

nary spontaneous decay, with all five types of the coupling in each case.

For very slow mesons w' vanishes in the case of charge retention with scalar or pseudovector coupling in the approximation $m=0$. The optimum numerical value of R' , for slow mesons, is provided by the theory of charge retention with pseudoscalar coupling and turns out to be (with $m=0$):

$$R' = 192\pi^2(\hbar/\mu c)^3 N.$$

Here N is the number of electrons per cm^3 of the medium. For $N \sim 10^{24}/\text{cm}^3$ the ratio R' is only of the order of magnitude 10^{-11} . Neither can the preparatory formation of the mesotronium yield significantly larger values for R' . The absence of the positron track at the end of some μ^+ -meson tracks must therefore be accounted for in another way.⁵

¹ S. Ogawa and S. Kamefuchi, *Prog. Theor. Phys.* 5, 311 (1950).

² J. Tiomno and J. A. Wheeler, *Revs. Modern Phys.* 21, 144 (1949).

³ This second event might also be observable (in spite of having presumably smaller probability than the first one) because it may be technically easier to detect in the photographic emulsion a splitting of the μ -meson track into two electron tracks than to find an unusually fast decay electron.

⁴ The value w_0 given by Ogawa and Kamefuchi, reference 1, is too large by a factor of 2.

⁵ C. Franzinetti, *Phil. Mag.* 41, 86 (1950); see especially p. 101. Further, C. F. Powell, *Cosmic Radiation* (Colston Papers, 1949), Vol. I, p. 85.

A Note on Time Reversal and the Dirac Equation*

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IT has been known for some time that in the customary formulation of the one-particle Dirac theory the bilinear form $\psi^\dagger\psi$ ($\psi^\dagger \equiv i\gamma^4\psi^*$) is invariant under continuous Lorentz transformations and space inversion but not under time reflection.¹ For a properly quantized field theory, however, $\psi^\dagger\psi$ can be made invariant to time reflection.² This behavior causes no difficulty in the one electron theory [it is necessary, in fact, in order that the normalization integral $\int (\psi^\dagger\gamma^4\psi) dx^1 dx^2 dx^3$ be a scalar under the extended group]. It might be of consequence, however, for interaction with other spin $\frac{1}{2}$ particles (e.g., β -decay).

If the invariance of $\psi^\dagger\psi$ to the extended group is postulated in addition to the usual postulates of the Dirac theory, some generalization must be made. A possible generalization is to consider equations with all real elements in which ψ is an 8-rowed column vector, and the matrices 8×8 . Motivation for this is the observation that the nonlinear operation³ of complex conjugation, K_0 , is adjointed to the Dirac matrix operations and plays an intimate role in the usual theory. By adjoining to the 4×4 Dirac matrices (multiplied by i if imaginary) the four matrices

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad K_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad K_1 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad K_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

we linearize all matrix operations. The algebra generated by this process has 64 elements, is irreducible, and is quite similar to the usual Dirac algebra. It is generated by the commutation relation:

$$\Gamma^\mu \Gamma^\nu + \Gamma^\nu \Gamma^\mu = 2g^{\mu\nu} (\mu, \nu = 0 \dots 5; \quad g^{\mu\nu} = \pm \delta^{\mu\nu}, \\ - \text{for } \mu = 0, 4, 5; \quad + \text{for } \mu = 1, 2, 3).$$

It is found possible to maintain all the postulates leading to the Dirac theory plus the additional requirement of the invariance of $\psi^\dagger\psi$. The wave equation is

$$(\Gamma^\lambda \partial_\lambda + \kappa)\psi = 0 (\lambda = 0 \dots 3; \quad \partial_\lambda = \partial/\partial x^\lambda; \quad x^0 = t; \quad \hbar = c = 1);$$

the adjoint equation is $\psi^\dagger(\Gamma^\lambda \partial_\lambda - \kappa) = 0$ with the adjoint uniquely determined to be $\psi^\dagger = \text{transpose}(\Gamma^0 \Gamma^4 \Gamma^5 \psi)$. To every wave function ψ there correspond three "conjugate" functions generated by the operators $\Gamma^6 \Gamma^4$, $\Gamma^6 \Gamma^5$, and $\Gamma^4 \Gamma^5$ ($\Gamma^6 \equiv \Gamma^0 \Gamma^1 \Gamma^2 \Gamma^3 \Gamma^4 \Gamma^5$). These conjugates are analogous to the usual charge conjugate solution; interactions with external fields distinguish among them. Three anti-commuting operators are available for interactions $\Gamma^4 \Gamma^5$, Γ^4 , Γ^5 . The proper operator for vector interactions is $\Gamma^4 \Gamma^5$, and

with this we get gauge invariance and a continuity equation. Gauge invariance (of the first kind) is lost, however, if the operators Γ^4 and/or Γ^5 appear in the wave equation with nonzero mass. The spin of the particles is readily shown to be $\frac{1}{2}$.

The time reversal operator in this formulation is $\Gamma^0 \Gamma^4$ (or $\Gamma^0 \Gamma^5$ equivalently). In the representation where $\Gamma^4 \Gamma^5$ corresponds to i (the usual Dirac formalism), the time reversal operator becomes $i\sigma_y K_0$.³⁻⁵ The reversal in sign of $\psi^\dagger\psi$ under time reflection in the Dirac theory is therefore due to the incorrect identification of $\gamma^1 \gamma^2 \gamma^3$ as the time reversal operator.

Because transpose ($\psi^\dagger O \psi$) = $\psi^\dagger O \psi$, the expectation value of many operators is identically zero. In particular, we may have only the S , A , P covariants⁵ with I (the identity); V , T with $\Gamma^4 \Gamma^5$; and T , A with Γ^4 and Γ^5 . As a result, the only nonvanishing vector is $j^\mu = \psi^\dagger \Gamma^4 \Gamma^5 \Gamma^\mu \psi$ ($\mu = 0 \dots 3$). Since $\Gamma^4 \Gamma^5$ is a scalar to all Lorentz transformations except time reversal, where it changes sign, the proper behavior for j^0 is obtained. Considering $\Gamma^4 \Gamma^5$ as the charge operator, one is led to the conclusion that changing sign under time reversal is a basic property of charge.

Besides the Dirac equation the proposed generalization includes other equations describing spin $\frac{1}{2}$ particles, the significance of which is yet to be explored.

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* This document is based on work performed for the Atomic Energy Project at Oak Ridge.

¹ W. Pauli, *Revs. Modern Phys.* 13, 203 (1941).

² J. Schwinger, *Lectures on field theory*, Harvard, 1948.

³ E. Wigner, *Göttinger Nachrichten* (1932), p. 546.

⁴ T. D. Newton and E. Wigner, *Revs. Modern Phys.* 21, 400 (1949).

⁵ C. L. Critchfield, *Phys. Rev.* 63, 417 (1943).

Neutron Yield for the Li+T Reactions*

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A PRELIMINARY investigation of neutron production from bombardment of natural metallic lithium by tritons with energies between 0.25 and 2.10 Mev has been made using the Los Alamos 2.5-Mev electrostatic accelerator.

Targets were made by evaporating lithium on thin aluminum or tungsten disks which could be attached to the target end of the accelerating tube. Thicknesses of the lithium films were obtained by measuring the neutron yields from $\text{Li}^7(p,n)\text{Be}^7$ for 2-Mev protons and using a value¹ of 0.025 barn for the differential cross section at 0° . Targets were about ~ 50 kev thick for 2-Mev protons.

Neutron yields were measured with a BF_3 "long counter"² placed about one meter from the target and at angles of 0° to 135°

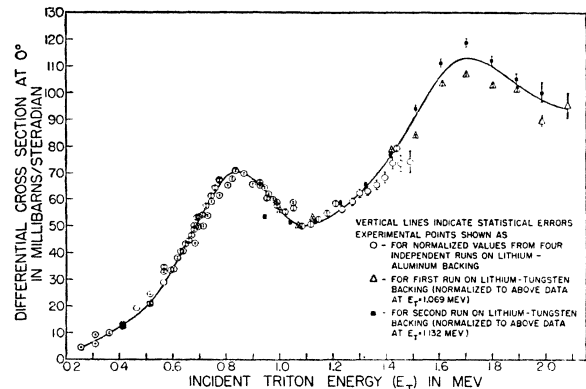


FIG. 1. Neutron yield at 0° in the laboratory system, expressed as a cross section, for the Li+T reactions.