measurements of the $C^{13}(d, \alpha)B^{11}$ Q-value is particularly fortunate, since this reaction is of critical importance in determining the mass of the nuclei lighter than B^{11} in terms of O^{16}. A mass table based on these accurately determined Q-values is being prepared for publication.

In addition to the ground-state transitions, a group of alpha-particles was observed which we have tentatively identified with the $C^{13}(d, \alpha)^{*}B^{11}$, leaving B^{11} excited by 2.107 ± 0.017 Mev above the ground state. The Q-value for this reaction is 3.057 ± 0.016 Mev. The existence of this lowest excited state in B^{11} has been previously reported by Bateson[4] and by Buechner and Van Patter[5] from a study of the $B^{10}(d, p)B^{11}$ proton groups.

The method of obtaining nuclear reaction cross sections from thick target spectra has been described previously.[6] Following this procedure, we find for 0.99-Mev deuterons a differential cross section at 90 degrees of 7 millibarns/steradian for $C^{13}(d, \alpha)B^{11}$, 2 millibarns/steradian for $C^{13}(d, t)C^{12}$. The high energy protons from $C^{13}(d, p)C^{14}$ were able to pass completely through the ZnS phosphor screen of the scintillation counter, producing a non-uniform pulse height distribution and making the counter efficiency uncertain, so that we are not able to give a value for this cross section.

We wish to thank Mr. John D. Seagrave for preparing the C^{13} targets; a detailed description of his targets will appear elsewhere.

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The Angular Dependence of Scattering and Reaction Cross Sections

JOHN M. BLATT
University of Illinois, Urbana, Illinois

AND

LAWRENCE C. BIEDENHARN
Oak Ridge National Laboratory, Oak Ridge, Tennessee
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THE general expression for the differential scattering or reaction cross sections for an unpolarized beam in terms of the scattering matrix has been given in the literature.[1] However, the practical evaluation of this expression runs into difficulties as soon as some of the particles involved have intrinsic spins. In that case, one must average over the spin directions in the incident channel and sum over the spin directions in the outgoing channel. The resulting sums over vector addition (Clebsch-Gordan) coefficients are quite tedious to evaluate directly. However, one should think that all sums over magnetic quantum numbers are essentially geometrical in character and can therefore be performed without a detailed knowledge of the particular collision process (of the elements of the scattering matrix). We wish to point out that this is indeed correct and leads to an explicit expression for the scattering or reaction cross sections free of all sums over magnetic quantum numbers. This explicit form has the additional advantage that it expresses the cross sections directly as sums of Legendre polynomials, with coefficients which are manifestly real numbers. The necessary formalism has been developed by Racah[2] in connection with the theory of complex atomic spectra.

The final result can be written most simply in terms of the quantities $Z(l_1J_1l_2J_2, sL)$ defined as follows:

$$Z(l_1J_1l_2J_2, sL) = (2l_1+1)^{\frac{1}{2}}(2J_1+1)^{\frac{1}{2}}(2l_2+1)^{\frac{1}{2}}(2J_2+1)^{\frac{1}{2}} \times W(l_1J_1l_2J_2, sL)s(l_1, l_2)_{L00}, \quad (1)$$

where W is the coefficient defined by Racah,[2] and $s(l_1, l_2)_{L00}$ is a vector addition coefficient in the notation of Wigner and Eisenbud.[1]

The differential cross section for a process leading from channel s to channel s' can then be written as follows:

$$d\sigma^{ss'} = (k_s)^{-2} \sum_{L=0}^{\infty} B_L^{ss'} P_L(\cos\theta) d\Omega_{s'}, \quad (2)$$

where $P_L(\cos\theta)$ is the Legendre polynomial defined in the usual way, and the coefficients $B_L^{ss'}$ are related to the scattering matrix $u_{sl;\,s'l'}^{J}$ of reference 1 as follows:

$$\begin{aligned} B_L^{ss'} = (-)^{j_{s'}-j_s}[4(2j_s+1)]^{-1} & \sum_{J=0}^{\infty} \sum_{l=|J-j_s|}^{J+j_s} \sum_{l'=|J-j_{s'}|}^{J+j_{s'}} \\ & Z(lJlJ, j_sL)Z(l'Jl'J, j_{s'}L)\,|\delta_{ss'}\delta_{ll'} - i^{l+l'}u_{sl;\,s'l'}^{J}|^2 \\ & + (-)^{j_{s'}-j_s}[2(2j_s+1)]^{-1} \sum_{J_1}\sum_{l_1}\sum_{l_1'} \Big(\sum_{J_2 \geq J_1} \sum_{l_2 \geq l_1} \sum_{l_2' \geq l_1'} \Big)' \\ & i^{-l_1+l_1'+l_2-l_2'} Z(l_1J_1l_2J_2, j_sL)Z(l_1'J_1l_2'J_2, j_{s'}L) \\ \times \text{Real Part of } & [(\delta_{ss'}\delta_{l_1l_1'} - i^{l_1+l_1'}u_{sl_1;\,s'l_1'}^{J_1})^{*} \\ & \times (\delta_{ss'}\delta_{l_2l_2'} - i^{l_2+l_2'}u_{sl_2;\,s'l_2'}^{J_2})], \end{aligned} \quad (3)$$

where the prime on the last three sums of the second term means that those terms are excluded for which all three inequalities become equalities, i.e., for which $J_2 = J_1$, $l_2 = l_1$, and $l_2' = l_1'$ simultaneously.

While one could hardly claim that (3) is a very simple expression, it is appreciably simpler than the one given in reference 1, since six sums over magnetic quantum numbers have been eliminated. Furthermore, it is satisfying that the rules about the limitations of the complexity of angular distributions[3] can be shown to follow from (3) by the use of Racah's selection rule for nonvanishing W-coefficients. All terms in (3) are manifestly real with the exception of $i^{-l_1+l_1'+l_2-l_2'}$ in the second sum of (3). This term, however, is also real because of the parity selection rule according to which $l_1 - l_1'$ and $l_2 - l_2'$ are either both even or both odd. Equation (3) simplifies considerably for a resonance reaction going through one and only one level of the compound nucleus. In that case the scattering matrix can be factored. This special case was considered already by Myers,[4] who failed to give explicit expressions for the Racah coefficients, however. The simplifications are only minor in the case of resonance scattering because of the interference between resonance and potential scattering. The formalism developed here is adequate for the description of scattering or reactions induced by neutral particles and for reactions or inelastic scattering of charged particles. In the case of elastic scattering of charged particles the series (2) contains the coulomb scattering and hence converges extremely slowly. It is possible, however, to subtract out the coulomb scattering to get a usable expression.

Further work in this connection is in progress and will be reported later in more detail. In particular, we are going to give recursion relations and tables of the Racah coefficients W and the Z-coefficients defined by (1). We would like to thank Dr. Stuart Lloyd of the University of Illinois for some valuable discussions in connection with the Racah coefficients.

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Study of Low Energy Gamma-Radiations Emitted from Pa^{231} and U^{234}

GERTRUDE SCHARFF-GOLDHABER AND MICHAEL MCKEOWN
Brookhaven National Laboratory, Upton, Long Island, New York*
(Received February 13, 1951)

MACKLIN AND KNIGHT[1] observed, with a Geiger counter, a low energy gamma-radiation from a thin source of U^{234}, an α-emitter. By absorption of this radiation in Al, they showed