

effect would increase the absolute value of the density gradient of helium over that computed on the basis of Eq. (1).⁸ However, the neon to argon ratio is not subject to this effect, since it is believed that these gases do not escape from the atmosphere.

Hence, it seems that mixing must prevail at least up to about 60 km and that McQueen's results must be interpreted in some other way than by diffusive separation in the atmosphere.

Details of the sampling procedure, etc., used by the writers will be published elsewhere. We are indebted to Professor Sydney Chapman for discussions in connection with our results.

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An Empirical Formula for the Microwave Spectrum of Ammonia

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THE inversion splitting for a double minimum potential was obtained by Dennison and Uhlenbeck¹ using the W-K-B approximation. They found

$$\Delta = 2h\nu \exp\left[(-2/\hbar) \int_0^{v_1} [2\mu(V-E)]^{1/2} dy\right]. \quad (1)$$

If this expression is a good approximation for the region, the microwave inversion spectrum might be more accurately represented by a formula

$$\nu = \nu_0 \exp[AJ(J+1) + BK^2 + CJ^2(J+1)^2 + DJ(J+1)K^2 + EK^4] \quad (2)$$

than by the usual expression

$$\nu = \nu_0 + A'J(J+1) + B'K^2 + C'J^2(J+1)^2 + D'J(J+1)K^2 + E'K^4. \quad (3)$$

TABLE I. Frequency of NH₃ lines.

J	K	Freq.	Error	Ref.	J	K	Freq.	Error	Ref.
1	1	23,694.49	0	2	9	9	27,478.00	-0.08	3
2	2	23,722.63	-0.01	2	9	8	23,657.48	-0.08	2
2	1	23,098.79	-0.07	2	9	7	20,735.44	+0.01	2
3	3	23,870.13	(-0.21)	2	9	6	18,499.5	-0.62	4
3	2	22,834.17	-0.05	2	9	5	16,798.3	-2.04	4
3	1	22,234.53	0	2	10	10	28,604.73	-0.18	3
4	4	24,139.41	+0.07	2	10	9	24,205.29	-0.10	2
4	3	22,688.29	(+1.76)	2	10	8	20,852.51	-0.04	2
4	2	21,703.36	+0.03	2	10	7	18,285.6	-1.08	4
4	1	21,134.29	+0.04	2	11	11	29,914.66	+0.12	3
5	5	24,532.98	+0.07	2	11	10	24,881.90	-0.01	2
5	4	22,653.00	-0.01	2	11	9	21,070.70	-0.01	2
5	3	21,985.27	(-7.03)	2	11	8	18,162.6	-1.59	4
5	2	20,371.46	+0.11	2	12	12	31,424.97	+0.51	3
5	1	19,838.4	+0.14	4	12	11	25,695.23	+0.36	2
6	6	25,056.02	+0.05	2	12	10	21,391.55	+0.19	3
6	5	22,732.43	+0.02	2	12	9	18,127.2	-2.36	4
6	4	20,994.61	+0.09	2	13	13	33,156.95	+1.22	3
6	3	19,757.57	(+21.18)	2	13	12	26,655.00	+1.27	3
6	2	18,884.9	+0.14	4	13	11	21,818.1	+0.93	4
6	1	18,391.6	-0.15	4	13	10	18,178.0	-2.82	4
7	7	25,715.17	-0.01	2	14	14	35,134.44	+3.38	3
7	6	22,924.94	-0.01	2	14	13	27,772.52	-2.79	3
7	5	20,804.83	+0.10	2	14	11	18,313.9	-3.16	4
7	4	19,218.52	0	2	15	15	37,385.18	+6.74	3
7	3	18,017.6	(-52.39)	4	15	14	29,061.14	+5.05	3
7	2	17,291.6	-0.82	4	15	12	18,535.1	-3.26	4
7	1	16,841.3	-0.95	4	16	16	39,941.54	+12.61	3
8	8	26,518.91	-0.13	3	16	14	23,777.4	+6.52	4
8	7	23,232.24	-0.04	2	16	13	18,842.9	-2.64	4
8	6	20,719.21	+0.06	2	17	15	24,680.1	+12.27	4
8	5	18,808.7	-0.09	4					
8	4	17,378.1	-1.18	4					

Formula (2) was fitted to 64 lines, as published²⁻⁴ to date. The six constants were obtained graphically using the differences of the logarithms of the frequencies, and were not adjusted to obtain a better fit for high J values. The equation obtained is

$$\nu = 23,785.88 \exp[-6.36996 \times 10^{-2} J(J+1) + 8.88986 \times 10^{-2} K^2 + 8.6922 \times 10^{-7} J^2(J+1)^2 - 1.7845 \times 10^{-6} J(J+1)K^2 + 5.3075 \times 10^{-7} K^4]. \quad (4)$$

The experimental frequencies are listed in Table I, together with the errors obtained by subtracting the calculated from the experimental frequencies.

The rms error (omitting $K=3$ lines) in fitting Eq. (4) to 60 lines is 2.9 Mc/sec and the average error 1.3 Mc/sec. The best fit so far obtained with a power series,³ such as Eq. (3), gives an average deviation of 26 Mc/sec, while a partially exponential equation⁵ gives 46 Mc/sec. With a power series employing sixth-power terms in J and K and ten constants,⁴ the average deviation is 7 Mc/sec, compared to 1.3 Mc/sec obtained with six constants above. The accuracy of Eq. (4) is best demonstrated by using only lines measured by Good and Coles.² Leaving out $J=12$, $K=11$, the rms error is then 0.05 Mc/sec.

The anomalous deviations of the 3,3; 4,3; 5,3; 6,3; 7,3 lines are -0.21; +1.76; -7.03; +21.18; -52.39 Mc/sec, respectively. Nielsen and Dennison⁶ have shown that the deviation to be expected theoretically is of the form

$$\Delta\nu = AF(J). \quad (5)$$

They calculated $A=0.258 \pm 10$ percent from molecular constants, and $F(J)=-1, 7, -28, 84, -210$, for $J=3$ to 7, inclusive. If $A=0.252$, the deviations given by Eq. (5) are -0.25, +1.76, -7.06, +21.18, -52.9, for $J=3$ to 7, respectively.

The agreement obtained indicates that the functional form of Eq. (4) is correct. Further, the second-order coefficients are about 10^{-4} of the first-order terms, a more reasonable value than 10^{-2} of the first-order in the power series expressions. The third-order terms necessary for a more accurate fit of high J values were not calculated accurately but are reduced by a further factor of 10^{-4} . Examination of the deviations calculated with Eq. (4) indicates that some of the frequencies given in reference 4 may be in error by about 1 Mc/sec.

¹ Dennison and Uhlenbeck, *Phys. Rev.*, **41**, 313 (1932).

² Good and Coles, *Phys. Rev.*, **71**, 383 (1947).

³ Simmons and Gordy, *Phys. Rev.*, **73**, 713 (1948).

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New Ferroelectric Tartrates

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ALTHOUGH several groups of ferroelectric crystals have been discovered in the thirty years since Valasek¹ first observed the phenomenon of ferroelectricity in Rochelle salt, the latter substance has so far occupied a special position among ferroelectric materials because of two distinctive features of its behavior: first, the existence of both a lower and an upper Curie point, and second, the fact that the only known ferroelectric crystals isomorphous with Rochelle salt were mixtures of this substance with other tartrates,² no other tartrate being known to be ferroelectric by itself. In reviewing this subject we have recently noticed that Scholz³ measurements on $\text{LiNH}_4\text{C}_4\text{H}_4\text{O}_6 \cdot \text{H}_2\text{O}$ at temperatures above 125°K show a steep rise in certain piezoelectric moduli for this crystal with decreasing temperature, a reasonable extrapolation indicating that the piezo-moduli become infinitely large in the neighborhood of 100°K. This behavior led us to expect the occurrence of ferroelectricity in lithium ammonium tartrate at low temperatures, a hypothesis which has now been confirmed by experiment.