The case of the Yukawa potential

$$V(|r_1-r_2|)=V(r_{12})=\exp(-r_{12}/r_0)/(r_{12}/r_0),$$

is different. For a vanishing r_0 (and appropriately increased V) it gives the Mayer limit; but if r_0 tends to infinity, not a square well but the coulomb potential is obtained, in which case the values of the F^k (for $k\geqslant 1$), while being smaller than that of F^0, are not negligible compared to it.

For the Yukawa potential, it is possible to calculate the F^k by elementary integrations, taking the coulomb-field wave functions without nodes

$$R_l(r)=N_l\, r^{l+1}\exp(-r/r_l),$$

with the aid of the addition theorem

$$\exp\left(-\frac{(r_1^2+r_2^2-2r_1r_2\cos\omega)^{\frac{1}{2}}}{r_0}\right)\Big/\frac{(r_1^2+r_2^2-2r_1r_2\cos\omega)^{\frac{1}{2}}}{r_0}$$

$$=\sum_{k=0}^{\infty}(2k+1)\frac{K_{k+\frac{1}{2}}(r_1/r_0)I_{k+\frac{1}{2}}(r_2/r_0)}{(r_1/r_0)^{\frac{1}{2}}(r_2/r_0)^{\frac{1}{2}}}\quad r_1\geqslant r_2.$$

The result, however elementary, is complicated: all the F^k are expressed as the quotient of two polynomials in the ratio $\lambda=r_0/r_l$. The numerator is a polynomial of the degree $4l+2$ (with coefficients that are complicated functions of k and l) multiplied by λ^3, and the denominator is $(1+2\lambda)^{4l+4}$.

Returning to the configuration $(d_{5/2})^3$, we find that the cross-over occurs at a value $\lambda=10$. We compute r_l in terms of the nuclear radius R by the relationship $R=\frac{1}{2}r_l[(4l+4)(4l+3)]^{\frac{1}{2}}$, which gives for d-nucleons $r_l=R/3.74$ and determines that the cross-over occurs at $R=r_0\times 3.74/10=\frac{3}{8}r_0$.

Taking for r_0 the value 1.4×10^{-13} cm, we see that the $J=5/2$ level is lower than the $J=3/2$ level for all nuclei having $R>0.53\times 10^{-13}$ cm which, in fact, applies to all cases where this configuration appears.

In order to compare this result with that cited above we must choose the wave functions so that they will give equal nuclear radii. For the oscillator wave functions, $R=\frac{1}{2}r_l(2l+3)^{\frac{1}{2}}$, and in this case $r_l=R/1.3$. The cross-over occurs at $\lambda=1.32$, or at $R\sim r_0$. The difference in the results can be attributed to the singularity of the Yukawa potential at the origin.

The above calculations support the assumption that the Mayer approximation is more reasonable than the "long range" one for the physically interesting cases.

I want to express my thanks to Professor W. Pauli for his stimulating interest and to Professor G. Racah for suggestions and helpful discussions.

* Hebrew University, Jerusalem, Israel.

[1] G. Racah, Phys. Rev. 78, 622 (1950).
[2] M. G. Mayer, Phys. Rev. 78, 22 (1950).
[3] Condon and Shortley, *Theory of Atomic Spectra* (Cambridge University Press, London, England), p. 174.
[4] R. D. Inglis, Phys. Rev. 38, 862 (1931).
[5] D. Kurath, Phys. Rev. 80, 98 (1950).

Adiabatic Magnetization of a Superconductor

R. L. Dolecek
Naval Research Laboratory, Washington, D. C.
(Received February 9, 1951)

It has been demonstrated[1,2] that the suppression of superconductivity by a magnetic field is accompanied by the absorption of energy. However, knowledge of possible changes in entropy upon application of subthreshold magnetic fields is necessary for thermodynamic treatment of this phenomenon.[3]

In initiating a study of the adiabatic magnetization of a superconductor, an upper limit of possible entropy change upon application of subthreshold magnetic fields has been established. This was done by measuring the temperature change accompanying the adiabatic magnetization of a superconducting polycrystalline tin sphere by subthreshold fields. For this geometry the transition from the superconducting to the intermediate state should be quite well defined at a threshold field equal to two-thirds the critical field for bulk material, and the application of a magnetic field smaller than this threshold value should not convert any portion of the sample to the intermediate state. A carbon thermometer was employed for the measurement of temperatures. Lack of complete thermal insulation of the sample, and small induction effects upon the application of the magnetic fields, limited the accuracy of measurement of temperature changes to approximately 10^{-4} degree K. To this accuracy, no measurable changes were observed upon the application of subthreshold magnetic fields. The result is in agreement with the observations of Keesom and Kok.[1] Sample data are given in Table I. For analysis, the temperature (T) at which the field was applied is expressed in terms of the transition temperature (T_c) of the sample, the upper limit of possible entropy change (ΔS) is expressed in percent of the theoretical entropy change ($\Delta S'$) that would be expected to accompany the complete suppression of superconductivity,[4] and the magnetic field applied (H) is expressed in terms of the threshold field (H_T) at the temperature (T). The data of de Haas and Engelkes[5] were used in calculating the critical fields. As evidenced by measurable entropy changes, the transition temperature and threshold fields for the sample used were in good agreement with these data.

Table I. Upper limit of entropy change produced by the application of subthreshold magnetic fields to a superconducting tin sphere.

Reduced temperature of sample T/T_c	Magnetic field applied H/H_T	Upper limit percent entropy change $100\times\Delta S/\Delta S'$
0.65	0.76	0.038
0.73	0.92	0.053
0.76	0.72	0.031
0.80	0.85	0.040
0.86	0.96	0.14
0.91	0.43	0.29

Data obtained in this investigation indicate little if any change in the entropy of a superconductor upon application of subthreshold magnetic fields. It is felt that, in thermodynamic treatments of the magnetization of superconductors, the assumption of zero entropy change upon application of subthreshold magnetic fields is valid. Study of the entropy change of a superconductor upon application of magnetic fields which exceed the threshold value is in process.

I am indebted to J. J. Madden for aid in the assembly and manipulation of the experimental equipment used.

[1] W. H. Keesom and J. A. Kok, Physica 1, 595 (1934).
[2] K. Mendelssohn and J. R. Moore, Nature 133, 413 (1934).
[3] C. J. Gorter, Arch. Teyler 7, 378 (1933).
[4] W. H. Keesom and P. H. VanLaer, Physica 5, 193 (1938).
[5] W. J. de Haas and A. D. Engelkes, Physica 4, 325 (1937).

Meson to Proton Mass Ratios*

Walter H. Barkas, F. M. Smith, and Eugene Gardner†
Radiation Laboratory, University of California, Berkeley, California
(Received February 9, 1951)

An extended series of measurements has been made on the ratios of meson masses to the mass of the proton. As in our previous work,[1] momenta and ranges of the mesons were measured, but a number of improvements over our earlier procedures have been devised to reduce systematic errors. Using prior knowledge of the approximate mass ratios, protons and mesons from separate targets in the 184-in. cyclotron were magnetically selected so as to lie in the same 5 percent velocity interval. The particles were stopped in the same nuclear emulsion, which they entered at a small angle to the surface. Relative momenta were calculated with errors of less than one part per thousand for the orbits,[2] which are approximately semi-circular. The rectified ranges in emulsion of the particles were carefully measured. For a small interval, the range-momentum relation is well represented by a power law: $R/m=c(p/m)^q$, where R is the range, m the particle mass, p the momentum, and c a constant of the emulsion. We have used the exponent $q=3.50$ derived from the range-energy relation[3]; the results, however, are insensitive to the value of q chosen. The utilization of protons with velocities distributed about the average meson velocity enabled us to evaluate c, and only momentum and range ratios entered into the determination of the mass ratios. Since all the particles are stopped in the same body of nuclear emulsion, the stopping power of the emulsion is eliminated. The momentum ratios are independent of the absolute value of the magnetic field intensity.

Other statistical errors are small in comparison to the range-straggling error of an individual observation. We have observed that for monoenergetic (π-μ-decay) particles the straggling of ranges has closely a normal distribution. The most probable mass is therefore obtained by averaging the individual observations of that function of the mass in which the range occurs linearly (i.e., $R/p^{3.5}$).

We find the following mass ratios:

$$(\pi^+/\text{proton})=0.1511\pm0.0006,$$
$$(\pi^-/\text{proton})=0.1504\pm0.0007.$$

If the proton to electron mass ratio is 1836.1, these figures correspond to 277.4 ± 1.1 and 276.1 ± 1.3, respectively, in units of the electron mass.

Particles[4] which were presumed to be μ^+ mesons originating from the decay of π^+ mesons stopping in the target were measured in the same experiment. The dispersion of apparent masses in this case, however, exceeds that to be expected if the particles were representatives of a single mass group, all of which comes from the target. μ^+ mesons which arise from decay of π^+ mesons in flight doubtlessly contribute to the distribution found, and we therefore must defer quoting a new μ^+ mass measurement until a better separation of the groups is obtained.

We wish to acknowledge the assistance we have received from numerous individuals of the Radiation Laboratory staff.

* This work was supported by the AEC.

† Dr. Gardner died on November 26, 1950, as a result of beryllium poisoning contracted while working on the Manhattan Project in 1942.

[1] F. M. Smith, *et al.*, Phys. Rev. **78**, 86 (1950).
[2] W. H. Barkas, Phys. Rev. **78**, 90 (1950).
[3] H. Bradner, *et al.*, Phys. Rev. **77**, 462 (1950).
[4] Burfening, Gardner, and Lattes, Phys. Rev. **75**, 382 (1949).

Erratum: Energy Dependence of Proton-Proton Scattering, 18.8 to 31.8 Mev

[Phys. Rev. **80**, 321 (1950)]

BRUCE CORK
Radiation Laboratory, Department of Physics, University of California, Berkeley, California

THE value given in row 8 of column 5 of Table I for the normalized triple should be 14.45 millibarns rather than 25.45 millibarns. The values given in Table IV are correct.

Recombination and the Helium Afterglow Spectrum

D. R. BATES
Department of Physics, University College, London, England

(Received December 26, 1950)

JOHNSON, McClure, and Holt[1] have recently made some important observations on the spectrum of a helium afterglow. They find that it consists of He_2 bands and that it does not contain He lines; they find also that the intensity of the luminosity is high, and over a considerable period is proportional to $[n(e)]^2$, the square of the electron concentration. These results might seem to be contradictory to the view[2] that electrons in such an afterglow disappear by dissociative recombination,

$$He_2^+ + e \rightarrow 2He. \quad (1)$$

However, in fact this is not necessarily the case. The absence of lines is to be expected: For the energy available from (1) is only about 21.4 ev, so that the atoms formed are limited to the 1^1S, 2^1S, 2^1P, 2^3S, 2^3P levels and in consequence do not radiate in the λ2000–8000A region studied.[3] Collisions involving them might, however, give rise to excited helium molecules and hence to band emission. Their rate of formation through (1) is proportional to $[n(e)]^2$ during the period in which He_2^+ is the principal ion, and, therefore, the intensity also follows this law. It is only necessary that their removal should be mainly due to the process suggested in order that a high photon yield should ensue. Phelps[4] finds that the rate of destruction of helium metastable atoms is proportional to the square of the gas pressure. The natural inference is that three-body collisions are the predominant cause of the destruction. These are likely to result in the production of molecules; it is not known whether they lead to the required excitation.

[1] Johnson, McClure, and Holt, Phys. Rev. **80**, 376 (1950).
[2] D. R. Bates, Phys. Rev. **77**, 718 (1950); **78**, 492 (1950).
[3] Although all the levels listed can be reached energetically, this does not mean that all are necessarily populated since other factors besides energy considerations enter. It would be of value to determine whether λ10,830A (2^3P-2^3S) is emitted.
[4] A. V. Phelps, *Conference on Gaseous Electronics* (American Physical Society, Division of Electron Physics, New York, October, 1950). Unfortunately the abstract of the paper read at the conference does not give the pressure range covered.

The Disintegration Scheme of I^{131}

P. R. BELL, JUDITH M. CASSIDY, AND G. G. KELLEY
*Oak Ridge National Laboratory, Oak Ridge, Tennessee**

(Received January 27, 1951)

COINCIDENT scintillation spectrometers have been applied to the study of 8-day I^{131}. Gamma-gamma and beta-gamma-coincidence spectra show that a consistent decay scheme can be made including the 720-kev gamma-ray recently found by Zeldes, Brosi, and Ketelle.[1]

The gamma-gamma-coincidence spectra were obtained using NaI-Tl1 phosphors and 5819 photo-multipliers. A thin sample was placed in a central hole in a 3-mm lead diaphragm between the two crystals, as shown, approximately to scale, in Fig. 1A. The lead absorber reduces the back scattering of photons by the Compton process from crystal to crystal. Curve A, Fig. 2 shows the gross gamma-ray spectrum. The positions of the six known gamma-rays[1,2] are indicated by arrows.

The spectrum of pulses that have a coincident pulse of any energy in the other spectrometer is shown in Fig. 2, curve B. This curve has been corrected for random coincidences which are shown in curve C. The random coincidences were measured by delaying one spectrometer pulse with respect to the other until immediate coincidences were impossible. The peak due to the 364-kev gamma-ray, as well as the bulge due to the 720-kev transition, is absent from the coincidence spectrum. The x-rays, the 80-kev, 284-kev, and 638-kev gamma-rays remain, showing that each is in coincidence with at least one other.

When the second spectrometer is set to count only pulses representing 525-kev energy or greater, the coincidence spectrum is that shown in Fig. 2D. The peak due to the 284-kev gamma-ray and, of course, the 638-kev peak are now absent. This result shows that the 284-kev transition is not in cascade with the 638-kev transition, and since it does appear in the total coincidence curve, it must be in cascade with that of 80 kev. The presence of the 80-kev peak (and the x-rays) in Fig. 2D shows that the 638-kev transition is in cascade with the 80-kev transition. The coincidence count at two points, with the second spectrometer set to count 675 kev and over, are shown at the bottom of Fig. 2 without subtracting the accidentals, together with the accidentals corresponding, showing that only a few x-ray coincidences remain.

These coincidence results lead to the decay scheme shown in Fig. 3. This is essentially that of Kern, Mitchell, and Zaffarano,[3] except for the 720-kev transition, which they did not see.