

The case of the Yukawa potential

$$V(|r_1 - r_2|) = V(r_{12}) = \exp(-r_{12}/r_0)/(r_{12}/r_0),$$

is different. For a vanishing  $r_0$  (and appropriately increased  $V$ ) it gives the Mayer limit; but if  $r_0$  tends to infinity, not a square well but the coulomb potential is obtained, in which case the values of the  $F^k$  (for  $k \geq 1$ ), while being smaller than that of  $F^0$ , are not negligible compared to it.

For the Yukawa potential, it is possible to calculate the  $F^k$  by elementary integrations, taking the coulomb-field wave functions without nodes

$$R_l(r) = N_l r^{l+1} \exp(-r/r_0),$$

with the aid of the addition theorem

$$\exp\left(-\frac{(r_1^2 + r_2^2 - 2r_1r_2 \cos\omega)^{1/2}}{r_0}\right) \bigg/ \frac{(r_1^2 + r_2^2 - 2r_1r_2 \cos\omega)^{1/2}}{r_0} \\ = \sum_{k=0}^{\infty} (2k+1) \frac{K_{k+1/2}(r_1/r_0) I_{k+1/2}(r_2/r_0)}{(r_1/r_0)^{1/2} (r_2/r_0)^{1/2}} \quad r_1 \geq r_2.$$

The result, however elementary, is complicated: all the  $F^k$  are expressed as the quotient of two polynomials in the ratio  $\lambda = r_0/r_l$ . The numerator is a polynomial of the degree  $4l+2$  (with coefficients that are complicated functions of  $k$  and  $l$ ) multiplied by  $\lambda^8$ , and the denominator is  $(1+2\lambda)^{4l+4}$ .

Returning to the configuration  $(d_{5/2})^2$ , we find that the cross-over occurs at a value  $\lambda=10$ . We compute  $r_l$  in terms of the nuclear radius  $R$  by the relationship  $R = \frac{1}{2}r_l[(4l+4)(4l+3)]^{1/2}$ , which gives for  $d$ -nucleons  $r_l = R/3.74$  and determines that the cross-over occurs at  $R = r_0 \times 3.74/10 = \frac{3}{8}r_0$ .

Taking for  $r_0$  the value  $1.4 \times 10^{-13}$  cm, we see that the  $J=5/2$  level is lower than the  $J=3/2$  level for all nuclei having  $R > 0.53 \times 10^{-13}$  cm which, in fact, applies to all cases where this configuration appears.

In order to compare this result with that cited above we must choose the wave functions so that they will give equal nuclear radii. For the oscillator wave functions,  $R = \frac{1}{2}r_l(2l+3)^{1/2}$ , and in this case  $r_l = R/1.3$ . The cross-over occurs at  $\lambda = 1.32$ , or at  $R \sim r_0$ . The difference in the results can be attributed to the singularity of the Yukawa potential at the origin.

The above calculations support the assumption that the Mayer approximation is more reasonable than the "long range" one for the physically interesting cases.

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<sup>1</sup> G. Racah, Phys. Rev. **78**, 622 (1950).

<sup>2</sup> M. G. Mayer, Phys. Rev. **78**, 22 (1950).

<sup>3</sup> Condon and Shortley, *Theory of Atomic Spectra* (Cambridge University Press, London, England), p. 174.

<sup>4</sup> R. D. Inglis, Phys. Rev. **38**, 862 (1931).

<sup>5</sup> D. Kurath, Phys. Rev. **80**, 98 (1950).

## Adiabatic Magnetization of a Superconductor

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IT has been demonstrated<sup>1,2</sup> that the suppression of superconductivity by a magnetic field is accompanied by the absorption of energy. However, knowledge of possible changes in entropy upon application of subthreshold magnetic fields is necessary for thermodynamic treatment of this phenomenon.<sup>3</sup>

In initiating a study of the adiabatic magnetization of a superconductor, an upper limit of possible entropy change upon application of subthreshold magnetic fields has been established. This was done by measuring the temperature change accompanying the adiabatic magnetization of a superconducting polycrystalline tin sphere by subthreshold fields. For this geometry the transition

from the superconducting to the intermediate state should be quite well defined at a threshold field equal to two-thirds the critical field for bulk material, and the application of a magnetic field smaller than this threshold value should not convert any portion of the sample to the intermediate state. A carbon thermometer was employed for the measurement of temperatures. Lack of complete thermal insulation of the sample, and small induction effects upon the application of the magnetic fields, limited the accuracy of measurement of temperature changes to approximately  $10^{-4}$  degree K. To this accuracy, no measurable changes were observed upon the application of subthreshold magnetic fields. The result is in agreement with the observations of Keesom and Kok.<sup>1</sup> Sample data are given in Table I. For analysis, the

TABLE I. Upper limit of entropy change produced by the application of subthreshold magnetic fields to a superconducting tin sphere.

Reduced temperature of sample $T/T_c$	Magnetic field applied $H/H_T$	Upper limit percent entropy change $100 \times \Delta S/\Delta S^0$
0.65	0.76	0.038
0.73	0.92	0.053
0.76	0.72	0.031
0.80	0.85	0.040
0.86	0.96	0.14
0.91	0.43	0.29

temperature ( $T$ ) at which the field was applied is expressed in terms of the transition temperature ( $T_c$ ) of the sample, the upper limit of possible entropy change ( $\Delta S$ ) is expressed in percent of the theoretical entropy change ( $\Delta S^0$ ) that would be expected to accompany the complete suppression of superconductivity,<sup>4</sup> and the magnetic field applied ( $H$ ) is expressed in terms of the threshold field ( $H_T$ ) at the temperature ( $T$ ). The data of de Haas and Engelkes<sup>5</sup> were used in calculating the critical fields. As evidenced by measurable entropy changes, the transition temperature and threshold fields for the sample used were in good agreement with these data.

Data obtained in this investigation indicate little if any change in the entropy of a superconductor upon application of subthreshold magnetic fields. It is felt that, in thermodynamic treatments of the magnetization of superconductors, the assumption of zero entropy change upon application of subthreshold magnetic fields is valid. Study of the entropy change of a superconductor upon application of magnetic fields which exceed the threshold value is in process.

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<sup>5</sup> W. J. de Haas and A. D. Engelkes, Physica **4**, 325 (1937).

## Meson to Proton Mass Ratios\*

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AN extended series of measurements has been made on the ratios of meson masses to the mass of the proton. As in our previous work,<sup>1</sup> momenta and ranges of the mesons were measured, but a number of improvements over our earlier procedures have been devised to reduce systematic errors. Using prior knowledge of the approximate mass ratios, protons and mesons from separate targets in the 184-in. cyclotron were magnetically selected so as to lie in the same 5 percent velocity interval. The particles were stopped in the same nuclear emulsion, which they entered at a small angle to the surface. Relative momenta were calculated with errors of less than one part per thousand for the orbits,<sup>2</sup> which are approximately semi-circular. The rectified ranges in emulsion of