Interpretation of the Decay Scheme of K^{40}

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The three ground states and one excited state of the three nuclei involved in the disintegration of K^{40} are assigned parity and angular momentum values which are essentially unique. The rates of all measured transitions and such energy differences as are well known are used for the assignment, which depends on the *j-j* coupling shell model, the Gamow-Teller selection rules, and certain empirical groupings of beta-ray lifetimes. Appreciable *K*-capture directly to the ground state of A^{40} can be excluded on these quasi-theoretical grounds.

I. INTRODUCTION

THE decay scheme of the natural radioactive isotope, K^{40}, has received very extensive study both because of the inherent interest of its unusual nuclear properties and because of its high importance for the history of the earth. From data now available, using very plausible assumptions based on the shell model and on general beta-decay theory, a consistent interpretation of the decay scheme is given, with an identification of the quantum numbers of the three ground states and the one excited state involved. This interpretation also serves as an aid in the examination of certain experimental results which are still ambiguous.

II. ANALYSIS OF THE DATA

The experimental data adopted are presented in Fig. 1. The experimental errors in the energy measurements are indicated. The assignment of the gamma-ray to the excited state, **e**, of the A^{40} nucleus is made both because no beta-rays have energies higher than that of the gamma-ray, and because of the experimental absence of beta-gamma coincidences.[1] The partial transition probabilities are given in Table I.

The two data most uncertain are the amount of *K*-capture to the A^{40} ground state, and the mass difference, δ, which, with the well-known gamma-ray energy, determines the energy of the excited state $(A^{40})_e$. Both of these numbers will be fixed more reliably on the basis of the present interpretation.

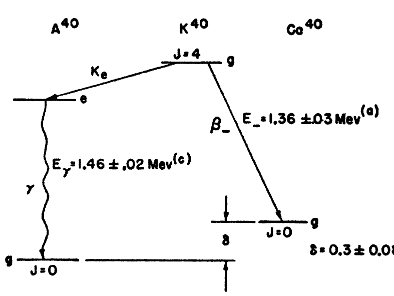

FIG. 1. The observed decay of K^{40}. The energies given are the measured values. (a) D. Alburger, Phys. Rev. **78**, 629 (1950); (b) T. Roberts and A. O. Nier, Phys. Rev. **79**, 198A (1950); (c) P. Bell and J. Cassidy, Phys. Rev. **79**, 173 (1950).

[1] Meyer, Schwachheim, and de Souza Santos, Phys. Rev. **71**, 908 (1947).

The shape of the beta-ray spectrum has been measured fairly accurately;[2] it is apparently, that to be expected for a third-forbidden transition of Greuling's types 3A or 3T. (Fourth-forbidden transitions involving the non-Gamow-Teller coupling types may lead to the same shape.) The lifetime is consistent with an empirically third-forbidden transition, although again this could arise from vector coupling, depending on parity changes.[3] In an interesting communication, which was unfortunately based on preliminary and now evidently misleading experimental data, Fireman[4] has shown how the ratio, λ_{Kg}/λ_+, which is almost independent of any nuclear matrix elements, depends on the total energy, E_+, which is available for positron emission. A recalculation of this ratio, using the approximation[3] for low values of Z, yields the results given in Table II, which differ slightly from those of Fireman.

Now from the shell model, say, in the *j-j* coupling form,[5] we can write the configurations for the three ground states concerned very plausibly as shown in Table III, using an obvious notation.

The change of parity indicated justifies the use of the Gamow-Teller coupling, assigning the transitions to the terms 3A or 3T. No way is available in this decay scheme for distinguishing between tensor and axial vector. Both shape and empirical lifetime fit well; and the transitions thus satisfy the selection rules: $\Delta J=3$, parity changes. The ground-state characters are then: Ca^{40} and A^{40}, $J=0$, *even*; and K^{40}, $J=4$, *odd*. The

TABLE I. Partial transition probabilities.

$K^{40}\rightarrow(Ca^{40})_g+\beta_-$, $\lambda_-=(1.54\pm0.06)\times10^{-17}$ sec^{-1}.[a]
$K^{40}\rightarrow(A^{40})_g+\beta_+$, λ_+: (not observed, less than $5\times10^{-4}\lambda_-$).[b]
$K^{40}+(e^-)_K\rightarrow(A^{40})_e$, $\lambda_{Ke}/\lambda_-=0.13\pm0.01$, from the observed gamma-ray yield.[a]
$K^{40}+(e^-)_K\rightarrow(A^{40})_g$, $\lambda_{Ke}+\lambda_{Kg}$, the total *K*-capture, is given by the measurement of Auger electrons,[a] and by the abundance of radiogenic argon. $(\lambda_{Ke}+\lambda_g)/\lambda_-=0.13\pm0.04$. λ_{Kg} is not observed separately.

[a] G. Sawyer and M. Wiedenbeck, Phys. Rev. **79**, 490 (1950).
[b] Private communication from Mr. S. Colgate. The very low limit $(\lambda_+/\lambda_-<2\times10^{-6})$ given in reference c to Fig. 1 could not be confirmed by recalculation from the published data.

[2] D. Alburger, Phys. Rev. **78**, 629 (1950).
[3] E. Greuling, Phys. Rev. **61**, 568 (1942).
[4] E. Fireman, Phys. Rev. **75**, 1447 (1949).
[5] M. Mayer, Phys. Rev. **78**, 16 (1950).

TABLE II. Dependence of λ_{Kg}/λ_+ on E_+.

$E_+(mc^2)$	λ_{Kg}/λ_+
2.1	100
2.3	60
2.5	30
$[(E_+)_{obs}=1.17\pm0.08$ Mev].	

transitions can be assigned to the change of a single particle state as follows:

(a) β_-: $f_{7/2}{}^n\rightarrow d_{3/2}{}^p$; (b) β_+(or K_g): $d_{3/2}{}^p\rightarrow f_{7/2}{}^n$.

It is now almost inescapable that one sets the nuclear matrix elements of these two similar transitions equal, or at least nearly so. Under this assumption we can compute quantities, A, which are proportional to the transition probabilities, as shown in Table IV.

From Tables II and IV and the observed energies it follows that the value of λ_{Kg}/λ_+ lies between 0.01 and 0.08. Since experiment[6,7] indicates that within the rather sizable error the total branching ratio to K-capture does not differ from that to gamma-emission, one infers that the lower value of around 0.01 is correct, and that the $K^{40}-(A^{40})_g$ atomic energy difference is

TABLE III. Ground state of configurations.

Nucleus	Z	$N(=A-Z)$	Configuration
Ca^{40}	20	20	closed shell
K^{40}	19	21	$(d_{3/2})^{-p}(f_{7/3})^{+n}$
A^{40}	18	22	$(d_{3/2})^{-2p}(f_{7/2})^{+2n}$

close to $3.1mc^2$. We conclude then that direct K-capture to the ground state *cannot* contribute heavily to the decay of K^{40}. At most, the process can give a few percent of all transitions. This conclusion is confirmed by the latest experiments,[8] and seems to provide an additional basis for disregarding the high upper limits given by several experimenters for possible K-capture. This conclusion is so decisive for certain geophysical problems that it is very satisfactory to find some theoretical argument to bolster the difficult experiments.

Now consider the K-capture to the excited state (even here, L_I electrons and those of all higher shells are unimportant), using the energy difference obtained

TABLE IV. Transition probabilities (unnormalized).

	λ_+		λ_-
$E_+(mc^2)$	A_+	$E_-(mc^2)$	A_-
2.1	2.4	3.6	8.2×10^3
2.3	6.7	3.7	1.15×10^4
2.5	21.2	3.8	1.8×10^4

[6] Hess, Brown, Patterson, and Ingraham, Phys. Rev. 81, 298A (1951).

[7] G. Sawyer and M. Wiedenbeck, Phys. Rev. 79, 490 (1950).

[8] T. Gráf, Phys. Rev. 79, 1014 (1950), and the references given there.

from the smallness of capture to the ground state. Assuming only as a zeroth approximation that the nuclear matrix elements are the same for the two K-capture transitions, we can write a relation for their lifetimes, in which the nuclear radius $R=1.4\times10^{-13}\cdot A^{\frac{1}{3}}$ cm enters:

$$\log_{10}(\lambda_{Ke}/\lambda_{Kg}) \\ =2\log_{10}N(n)+(2n+2)\log(E_{Kg}/E_{Ke})-1.4(2n-6).$$

Here n is the degree of forbiddenness for the excited state K-capture, and $N(n)=1\cdot3\cdots(2n+1)/1\cdot3\cdot5\cdot7$. Evaluating this expression for various choices of n, we can conclude that the transition is at least first forbidden, and not as slow as second forbidden. It is very slow among transitions with $n=1$; it is in fact about intermediate between $n=1$ and $n=2$. The value of the familiar product $\log_{10}(ft_{1/2})$ is about 8, with f calculated as for an allowed transition. Nordheim[9] and Shull and Feenberg[10] have pointed out that a whole set of beta-transitions exists with values of $ft_{1/2}$ in this range. Such transitions correspond to $\Delta J=2$, but with parity change, which can be described in the language of the j–j coupling model as $\Delta J=2$, $\Delta L=1$. Now, if the first

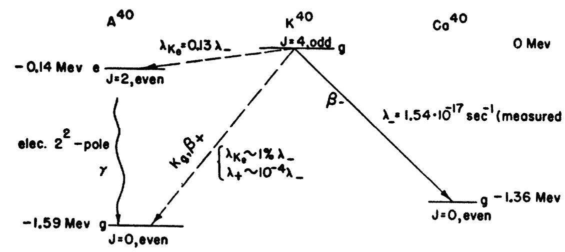

FIG. 2. Complete decay scheme and level assignments for K^{40}. The estimated atomic mass differences are shown.

excited state in A^{40} belongs to the same configuration as does the ground state, as one might expect, the transition is just $d_{3/2}\rightarrow f_{7/2}$ with $\Delta L=1$. One of the possible equivalent levels in this configuration, indeed one of the most plausible, is that with $J=2$, *even.* This assignment then fulfills the empirical requirements of the beta-decay theory, $\Delta J=2$, $\Delta L=1$.

III. THE DECAY SCHEME OF K^{40}

The proposed decay scheme, which satisfies all demands of the beta-decay theory with Gamow-Teller coupling and of the j–j coupling shell model, allows the assignment of all four nuclear states and determines all lifetimes at least approximately. This assignment is given in Fig. 2.

The gamma-ray is an ordinary electric quadrupole transition on this scheme, with an expected lifetime of about 10^{-12} second. It is known empirically only that its lifetime is short compared with the time required for diffusion of argon out of a beaker full of boiling KOH,

[9] L. Nordheim, Phys. Rev. 78, 294 (1950).

[10] F. Shull and E. Feenburg, Phys. Rev. 75, 1768 (1949).

say, a few seconds.[11] This is enough to exclude the possible assignment $J=6$ for the level **e**, which would otherwise be acceptable. The internal conversion of the quadrupole gamma-ray by electron or pair emission is very improbable. No positrons have yet been observed in spite of fairly careful search. Further confirmation of the scheme seems possible but not easy, either by looking still more closely for the predicted positrons or by improving the accuracy of the K-capture and gamma-ray yield measurements considerable.

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[11] See reference b of Table I.

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On Fermi's Theory of the Origin of Cosmic Radiation

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Starting from Fermi's theory of the origin of cosmic radiation, an explanation of the observed shape of the energy spectrum of the proton component is suggested. On this basis there seems to be some indication that the majority of the protons originate from the nucleus of our galaxy. Assuming that the nucleus of the galaxy is a cluster of stars, the origin of the heavier nuclear components and protons of cosmic radiation can be explained on the same basis providing that an ordinary star can eject high energy particles (up to one Bev per nucleon). The observation of the effect of the sun on the heavier nuclear components may give a critical test of the theory.

The trapping magnetic field, if any, of the galaxy has a negligible effect on the energy spectra of cosmic radiation observed on the earth. The age of the galaxy is estimated to be greater than two billion years.

I. INTRODUCTION

RECENT measurements of the proton component of the cosmic radiation[1,2] show that the differential energy spectrum indicates a maximum at the low energy end with a gradually increasing slope toward high energies. In this paper, which is based on Fermi's theory of the origin of cosmic radiation,[3] an explanation of this effect is suggested and a hypothetical picture describing the origin of cosmic radiation is then deduced.

Fermi's theory is based on the assumed existence of wandering magnetic clouds in interstellar space. According to this theory a charged particle will gain energy by a "head-on collision" with a magnetic cloud and will lose energy by an "over-taking collision;" but since the chance of a collision of the first type is larger than one of the second type, there is an average energy gain per collision. This is given by $\delta w = B^2 w$, where w is the energy of the particle (including rest energy) before the collision, and B is the velocity of the wandering cloud in units of the velocity of light. If the initial energy of the particle exceeds a certain threshold value so that the loss of energy in passing through the interstellar medium between collisions can be compensated by the gain, the particle will be accelerated and will become a cosmic-ray particle.

The wandering magnetic clouds are presumably eddies in interstellar space. They are stirred up by the rotation of the galaxy. Recent reports on the polarization of starlight when passing through interstellar clouds seem to support the existence of the magnetic nature of these clouds.[4–6] The existence of the wandering magnetic clouds indicates that Fermi's mechanism must account for at least a part of the cosmic radiation. We shall start with Fermi's mechanism as a fundamental basis and investigate the spatial distribution of sources of protons which will produce the observed spectrum of the primary proton component.

Assume that protons originate from some source at a distance r from the earth and that some of them reach the earth after many collisions. Applying the principle of random flight, we are able to deduce the energy distribution of these protons as a function of r. Thus, the energy spectrum of the proton component is determined by the spatial distribution of the proton sources. Several types of spatial distribution were investigated, and it was found that the observed energy spectrum seems to indicate that the majority of cosmic-ray particles originate from the nucleus of our galaxy.

The nucleus of our galaxy is presumably a cluster of stars. If stars are able to eject high energy particles (up to one Bev per nucleon), as proposed by different authors,[7,8] then our finding might mean that cosmic-ray particles originate from stars and are then accelerated in interstellar space according to Fermi's mechanism. However, if normal stars are unable to eject high energy particles, we are forced to assume that a series of some

[1] Winckler, Stix, Dwight, and Sabin, Phys. Rev. **79**, 656 (1950).
[2] J. A. Van Allen and S. F. Singer, Phys. Rev. **78**, 819 (1950).
[3] E. Fermi, Phys. Rev. **75**, 1169 (1949).
[4] J. S. Hall, Science **109**, 166 (1949).
[5] W. A. Hiltner, Science **109**, 165 (1949).
[6] L. Spitzer and J. W. Tukey, Science **109**, 461 (1949).
[7] F. Hoyle, Monthly Notices Roy. Astron. Soc. **106**, 384 (1947).
[8] K. Kiepenheuer, Phys. Rev. **78**, 809 (1950).