolving the ambiguities was to examine in detail the distributions of charge and spin corresponding to a $\{\nu_4\nu_3\nu_2\nu_1\}$. Suppose we have an f-matrix containing (STY) projections for which $S>3$ so that we need to determine how many (222) and how many (220) supermultiplets are contained in it. We consider the total of all distributions with $\{\nu_4\nu_3\nu_2\nu_1\}$ that can be achieved with $N-P=4$ and $\alpha-\beta=4(S_z=T_\zeta=2)$ and calculate Y_ζ for each. There will be so many states, q_2, for $Y_\zeta=2$. Of these a certain number, r_2, will be substates of supermultiplets with $S>2$, which we have determined by Wigner's method cited earlier. Hence, the number of (222) supermultiplets must be q_2-r_2. As mentioned previously, the number of examples requiring such explicit consideration is so small as to make the method of expanding in terms of projections on the S–T plane convenient.

IV. CLASSIFICATION OF STATES

The lowest state of 20 nucleons is assumed to be a $(1s)^4(2p)^{12}(3d)^4=s^4p^{12}d^4$ configuration. All other configurations obtained by promoting nucleons into orbits of higher kinetic energy and having a total increase of kinetic energy less than 25 Mev were determined. The number of ways of constructing states of given L_z and the supermultiplet composition of each was determined by the method described above. If the sum of the $(STY)_i$ for a certain L_z be denoted by $\eta(L_z)$, the number of states of definite L is $\eta(L)$,

$$\eta(L)=\eta(L_z)-\eta(L_z+1).$$

States of $L=0,\ 1,\ 2\cdots$ are designated $S,\ P,\ D\cdots$, respectively, in the customary way. The potential energy of each supermultiplet was calculated from

TABLE II. Classification of states of excitation less than 25 Mev.

Configuration	Kinetic energy (Mev)	Potential energy (Mev)	Total energy (Mev)	Supermultiplet	L states
$s^4p^{12}d^4$	0	0	0	(000)	SD^2G^2HIL
$s^4p^{12}d^3s$	5.4	0	5.4	(000)	$SDFGI$
$s^4p^{12}d^4$	0	8.0	8.0	(110)	$P^2D^2F^3G^2H^2IK$
$s^4p^{12}d^2s^2$	10.8	0	10.8	(000)	SDG
$s^4p^{12}d^4$	0	12.0	12.0	(200)	$S^2D^2FG^2I$
$s^4p^{11}d^5$	12.8	0	12.8	(000)	$S^2P^6D^9F^{11}G^{11}H^{10}I^8K^6L^4M^2N$
$s^4p^{12}d^3s$	5.4	8.0	13.4	(110)	$SPD^3F^2G^2HI$
$s^4p^{12}d^3f$	14.5	0	14.5	(000)	$SP^3D^3F^5G^4H^4I^3K^2LM$
$s^4p^{12}d^4$	0	16.0	16.0	(211)	P^2DF^2GH
$s^4p^{12}ds^3$	16.2	0	16.2	(000)	D
$s^4p^{12}d^3s$	5.4	12.0	17.4	(200)	PD^2FGH
$s^4p^{11}d^4s$	18.2	0	18.2	(000)	$S^2P^7D^9F^{11}G^{10}H^9I^6K^4L^2M$
$s^4p^{12}d^2s^2$	10.8	8.0	18.8	(110)	$SPDFG$
$s^4p^{12}d^2sf$	19.9	0	19.9	(000)	$P^2D^2F^3G^2H^2IK$
$s^4p^{11}d^5$	12.8	8.0	20.8	(110)	$S^7P^{18}D^{27}F^{30}G^{30}H^{25}I^{19}K^{12}L^7M^3N$
$s^4p^{12}d^3s$	5.4	16.0	21.4	(211)	$P^2D^2F^2GH$
$s^4p^{12}s^4$	21.6	0	21.6	(000)	S
$s^4p^{12}d^3f$	14.5	8.0	22.5	(110)	$S^2P^7D^9F^{11}G^{10}H^9I^6K^4L^2M$
$s^4p^{12}d^2s^2$	10.8	12.0	22.8	(200)	SDG
$s^3p^{12}d^5$	23.1	0	23.1	(000)	$SP^2D^3F^4G^4H^3I^3K^2LM$
$s^4p^{11}d^3s^2$	23.6	0	23.6	(000)	$SP^5D^6F^7G^5H^4I^2K$
$s^4p^{12}d^4$	0	24.0	24.0	(222)	D
$s^4p^{12}ds^3$	16.2	8.0	24.2	(110)	D
$s^4p^{11}d^5$	12.8	12.0	24.8	(200)	$S^3P^{12}D^{15}F^{18}G^{15}H^{13}I^8K^5L^2M$

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[10] The next such relation is Projection[(330)+(211)+(200)] = Projection[(332)+(310)+(000)].

TABLE III. Double multiplet composition.

(STY)	$(2T+1)(2S+1)$			
(000)	11			
(110)	13	31 33		
(200)	11 15	33	51	
(211)	13	31 33 35	53	
(220)	11 13	31 $(33)^2$ 35	53 55	
(222)	11	33	55	
(310)	13 15 17	31 $(33)^2$ $(35)^2$ 37	51 $(53)^2$ 55	71 73
(321)	13 15	31 $(33)^2$ $(35)^2$ 37	51 $(53)^2$ $(55)^2$ 57	73 75
(332)	13	31 33 35	53 55 57	75 77

Eq. (6) and added to the kinetic energy of excitation. Restriction of the total excitation to 25 Mev eliminates most of the states that have any appreciable kinetic energy because of the high potential energy of states of low symmetry. The results on symmetry character and L values are given in Table II.

The remaining step in the calculation is the combination of the possible spins in each (STY) with the orbital angular momenta in order to form states of definite total angular momentum, j. For this purpose it is convenient to use the "double multiplet" expansion[11] of the (STY); i.e., express the projections, Eq. (9), as sums of products of the isotopic and ordinary spin multiplets $(T, T-1, \cdots, -T)\times(S, S-1, \cdots, -S)$ and designate them by the double number $(2T+1)\times(2S+1)$. The components of (STY) of interest are shown in Table III. In applying the results in Table III to Ne^{20*}, all spin values are to be used and added vectorially to the L's with which each occurs in Table II (the exponents in Table II mean the number of distinct states of given L), since all isotopic multiplets contain $T_\zeta=0$. In F^{20} (and Na^{20}), however, $T_\zeta=1$ and the isotopic singlets cannot contribute. In O^{20} (and Mg^{20}), only the isotopic quintets and higher contribute. The resulting numbers of levels at each excitation in Ne^{20} are shown in Table IV for j values 0 to 6. In Fig. 1 we plot the total number of levels with $j=0$ and excitation equal to or less than U. For comparison, the experimental value for Ne^{20} is shown as a point (25 levels) at 15 Mev and the prediction of Bethe's statistical theory[7]

TABLE IV. Number of states of particular J value at a given excitation.

Configuration	Total energy (Mev)	$J=0$	$J=1$	$J=2$	$J=3$	$J=4$	$J=5$	$J=6$
$s^4p^{12}d^4$	0	1		2		2	1	1
$s^4p^{12}d^3s$	5.4	1		1	1	1		1
$s^4p^{12}d^4$	8.0	4	10	16	17	16	12	9
$s^4p^{12}d^2s^2$	10.8	1		1		1		
$s^4p^{12}d^4$	12.0	6	7	14	12	13	7	6
$s^4p^{11}d^5$	12.8	2	6	9	11	11	10	8
$s^4p^{12}d^3s$	13.4	3	11	15	16	12	9	5
$s^4p^{12}d^3f$	14.5	1	3	3	5	4	4	3
$s^4p^{12}d^4$	16.0	7	16	22	21	18	11	5
$s^4p^{12}ds^3$	16.2			1				
$s^4p^{12}d^3s$	17.4	3	9	13	12	10	7	3
$s^4p^{11}d^4s$	18.2	2	7	9	11	10	9	6
$s^4p^{12}d^2s^2$	18.8	3	7	7	7	5	2	
$s^4p^{12}d^2sf$	19.9		2	2	3	2	2	1
$s^4p^{11}d^5$	20.8	43	122	177	204	200	173	131
$s^4p^{12}d^3s$	21.4	8	20	27	25	19	11	5
$s^4p^{12}s^4$	21.6	1						
$s^4p^{12}d^3f$	22.5	16	43	63	71	70	59	44
$s^4p^{12}d^2s^2$	22.8	3	3	6	4	5	2	1
$s^3p^{12}d^5$	23.1	1	2	3	4	4	3	3
$s^4p^{11}d^3s^2$	23.6	1	5	6	7	5	4	2
$s^4p^{12}d^4$	24.0	1	2	3	2	1		
$s^4p^{12}ds^3$	24.2		2	3	2			
$s^4p^{11}d^5$	24.8	33	99	138	157	145	121	85

$$I_B(U)=\int_1^U \rho_B(x)dx,$$
$$\rho_B(U)=0.6S^{-4}e^S,\quad S=(AU/2.30)^{\frac{1}{2}}. \tag{12}$$

The agreement between theory and the experimental point in Fig. 1 is partly accidental, inasmuch as it cannot be claimed either that all levels in Ne^{20} below 15 Mev have been found experimentally, or that all the levels which have been found have $j=0$. The two deviations, however, tend to cancel rather than add. The agreement between our results and Bethe's calculation for $j=0$ is also astonishingly good, indicating that disregarding the charge exchange is a reasonable device for allowing for symmetry effects in level density.

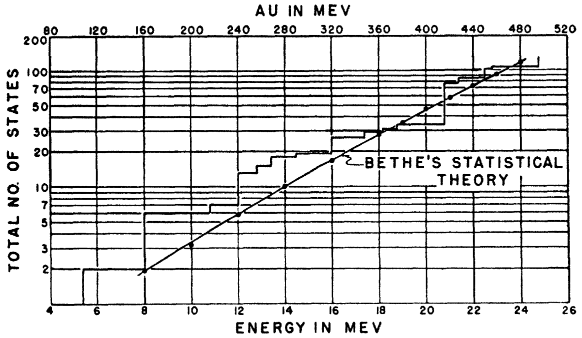

FIG. 1. Total number of states in Ne^{20} for $j=0$ and both parities. The point (■) indicates the total number of levels observed to date.

On the other hand, the statistical theory predicts a factor $(2j+1)$ in the level density of higher j, and, as found in the previous work of Feenberg *et al.*, the actual ratio of the number for $j=0$ is much less for high j and low excitation. One is usually interested in the total number of levels up to a certain value of j. Hence, we present in Table V the ratio of the total number of levels of j less than or equal to $j_{\max}$ to the total number of $j=0$ for several energies of excitation.

In using the information in Table V for the interpretation of nuclear reactions one must bear in mind the influence of potential barriers in favoring certain angular momenta and parity.

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[11] F. Hund, Z. Physik **105**, 202 (1937); H. A. Jahn, Proc. Roy. Soc. (London) **201**, 516 (1950).

TABLE V. Ratio of total number at levels with $j \leq j_{\max}$ to number with $j=0$.

AU	$j_{\max}=0$	1	2	3	4	5	6
200	1.0	2.0	4.1	6.0	8.0	9.0	9.9
280	1.0	3.8	7.6	11.7	15.2	17.8	19.4
360	1.0	4.1	8.1	12.2	15.6	18.1	19.4
440	1.0	3.7	7.6	11.9	16.0	19.4	21.9
∞	1.0	4.0	9.0	16.0	25.0	36.0	49.0

V. APPLICATION TO NUCLEI IN GENERAL

The computations for Ne^{20} have been made in the hope that they might serve as a model for the interpretation of other nuclei. The same detail can be presented, of course, on the basis of the work above, for F^{20} and O^{20}. We present, however, only a comparison between total numbers of levels with $j=0$ and both parities at various excitations in Table VI. It will be noted that F^{20} has a higher level density than Ne^{20} or O^{20}. This effect was shown first by Bardeen and Feenberg.[2] It might be thought, off hand, that the omission of the isotopic singlet states in the enumeration for F^{20}, and both singlets and triplets for O^{20}, would reduce the number of levels below that of Ne^{20}. Actually, this effect is counterbalanced by the fact that the lowest states in F^{20} and O^{20} have lower symmetry than does that of Ne^{20}. Hence, a given excitation falls in a region accessible to higher supermultiplets and greater multiplicity.

We shall generalize the application to nuclei for which the ground state is not a (000) state in the following way. If the ground state is $(S_0 T_0 Y_0)$, the over-all level density is related to that for a (000) nucleus by adding the Majorana potential to the excitation in the latter and finding $\rho_0(U')$ instead of $\rho_0(U)$, where

$$U' = U + (20/A)[S_0(S_0+4) + T_0(T_0+2) + Y_0^2]. \quad (13)$$

On the other hand, the true level density is less than $\rho_0(U')$, because the lower double multiplets no longer contribute. Let us approximate the relative number of double multiplets by the factor

$$\exp\{-\lambda(S_0^2+T_0^2)/AU'\}, \quad (14)$$

where λ is an unknown, constant energy. The expected order of magnitude of λ is several hundred Mev, since with $A=20$ the double multiplets are significantly reduced in number if $U'<30$.

TABLE VI. Comparison of $j=0$ level numbers for isobars.

Excitation (Mev)	Ne^{20} $T_\zeta=0$	F^{20} 1 Na^{20} −1	O^{20} 2 Mg^{20} −2
		Total number of states	
0	1	2	2
8	6	11	5
12	13	14	11
16	26	56	16
20	34	114	72
24	108	255	163

As for an empirical form for $I_0(U)$, we have assumed the following

$$\ln[I_0(U)] = a + b \ln U + cU^{\frac{1}{2}} \quad (15)$$

and have determined the best fit to the step-curve in Fig. 1, by the method of least squares. The resulting values are

$$a=-2.06, \quad b=-1.31, \quad c=2.21.$$

In order to generalize Eq. (15) to nuclei of other values of A and (000) ground states we note that Bethe's result for I_B for the case $j=0$ [Eq. (12)] is A^{-1} times a function of the product AU. We assume $I_0(U)$ to have the same functional dependence and obtain

$$\ln[I_0(U)] = 4.87 - 1.31 \ln AU + 0.49(AU)^{\frac{1}{2}} - \ln A. \quad (16)$$

Generalizing to other values of A and other than (000) ground states, we substitute U' from Eq. (13) for U and add the term given in Eq. (14). We determine λ to give a reasonable fit to the calculations for F^{20} and O^{20}. The result is

$$\ln[I(U)] = 4.87 - 1.31 \ln AU' + 0.49(AU')^{\frac{1}{2}} - \ln A - 200(S_0^2+T_0^2)/AU'. \quad (17)$$

We estimate the validity of Eq. (17) to be restricted to values of $AU'<800$. The total number of levels with $j=0$ and with a given parity is then $\frac{1}{2}I(U)$, and the total number with higher j is to be estimated by use of Table V. Finally, the empirical formula for the density of levels of $j=0$ and given parity is

$$\rho(U) = \frac{1}{2U'}\left[0.245(AU')^{\frac{1}{2}} + 200\frac{(S_0^2+T_0^2)}{AU'} - 1.31\right] I(U). \quad (18)$$